1

$Ax = b$ $\qquad \begin{bmatrix} 3 & 1 & 2 \\ 5 & 4 & 3 \\ -1 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 16 \\ 2 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 3 & 1 & 2 & 7 \\ 5 & 4 & 3 & 16 \\ -1 & 2 & -1 & 2 \end{array}\right] \begin{array}{l} -r_3 \\ r_2+5r_3 \\ r_1+3r_3 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & -2 \\ 0 & 14 & -2 & 26 \\ 0 & 7 & -1 & 13 \end{array}\right] \begin{array}{l} 7r_1+2r_3 \\ r_3 \\ \boxed{r_2-2r_3 \to 0} \end{array} \rightarrow \left[\begin{array}{ccc|c} 7 & 0 & 5 & 12 \\ 0 & 7 & -1 & 13 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

parametr

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \boxed{\begin{bmatrix} 12/7 \\ 13/7 \\ 0 \end{bmatrix}} + p \boxed{\begin{bmatrix} -5/7 \\ 1/7 \\ 1 \end{bmatrix}}, \quad p \in \mathbb{R}.$$

2

$$\left[\begin{array}{ccc|ccc} -2 & 2 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} r_2 \\ r_1+2r_2 \\ r_3-r_2 \end{array} \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 & 2 & 0 \\ 0 & 3 & 1 & 0 & -1 & 1 \end{array}\right] \begin{array}{l} \\ r_2/2 \\ r_3 - \frac{3}{2} r_2 \end{array}$$

$$\rightarrow \left[I \;\middle|\; \underbrace{\begin{array}{ccc} 0 & 1 & 0 \\ 1/2 & 1 & 0 \\ -3/2 & -4 & 1 \end{array}}_{A^{-1}} \right]$$

3

$0 = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3 + \alpha_4 p_4 \iff 0x^3 + 0x^2 + 0x + 0 = (\alpha_1 - \alpha_2 + \alpha_3 + 3\alpha_4)x^3 + \dots$

$$\begin{array}{c} x^3 \\ x^2 \\ x \\ 1 \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 1 & 3 & 0 \\ 2 & 1 & 1 & 2 & 0 \\ -1 & -1 & -1 & 1 & 0 \\ 2 & 1 & -1 & 6 & 0 \end{array}\right] \begin{array}{l} \\ r_2-2r_1 \\ r_3+r_1 \\ r_4-2r_1 \end{array} \rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 1 & 3 & 0 \\ 0 & 3 & -1 & -4 & 0 \\ 0 & -2 & 0 & 4 & 0 \\ 0 & 3 & -3 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \\ 3r_3+2r_2 \\ r_4-r_2 \end{array} \rightarrow \left[\begin{array}{cccc} 1 & -1 & 1 & 3 \\ 0 & 3 & -1 & -4 \\ 0 & 0 & -2 & 4 \\ \cancel{0} & \cancel{0} & \cancel{-2} & \cancel{4} \end{array}\right]$$

∞ řešení!
$\Downarrow$
$p_1, \dots, p_4$ jsou LZ.

4

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} \oplus \boxed{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}} = \begin{bmatrix} \sin(x_1 y_2^2) \\ e^{x_2+y_1} \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin(1) \\ e^0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}} \oplus \boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} \overset{?}{=} \begin{bmatrix} \sin(y_1 \cancel{x_2}^2) \\ e^{\cancel{x_2}+\cancel{x_1}} \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 1 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \sin(0 \cdot 0) \\ e^{1+1} \end{bmatrix}$$

není

5

$$A(\underset{\begin{bmatrix} -2 \\ -14 \end{bmatrix}}{v}) = A\left(\alpha_1 \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \alpha_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) \overset{L.Z.}{=} -4\, A\left(\begin{bmatrix} 2 \\ 2 \end{bmatrix}\right) + 6 A\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$$

$$\left[\begin{array}{cc|c} 2 & 1 & -2 \\ 2 & -1 & -14 \end{array}\right] \rightarrow \boxed{\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}} = \boxed{\begin{bmatrix} -4 \\ 6 \end{bmatrix}}$$

$$-4 \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + 6 \cdot \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -16 & -8 \\ -12 & -22 \end{bmatrix}$$