DETERMINANTY

Def: (rekurzivní)

$A \in \mathbb{R}^{1\times 1}$ ... $\det(\underset{[a]}{A}) = a$

$A \in \mathbb{R}^{(n+1)\times(n+1)}$ ... $\det A = a_{11}(-1)^{1+1}\det A_{11} + a_{12}(-1)^{1+2}\det A_{12} + \dots + a_{1(n+1)}(-1)^{1+(n+1)}\det A_{1(n+1)}$

rozvoj dle 1. řádku

$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} & a_{1(n+1)} \\ a_{21} & & A_{11} & & \\ \vdots & & & & \end{bmatrix}$ A_{12}

Př: dle definice odvoďte vzorec pro

$\det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}\cdot(-1)^2\det[a_{22}] + a_{12}(-1)^3\det[a_{21}] = a_{11}a_{22} - a_{12}a_{21}$

3×3

$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$
$\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{matrix}$

$= a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{13}a_{22}a_{31} - a_{23}a_{32}a_{11} - a_{33}a_{12}a_{21}$

!! pro 4×4, 5×5, atd. je tam mnohem více členů

ERÚ & determinant

- $\det\begin{bmatrix} \vec{r}_1 \\ \vdots \\ \vec{r}_i \\ \vdots \\ \vec{r}_m \end{bmatrix} = \frac{1}{\alpha}\det\begin{bmatrix} \vec{r}_1 \\ \vdots \\ \alpha\vec{r}_i \\ \vdots \\ \vec{r}_m \end{bmatrix}$

- $\det\begin{bmatrix} \vec{r}_1 \\ \vdots \\ \vec{r}_i \\ \vdots \\ \vec{r}_j \\ \vdots \\ \vec{r}_m \end{bmatrix} = -\det\begin{bmatrix} \vec{r}_1 \\ \vdots \\ \vec{r}_j \\ \vdots \\ \vec{r}_i \\ \vdots \\ \vec{r}_m \end{bmatrix}$

- $\det\begin{bmatrix} \vec{r}_1 \\ \vdots \\ \vec{r}_i \\ \vdots \\ \vec{r}_j \\ \vdots \\ \vec{r}_m \end{bmatrix} = \det\begin{bmatrix} \vec{r}_1 \\ \vdots \\ \vec{r}_i + \alpha\vec{r}_j \\ \vdots \\ \vec{r}_j \\ \vdots \\ \vec{r}_m \end{bmatrix}$

Př:

$\det\begin{bmatrix} 3 & -2 & 4 & -1 \\ 2 & 3 & -2 & -1 \\ -3 & 4 & 1 & -2 \\ -3 & 1 & -1 & 4 \end{bmatrix} \begin{matrix} 2r_1 \\ 3r_2 - 2r_1 \\ r_3 + r_1 \\ r_4 + r_1 \end{matrix} = \frac{1}{2\cdot 3}\det\begin{bmatrix} 6 & -4 & 8 & -2 \\ 0 & 13 & -14 & -1 \\ 0 & 2 & 5 & -3 \\ 0 & -1 & 3 & 3 \end{bmatrix} \begin{matrix} -r_4 \\ r_3 + 2r_4 \\ r_2 + 13r_4 \end{matrix} =$

$= \frac{1}{6}(-1)(-1)\det\begin{bmatrix} 6 & -4 & 8 & -2 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 11 & 3 \\ 0 & 0 & 25 & 38 \end{bmatrix} 11r_4 - 25r_3 = \frac{1}{6}\cdot\frac{1}{11}\cdot\det\begin{bmatrix} 6 & -4 & 8 & -2 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 11 & 3 \\ 0 & 0 & 0 & 343 \end{bmatrix}$ X

0

$= \frac{1}{\cancel{6\cdot 11}}\cdot(\cancel{6}\cdot 1\cdot\cancel{11}\cdot 343) = \underline{\underline{343}}$

Př $V = \langle \overbrace{x^4+3x^2+2x+1}^{v_1}, \overbrace{x^4-x-1}^{v_2}, \overbrace{x^3-x^2}^{v_3} \rangle$

podprostor $P_5 = \{ax^4+bx^3+cx^2+dx+e \mid a,b,c,d,e \in \mathbb{R}\}$

skal. součin $(\cdot,\cdot)_*$ na P_5

$(\underline{u}, \underline{v})_* = [a\,b\,c\,d\,e]\begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & 2 & \\ 0 & & & & 2 \end{bmatrix}\begin{bmatrix} m \\ n \\ o \\ p \\ q \end{bmatrix}$

$u = ax^4+bx^3+cx^2+dx+e$, $v = mx^4+nx^3+ox^2+px+q$

Pomocí Grammova–Schmidtova ortonormalizačního procesu utvořte ortonormální bázi V.

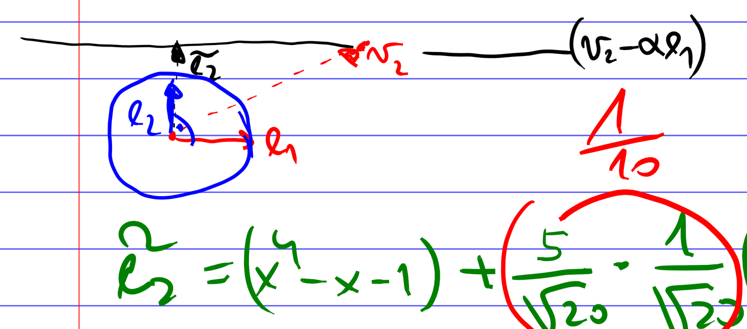
$\mathcal{E} = (e_1, e_2, e_3)$: $(e_i, e_j)_* = \begin{cases} 1 & \dots\ i=j \\ 0 & \dots\ i\neq j \end{cases}$

① $\tilde{e}_1 = v_1$ $\qquad e_1 = \frac{1}{\|\tilde{e}_1\|}\tilde{e}_1$

$\|\tilde{e}_1\|^2 = (\tilde{e}_1, \tilde{e}_1)_* = [1\,0\,3\,2\,1]\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 2 & \\ & & & & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 3 \\ 2 \\ 1 \end{bmatrix} = 1+9+8+2 = 20$

$e_1 = \frac{1}{\sqrt{20}}(x^4+3x^2+2x+1)$

② $\tilde{e}_2 = v_2 - \alpha e_1$: $(\tilde{e}_2, e_1)_* = 0$



$(v_2 - \alpha e_1, e_1)_* = (v_2, e_1)_* - \alpha\underbrace{(e_1, e_1)_*}_{1}$

$\alpha = (v_2, e_1)_* = [1\ 0\ 0\ {-1}\ {-1}]\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 2 & \\ & & & & 2 \end{bmatrix}\frac{1}{\sqrt{20}}\begin{bmatrix} 1 \\ 0 \\ 3 \\ 2 \\ 1 \end{bmatrix}$

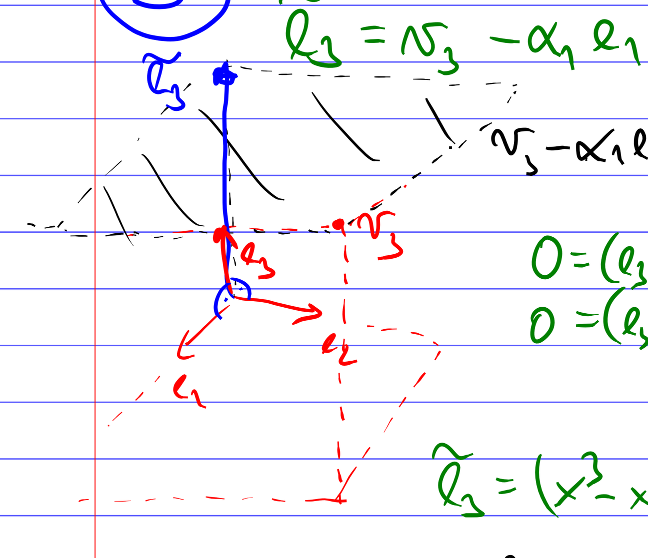
$\frac{1}{\sqrt{20}}(1 + (-1)\cdot 2\cdot 2 + (-1)\cdot 2\cdot 1) = -\frac{5}{\sqrt{20}}$

$\tilde{e}_2 = (x^4-x-1) + \underbrace{\frac{5}{\sqrt{20}}\cdot\frac{1}{\sqrt{20}}}_{\frac{1}{4}}(x^4+3x^2+2x+1)$

$= \frac{5}{4}x^4 + \frac{3}{4}x^2 - \frac{2}{4}x - \frac{3}{4}$

$\|\tilde{e}_2\|^2 = \frac{1}{4}[5\,0\,3\,{-2}\,{-3}]\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 2 & \\ & & & & 2 \end{bmatrix}\frac{1}{4}\begin{bmatrix} 5 \\ 0 \\ 3 \\ -2 \\ -3 \end{bmatrix} = \frac{60}{8} \Rightarrow e_2 = \frac{1}{\sqrt{60}}(5x^4+3x^2-2x-3)$

③ $\tilde{e}_3 = v_3 - \alpha_1 e_1 - \alpha_2 e_2$: $(\tilde{e}_3, e_1)_* = 0$, $(\tilde{e}_3, e_2) = 0$



$0 = (e_3, e_1)_* = (v_3, e_1)_* - \alpha_1\overbrace{(e_1, e_1)_*}^{1} - \alpha_2\overbrace{(e_2, e_1)_*}^{0} \Rightarrow \alpha_1 = (v_3, e_1) = [0\,1\,{-1}\,0\,0]\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 2 & \\ & & & & 2 \end{bmatrix}\frac{1}{\sqrt{20}}\begin{bmatrix} 1 \\ 0 \\ 3 \\ 2 \\ 1 \end{bmatrix} = -\frac{3}{\sqrt{20}}$

$0 = (e_3, e_2)_* = (v_3, e_2)_* - \alpha_1\underbrace{(e_1, e_2)_*}_{0} - \alpha_2\underbrace{(e_2, e_2)_*}_{1} \Rightarrow \alpha_2 = (v_3, e_2) = [0\,1\,{-1}\,0\,0]\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 2 & \\ & & & & 2 \end{bmatrix}\frac{1}{\sqrt{60}}\begin{bmatrix} 5 \\ 0 \\ 3 \\ -2 \\ -3 \end{bmatrix} = -\frac{3}{\sqrt{60}}$

$\tilde{e}_3 = (x^3-x^2) + \frac{3}{\sqrt{20}}\cdot\frac{1}{\sqrt{20}}(x^4+3x^2+2x+1) + \frac{3}{\sqrt{60}}\frac{1}{\sqrt{60}}(5x^4+3x^2-2x-3) = \frac{2}{5}x^4 + \frac{5}{5}x^3 - \frac{2}{5}x^2 + \frac{1}{5}x$

$\|\tilde{e}_3\|^2 = \frac{1}{5}[2\,5\,{-2}\,1\,0]\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 2 & \\ & & & & 2 \end{bmatrix}\frac{1}{5}\begin{bmatrix} 2 \\ 5 \\ -2 \\ 1 \\ 0 \end{bmatrix} = \frac{35}{25} \Rightarrow e_3 = \frac{1}{\sqrt{35}}(2x^4+5x^3-2x^2+x)$

$\mathcal{E} = \left(\frac{1}{\sqrt{20}}(x^4+3x^2+2x+1), \frac{1}{\sqrt{60}}(5x^4+3x^2-2x-3), \frac{1}{\sqrt{35}}(2x^4+5x^3-2x^2+x)\right)$