Př. Klasifikujte kv. formu

• čtvercovou, sym. matici:

zadáno

$$\left[\begin{array}{ccc|ccc} 2 & -1 & 1 & 1 & 0 & 0 \\ -1 & 5 & -2 & 0 & 1 & 0 \\ 1 & -2 & 1 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} 2v_2+v_1 \\ 2v_3-v_1 \end{array}$$

$A \qquad I$

$$\left[\begin{array}{ccc|ccc} 2 & -1 & 1 & 1 & 0 & 0 \\ 0 & 9 & -3 & 1 & 2 & 0 \\ 0 & -3 & 1 & -1 & 0 & 2 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 18 & -6 & 1 & 2 & 0 \\ 0 & -6 & 2 & -1 & 0 & 2 \end{array}\right] \xrightarrow{3v_3+v_2}$$

$T_1A \quad T_1 \qquad T_1AT_1^T \quad T_1$

sloupcově

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 18 & -6 & 1 & 2 & 0 \\ 0 & 0 & 0 & -2 & 2 & 6 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 18 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & -2 & 2 & 6 \end{array}\right]$$

$T_2T_1AT_1^T \quad T_2T_1 \qquad T_2T_1AT_1^TT_2^T \quad T_2T_1$

$B(u,v) = [u]_\mathcal{E}^T [B]_\mathcal{E} [v]_\mathcal{E}$

$Q(u) = [u]_\mathcal{E}^T \boxed{[Q]_\mathcal{E}} [u]_\mathcal{E}$

sym

$= [u]_\mathcal{F}^T [Q]_\mathcal{F} [u]_\mathcal{F}$

$= [u]_\mathcal{E}^T \boxed{P^T [Q]_\mathcal{F} P} [u]_\mathcal{E}$

Vl. čísla, vlastní vektory

A: dvojici (λ, v) nazveme $\iff Av = \lambda \cdot v$

vlastní vektor

vlastní číslo

Př: $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$ $\quad v_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $\quad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Rozhodněte, zda v_1, v_2 jsou vl. vektory A. Pokud ano, najděte příslušná vl. čísla: $Av_1 = \begin{bmatrix} -2 \\ 4 \end{bmatrix} \neq \lambda v_1$ není! $\qquad A \cdot v_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 \cdot v_2$ je

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Př: Najděte vl. čísla a vl. vektory $A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$\underline{A}\underline{v} = \lambda \cdot \underline{v} \iff \underline{0} = (\underline{A} - \lambda \underline{I})\underline{v}$

$$\det(A - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} 1-\lambda & 3 & 0 \\ 3 & 2-\lambda & 0 \\ 0 & 0 & 4-\lambda \end{bmatrix}\right) = 0 \cdot |\ | + 0 \cdot |\ | + (4-\lambda)\begin{vmatrix} 1-\lambda & 3 \\ 3 & 2-\lambda \end{vmatrix}$$

$$= (4-\lambda)\left[(1-\lambda)(2-\lambda) - 3 \cdot 3\right] = (4-\lambda)\left(\lambda - \frac{3+\sqrt{37}}{2}\right)\left(\lambda - \frac{3-\sqrt{37}}{2}\right)$$

$+\lambda^2 - 3\lambda - 7 \qquad \lambda_{1,2} = \frac{3 \pm \sqrt{9+4 \cdot 7}}{2} = \frac{3 \pm \sqrt{37}}{2}$

vl. čísla: $\lambda \in \left\{4, \frac{3+\sqrt{37}}{2}, \frac{3-\sqrt{37}}{2}\right\}$

$\boxed{\lambda_1 = 4}$ Hledáme v_1: $A \cdot v_1 = 4v_1$ tj. $(A - 4I)v_1 = 0$

$$\left[\begin{array}{ccc|c} -3 & 3 & 0 & 0 \\ 3 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \cdots \quad \tilde{v}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} p \quad \cdots \quad v_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\boxed{\lambda_2 = \frac{3+\sqrt{37}}{2}}$

$$\left[\begin{array}{ccc|c} \frac{-1-\sqrt{37}}{2} & 3 & 0 & 0 \\ 3 & \cancel{\frac{1-\sqrt{37}}{2}} & 0 & 0 \\ 0 & 0 & \frac{5-\sqrt{37}}{2} & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} -1-\sqrt{37} & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\tilde{v}_2 = \begin{bmatrix} 6 \\ 1+\sqrt{37} \\ 0 \end{bmatrix} p \implies$

$\|\tilde{v}_2\|^2 = p^2\left[6^2 + (1+\sqrt{37})^2\right] = p^2\left[74 + 2\sqrt{37}\right]$

$v_2 = \frac{1}{\sqrt{2(37+\sqrt{37})}} \begin{bmatrix} 6 \\ 1+\sqrt{37} \\ 0 \end{bmatrix}$

$\boxed{\lambda_3 = \frac{3-\sqrt{37}}{2}}$

$$\left[\begin{array}{ccc|c} \frac{-1+\sqrt{37}}{2} & 3 & 0 & 0 \\ \cancel{3} & \cancel{\frac{1+\sqrt{37}}{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \text{ c.r.} \quad \tilde{v}_3 = \begin{bmatrix} 6 \\ 1-\sqrt{37} \\ 0 \end{bmatrix} p \implies v_3 = \frac{1}{\sqrt{2(37-\sqrt{37})}} \begin{bmatrix} 6 \\ 1-\sqrt{37} \\ 0 \end{bmatrix}$$

$\|\tilde{v}_3\|^2 = p^2\left((1-\sqrt{37})^2 + 36\right)$

$$A \cdot \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \begin{bmatrix} \lambda_1 v_1 & \lambda_2 v_2 & \lambda_3 v_3 \end{bmatrix} = \underbrace{\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}}_{\text{ortonormální matice}} \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix}$$

$$A = \overbrace{V}^{\text{orto}} \overbrace{[\Lambda]}^{\text{diag}} \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}^{-1}$$

$$V^T = V^{-1}$$

$$\begin{bmatrix} 0 & 6/\sqrt{2(37+\sqrt{37})} & 6/\sqrt{2(37-\sqrt{37})} \\ 0 & (1+\sqrt{37})/\sqrt{2(37+\sqrt{37})} & (1-\sqrt{37})/\sqrt{2(37-\sqrt{37})} \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & \frac{3+\sqrt{37}}{2} & 0 \\ 0 & 0 & \frac{3-\sqrt{37}}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 6/\sqrt{2(37+\sqrt{37})} & (1+\sqrt{37})/\sqrt{2(37+\sqrt{37})} & 0 \\ 6/\sqrt{2(37-\sqrt{37})} & (1-\sqrt{37})/\sqrt{2(37-\sqrt{37})} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$