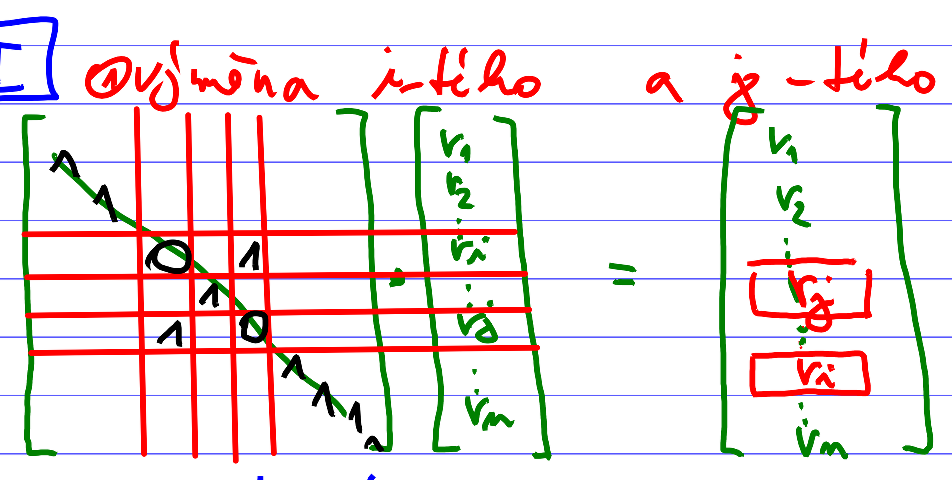
02.

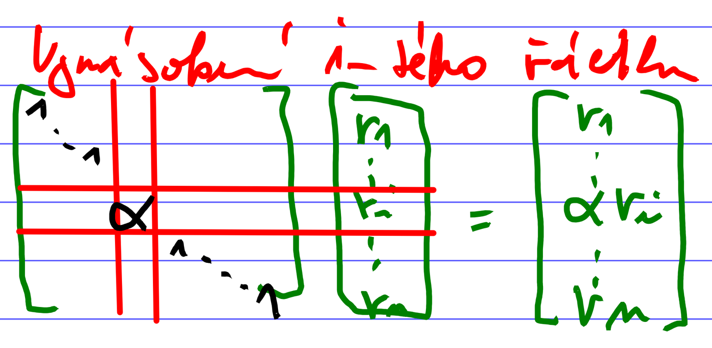
- I • Maticový zápis elementárních řádkových úprav (ERÚ)
- II • Inverzní matice
- III • Soustavy lineárních rovnic: řešitelnost, parametrické řešení

I ① Výměna i-tého a j-tého řádku

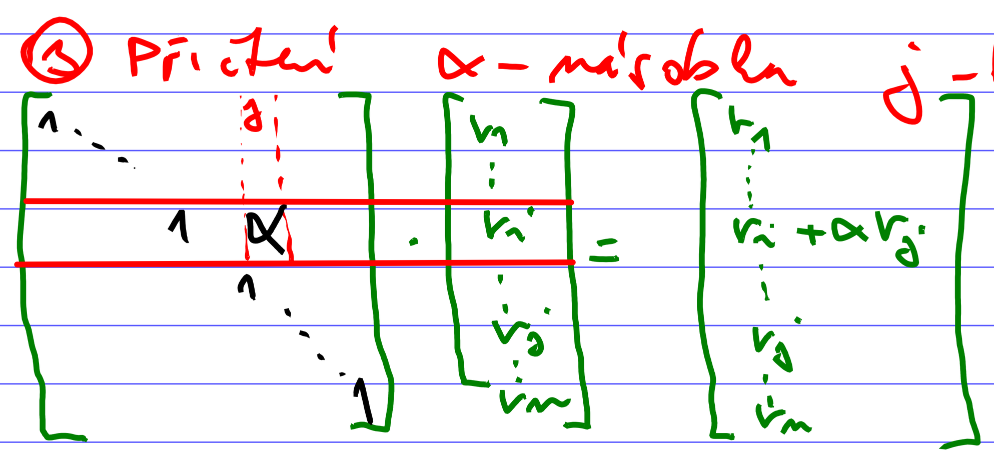


V jednotkové matici vyměníme i-tý a j-tý řádek

② Vynásobení i-tého řádku číslem $\alpha \neq 0$.



③ Přičtení α-násobku j-tého řádku k i-tému



Př: Pomocí maticové podoby ERÚ upravte matici A na jednotkovou:

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 7 & -4 & 2 \\ 8 & 7 & 6 \end{bmatrix}$$

$$\left[\, A \mid I \,\right] = \underbrace{\begin{bmatrix} 3 & 2 & 1 \\ 7 & -4 & 2 \\ 8 & 7 & 6 \end{bmatrix}}_{A}\left|\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{I}\right. \begin{array}{l} r_1 \\ 3r_2 \\ 3r_3 \end{array} \to \underbrace{\begin{bmatrix} 3 & 2 & 1 \\ 21 & -12 & 6 \\ 24 & 21 & 18 \end{bmatrix}}_{T_2 \cdot T_1 \cdot A}\left|\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}}_{T_2 \cdot T_1}\right. \begin{array}{l} \\ r_2 - 7r_1 \\ r_3 - 8r_1 \end{array}$$

$$\underbrace{\begin{bmatrix} 3 & 2 & 1 \\ 0 & -26 & -1 \\ 0 & 5 & 10 \end{bmatrix}}_{T_4 \cdot T_3 \cdot T_2 \cdot T_1 \cdot A}\left|\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -7 & 3 & 0 \\ -8 & 0 & 3 \end{bmatrix}}_{T_4 \cdots T_1}\right. \begin{array}{l} 26 \cdot r_1 \\ r_2 \\ 26 \cdot r_3 \end{array} \to \underbrace{\begin{bmatrix} 78 & 52 & 26 \\ 0 & -26 & -1 \\ 0 & 130 & 260 \end{bmatrix}}_{T_6 \cdot T_5 \cdots T_1 \cdot A}\left|\underbrace{\begin{bmatrix} 26 & 0 & 0 \\ -7 & 3 & 0 \\ -208 & 0 & 78 \end{bmatrix}}_{T_6 \cdots T_1}\right. \begin{array}{l} r_1 + 2r_2 \\ \\ r_3 + 5r_2 \end{array}$$

$$\underbrace{\begin{bmatrix} 78 & 0 & 24 \\ 0 & -26 & -1 \\ 0 & 0 & 255 \end{bmatrix}}_{T_8 T_7 T_6 \cdots T_1 A}\left|\underbrace{\begin{bmatrix} 12 & 6 & 0 \\ -7 & 3 & 0 \\ -243 & 15 & 78 \end{bmatrix}}_{T_8 T_7 \cdot T_6 \cdots T_1}\right. \begin{array}{l} \frac{85}{6} r_1 \\ 85\, r_2 \\ \frac{1}{3} r_3 \end{array} \to \underbrace{\begin{bmatrix} 1105 & 0 & 340 \\ 0 & -2210 & -85 \\ 0 & 0 & 85 \end{bmatrix}}_{T_{11} T_{10} T_9 \cdots T_1 A}\left|\underbrace{\begin{bmatrix} 170 & 85 & 0 \\ -595 & 255 & 0 \\ -81 & 5 & 26 \end{bmatrix}}_{T_{11} \cdots T_1}\right. \begin{array}{l} r_1 - 4r_3 \\ r_2 + r_3 \\ \\ \end{array}$$

$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $T_3 = \begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $T_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -8 & 0 & 1 \end{bmatrix}$, $T_5 = \begin{bmatrix} 26 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $T_6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 26 \end{bmatrix}$, $T_7 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix}$

$T_8 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $T_9 = \begin{bmatrix} 1 & & \\ & 1 & \\ & & \frac{1}{3} \end{bmatrix}$, $T_{10} = \begin{bmatrix} 1 & & \\ & 85 & \\ & & 1 \end{bmatrix}$, $T_{11} = \begin{bmatrix} 85/6 & & \\ & 1 & \\ & & 1 \end{bmatrix}$, $T_{12} = \begin{bmatrix} 1 & & \\ & 1 & 1 \\ & & 1 \end{bmatrix}$, $T_{13} = \begin{bmatrix} 1 & & -4 \\ & 1 & \\ & & 1 \end{bmatrix}$

$$\underbrace{\begin{bmatrix} 1105 & 0 & 0 \\ 0 & -2210 & 0 \\ 0 & 0 & 85 \end{bmatrix}}_{T_{13} \cdot T_{12} \cdots T_1 A}\left|\underbrace{\begin{bmatrix} 494 & 65 & -104 \\ -676 & 260 & 26 \\ -81 & 5 & 26 \end{bmatrix}}_{T_{13} \cdot T_{12} \cdots T_1}\right. \begin{array}{l} \frac{1}{85 \cdot 13} r_1 \\ \frac{-1}{85 \cdot 26} r_2 \\ \frac{1}{85} r_3 \end{array} \longrightarrow \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T_{16} \cdot T_{15} \cdot T_{14} \cdots T_1 A}\left|\underbrace{\frac{1}{85}\begin{bmatrix} 38 & 5 & -8 \\ 26 & -10 & -1 \\ -81 & 5 & 26 \end{bmatrix}}_{\underbrace{T_{16} \cdots T_1}_{A^{-1}}}\right.$$

$T_{14} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1/85 \end{bmatrix}$, $T_{15} = \begin{bmatrix} 1 & & \\ & -\frac{1}{2210} & \\ & & 1 \end{bmatrix}$, $T_{16} = \begin{bmatrix} \frac{1}{1105} & & \\ & 1 & \\ & & 1 \end{bmatrix}$

$$A^{-1} \cdot A = \frac{1}{85}\begin{bmatrix} 38 & 5 & -8 \\ 26 & -10 & -1 \\ -81 & 5 & 26 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 & 1 \\ 7 & -4 & 2 \\ 8 & 7 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Kdybychom řešili: $Ax = \begin{bmatrix} 10 \\ 5 \\ 40 \end{bmatrix}$

$$\underbrace{\underbrace{A^{-1} \cdot A}_{I}\, x}_{x} = A^{-1} b = \frac{1}{85}\begin{bmatrix} 38 & 5 & -8 \\ 26 & -10 & -1 \\ -81 & 5 & 26 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 5 \\ 40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Př: Dokažte, že $(\underbrace{A_1 \cdot A_2 \cdots A_n}_{X})^{-1} = A_n^{-1} A_{n-1}^{-1} \cdots A_2^{-1} A_1^{-1}$

D: $X^{-1} \cdot X = A_n^{-1} A_{n-1}^{-1} \cdots A_2^{-1} \underbrace{A_1^{-1} \cdot A_1}_{I} \cdot A_2 \cdots A_{n-1} A_n = A_n^{-1} A_{n-1}^{-1} \cdots \underbrace{A_2^{-1} A_2}_{I} \cdots A_{n-1} A_n$

$= \cdots = A_n^{-1} \underbrace{A_{n-1}^{-1} A_{n-1}}_{I} A_n = A_n^{-1} A_n = I$

□

③ **Př** Řešte soustavu $Ax = b$ (3 rovnice o 7 neznámých)

$$\left[\begin{array}{ccccccc|c} 0 & 7 & 1 & 6 & 6 & 28 & 34 & -16 \\ 0 & 21 & 3 & -3 & 4 & 14 & 18 & -13 \\ 0 & 7 & 1 & 3 & 2 & 12 & 15 & -9 \end{array}\right] \begin{array}{l} r_1 \\ r_2 - 3r_1 \\ r_3 - r_1 \end{array} \to \left[\begin{array}{ccccccc|c} 0 & 7 & 1 & 6 & 6 & 28 & 34 & -16 \\ 0 & 0 & 0 & -21 & -14 & -70 & -84 & 35 \\ 0 & 0 & 0 & -3 & -4 & -16 & -19 & 7 \end{array}\right] \begin{array}{l} r_1 + \frac{2}{7} r_2 \\ -\frac{1}{7} r_2 \\ r_3 - \frac{1}{7} r_2 \end{array} \to$$

$$\left[\begin{array}{ccccccc|c} 0 & 7 & 1 & 0 & 2 & 8 & 10 & -6 \\ 0 & 0 & 0 & 3 & 2 & 10 & 12 & -5 \\ 0 & 0 & 0 & 0 & -2 & -6 & -7 & 2 \end{array}\right] \begin{array}{l} r_1 + r_3 \\ r_2 + r_3 \\ -r_3 \end{array} \to \begin{array}{c} \begin{array}{ccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{array} \\ \left[\begin{array}{ccccccc|c} 0 & 7 & 1 & 0 & 0 & 2 & 3 & -4 \\ 0 & 0 & 0 & 3 & 0 & 4 & 5 & -3 \\ 0 & 0 & 0 & 0 & 2 & 6 & 7 & -2 \end{array}\right] \end{array}$$

$\Downarrow$

x_2	x_4	x_5			x_1 (p_1)	x_3 (p_2)	x_6 (p_3)	x_7 (p_4)
7	0	0	=	-4	0	-1	-2	-3
0	3	0		-3	0	0	-4	-5
0	0	2		-2	0	0	-6	-7

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 0 \\ -4/7 \\ 0 \\ -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} + p_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + p_2 \begin{bmatrix} 0 \\ -1/7 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + p_3 \begin{bmatrix} 0 \\ -2/7 \\ 0 \\ -4/3 \\ -3 \\ 1 \\ 0 \end{bmatrix} + p_4 \begin{bmatrix} 0 \\ -3/7 \\ 0 \\ -5/3 \\ -7/2 \\ 0 \\ 1 \end{bmatrix}, \quad p_1, p_2, p_3, p_4 \in \mathbb{R}$$