2. cvičení z FKP

1) Znázorněte v Gaussově rovině množinu

a) $\{z \in \mathbb{C} : \ |z+i| \leq 2\}$,

b) $\left\{z \in \mathbb{C} : \ |z-1+i| > \sqrt{2}\right\}$,

c) $\left\{z \in \mathbb{C} : \ 0 < \operatorname{Re} z < 1 \ \wedge \ \arg z \in \left(0, \frac{\pi}{4}\right)\right\}$,

d) $\left\{z \in \mathbb{C} : \ \frac{\pi}{4} \leq \arg(z-2i) \leq \pi\right\}$,

e) $\{z \in \mathbb{C} : \ 1 \leq |z-i| \leq 2\} \cup \left\{z \in \mathbb{C} : \ -\frac{\pi}{3} \leq \arg(z-i) \leq \frac{2}{3}\pi\right\}$,

f) $\left\{z \in \mathbb{C} : \ \operatorname{Re} \frac{1}{z} = \frac{1}{6}\right\}$,

g) $\left\{z \in \mathbb{C} : \ |z| + \operatorname{Im} \overline{z} > 1 \ \wedge \ \frac{\pi}{2} \leq \arg(z+i) \leq \pi\right\}$,

h) $\left\{z \in \mathbb{C} : \ -\frac{2}{3}\pi \leq \arg(-z+2i) < 0\right\}$,

i) $\{z \in \mathbb{C} : \ |z+2| < 2 \ \wedge \ \operatorname{Re} z < \operatorname{Im} z\}$,

j) $\{z \in \mathbb{C} : \ \operatorname{Re}(iz) > \operatorname{Re} z - 1\}$,

k) $\{z \in \mathbb{C} : \ \arg(z-1) \neq \pi\}$,

ℓ) $\{z \in \mathbb{C} : \ |z|^2 = \operatorname{Im} \overline{z}\}$,

m) $\{z \in \mathbb{C} : \ |z+2i| = |z|\}$,

n) $\left\{z \in \mathbb{C} : \ \operatorname{Re} \frac{z+1}{z-1} \leq 0\right\}$,

o) $\left\{z \in \mathbb{C} : \ \operatorname{Im} \frac{z+1}{z-1} = 0\right\}$,

p) $\{z \in \mathbb{C} : \ |z-i| = 2|z+i|\}$.

2) Určete $\operatorname{Re} z^m$ a $\operatorname{Im} z^m$, je-li

a) $z = 1+i, \ m = 6$,

b) $z = \frac{\sqrt{3}}{2} - \frac{i}{2}, \ m = 5$,

c) $z = -\frac{\sqrt{2}}{1+i}, \ m = 16$,

d) $z = \frac{3\sqrt{3}+5i}{2\sqrt{3}-i}, \ m = 9$.

3) Vypočtěte

a) $\lim \frac{i^n}{n}$,

b) $\lim \frac{3n^2+5}{in^2+2in-1}$,

c) $\lim \frac{3n+5}{in^2-4n}$,

d) $\lim (2+5i)^n$,

e) $\lim z^n$ pro $|z|<1$,

f) $\lim \left(\frac{-\sqrt{3}+i}{2}\right)^n$,

g) $\lim \left(\frac{1-i}{\sqrt{2}}\right)^{8n}$.