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... (Introduction, will be inserted by the editor later) ...

Abstracts

Computational shape and topology optimization with applications to 3–dimensional magnetostatics

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(joint work with U. Langer, E. Lindner, R. Stainko, J. Pištora)

In the talk we mainly discussed computational aspects of shape and topology optimization governed with 3–dimensional linear and nonlinear magnetostatics, respectively. This is covered in the speaker's thesis [2] and in [3]. The acknowledgment is due to the Special Research Initiative SFB F013 "Numerical and symbolic scientific computing", subproject "Multilevel solvers for large scale discretized optimization problems" at the University of Linz, Austria. The speaker especially thanks to Dr. Joachim Schöberl for his kind software support during the week in Oberwolfach.

The presentation started with a motivation from physics. We described electromagnets that are used for measurements of magnetooptic effects on thin layers. We aim at designing their optimal topology and shape so that in the area where the measurements take place the magnetic field is as constant as possible and above a prescribed magnitude. Throughout the presentation we instantiate the ideas for this application.

Next, we recalled an abstract optimal shape design problem, its finite element approximation and we discussed the existence and convergence issues following the theory in [1], which is based on the compactness and continuity arguments. We optimize the interface between the air and ferromagnetics, rather than the boundary of the computational domain as usual in mechanics. We pointed out a drawback that on fine discretizations the non–design grid nodes cannot follow large perturbations of the design shape. The mapping from the shape to the grid nodes is carried over an artificial linear elasticity problem with the prescribed displacements along the design shape interface. Then, we presented the algebraic approach to the shape sensitivity analysis and its efficient software implementation, see [5]. The user is only supposed to dessignate the shape and to code the objective in terms of the state solution. The underlying finite element code provides the sensitivity of element contributions to the bilinear form with respect to the grid displacements. The optimization package is now to be included into the NgSolve, see [7].

Further, we presented numerical results for both 2– and 3–dimensional shape optimization problems. After the 2d optimized design the electromagnets were manufactured and the measurements of the magnetic field showed the 4.5–times improvements in terms of the objective functional, compared to the initial design.

We presented a multilevel optimization approach. Here, hierarchies of discretizations of both the state and design space are considered. We begin with the optimization on a coarse discretization for only two design parameters. The multilevel algorithm then proceeds such that the optimized shape is used on a finer level as the initial guess. Moreover, we prolonged the 2d coarse optimized shape to the third dimension and used that as the initial guess in the multilevel 3d optimization. In the 2d case for 7 design and 12.000 state unknowns we achieved the speedup 4.5. In the 3d case for 12 design and 30.000 state unknowns the speedup was more than 10–times.

Finally, we formulated a corresponding topology optimization problem governed by nonlinear magnetostatics. In the 2d case we solved for 3.920 design variables with 4.832 state ones and the computation typically proceeded within 8 steepest descent iterations and 8 nested nonlinear state Newton iterations. Just during the week in Oberwolfach we managed to run 3d topology optimization governed by linear magnetostatics and we were able to solve problems of up to 1 million design unknowns in hours. The optimal design is close to a sphere around the area where the constant magnetic field is required. The talk was ended with the outlook concerned on using nonlinear multigrid techniques, all–at–once optimization approach and preconditioning techniques for the arising KKT–systems and adaptivity with respect to the cost functional.

Edited by Ralf Hiptmair

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[7] *J. Schöberl et al.*, *NgSolve – finite element multi–purpose package*, http://www.hpfem.jku.at.