Multilevel solvers for 3–dimensional optimal shape design with an application to magneto–optics

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ABSTRACT

This paper presents a new numerical technique for solving design shape optimization problems. The idea is to apply a standard optimization algorithm within a hierarchy of discretizations such that a coarse optimized design is used as the initial guess at the next finer discretized level. We give a comparison with the standard optimization approach which proceeds only on the finest discretization. The method is used for 3–dimensional optimal shape design of an electromagnet that arises in the research on magneto–optic effects.

Keywords: Multilevel methods, shape optimization, magnetostatics, magneto–optics

1. INTRODUCTION

Nowadays, both scientific and commercial software computing tools are used in the design process. Researchers and/or developers are usually modelling a new device on a computer, doing some calculations, and thinking what parameters and how to shift to achieve better properties of the device. As far as the direct simulation, e.g., calculation of the magnetic field distribution, is fast enough, it is straightforward to automatize also the design process. To this end, we have to exactly formulate an objective criterion $\mathcal{J} : \mathbb{R}^{n_\mathrm{d}} \mapsto \mathbb{R}$ saying what design is better, specify design parameters $\mathbf{p} \in \mathbb{R}^{n_\mathrm{d}}$ that can be changed within some interval $[\mathbf{p}_\mathrm{l}, \mathbf{p}_\mathrm{u}]$, and introduce some additional constraining criteria $\mathbf{c} : \mathbb{R}^{n_\mathrm{d}} \mapsto \mathbb{R}^{n_\mathrm{c}}$ that the device must satisfy. Moreover, the direct problem cannot be solved exactly, therefore, we employ a discretization technique, the finite element method in this case. We superscribe all the symbols with a positive discretization parameter h. The discretized optimization problem can be then stated as follows:

$$\left.\begin{array}{l}\text{Find } \mathbf{p}^h_{\mathrm{opt}} \in \Upsilon^h\text{:}\\ \mathcal{J}^h(\mathbf{p}^h_{\mathrm{opt}}) \le \mathcal{J}^h(\mathbf{p}^h) \quad \forall \mathbf{p}^h \in \Upsilon^h\end{array}\right\}, \qquad (P^h)$$

where

$$\Upsilon^h := \{\mathbf{p}^h \in \mathbb{R}^{n^h_\mathrm{d}} \mid \mathbf{p}^h_\mathrm{l} \le \mathbf{p}^h \le \mathbf{p}^h_\mathrm{u} \text{ and } \mathbf{c}^h(\mathbf{p}^h) \le \mathbf{0}\}.$$

We aim at a fast solution of (P^h) while the discretization is fine enough, i.e., the direct simulation problem is calculated under a small computational error. A standard approach is that we first choose a small enough discretization parameter h and then solve (P^h) by an optimization algorithm. Typically, the optimization algorithm, e.g., Newton–like, takes initial design parameters as an input, then, for a number of modifications of the design parameters the direct simulation problem is calculated, which is involved in $\mathcal{J}^h$ and $\mathbf{c}^h$, and the algorithm ends up with optimized design parameters. This is very time consuming. The idea of the multilevel approach is to apply a standard optimization algorithm such that we first solve a coarse discretized problem (P^{h_1}), the coarse optimized design parameters are used as the initial ones at the next level where a finer discretized problem (P^{h_2}) is solved, i.e., $h_2 < h_1$, and so further. Doing it in a proper way, we can sufficiently approximate the solution at coarse levels and decrease the number of time consuming direct simulations at fine levels. In[1] the first numerical test was published. Being inspired by the monograph,[2] in[3] we treat theoretical as well as computational issues of shape optimization in magnetostatics. All the results are summarized in the thesis.[4]

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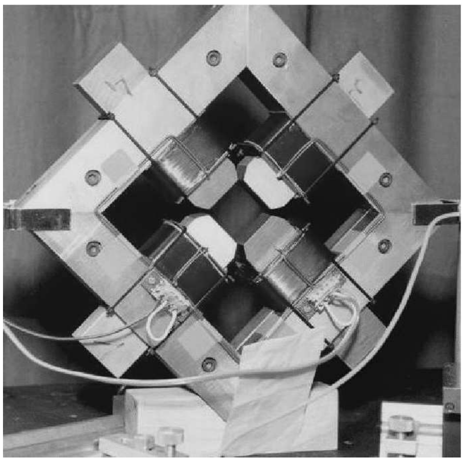
Our research on multilevel optimization techniques was initiated by Professor Ulrich Langer at the University of Linz in Austria, based on the world–leading results[5,6] concerning multigrid methods for linear systems of equations which were achieved within his research group. The multilevel techniques have been recently used as adaptive optimization methods[7,8] where the error of the objective function approximation $\mathcal{J}^h$ is estimated by the aposteriori finite element error analysis. In the paper[9] a multilevel method is used for ill–posed inverse problems.

2. OPTIMAL SHAPE DESIGN OF AN ELECTROMAGNET

Let us first describe an application which we will use for testing the multilevel approach. We consider an electromagnet of the Maltese Cross (MC) geometry, as depicted in Fig. 1. It consists of a ferromagnetic yoke and 4 poles completed with coils which are pumped with direct electric currents. The electromagnets are used



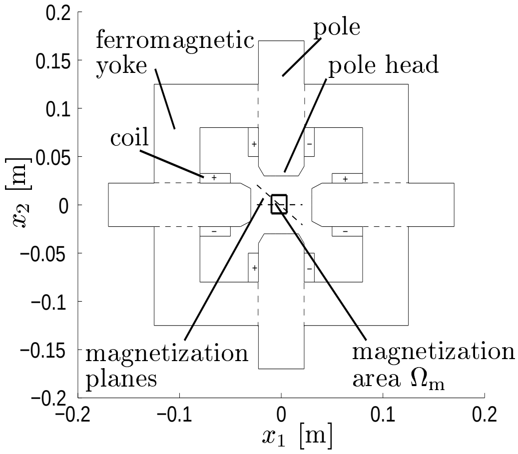


Figure 1. The Maltese Cross electromagnet and its cross–section

for measurements of Kerr magneto–optic effects.[10] They require the magnetic field as homogeneous, i.e., as constant as possible in a given normal direction. Let us note that the magneto–optic effects are investigated for applications in high capacity data storage media, like a development of new media materials for magnetic or compact discs recording. Let us also note that the electromagnets have been developed at the Institute of Physics, VŠB–Technical University of Ostrava, Czech Republic in the research group of Professor Jaromír Pištora. Some instances have been already delivered to the following laboratories: Institute of Physics, Charles University Prague, Czech Republic, National Institute of Applied Sciences INSA in Toulouse, France, Department of Physics, Simon Fraser University in Vancouver, Canada, Department of Chemistry, Simon Fraser University in Vancouver, Canada, and University Paris VI., France. In[11] more details can be found.

First, we describe how the Kerr magneto–optic effect is measured. A sample of a magnetic material is placed into the magnetization area which is located in the middle among the pole heads. In this area the magnetic field is homogeneous enough with respect to the normal vector of some polarization plane, see Fig. 1. We pass an optical (light) beam of a given polarization vector to the sample. There it reflects and components of the reflected polarization vector are measured in terms of the Kerr rotation and ellipticity. Briefly saying, we measure the polarization state of the reflected beam. The Kerr rotation means the difference between the angle of the main ellipticity axis of the reflected beam from that one before the reflection.

In[12,13] the anisotropy of Kerr effects is discussed. It follows that the measurements should be done in as many directions as possible. One has either to rotate the sample in the magnetic field, rotate the electromagnet while the sample is fixed, or rotate the magnetic field itself while both the sample and electromagnet are fixed. Certainly, the last variant is most preferred. The electromagnets have been developed such that they are capable to generate magnetic fields homogeneous in step–by–step different directions just by switching some currents in coils on or off, or by switching their senses. The more coils we have, the more directions the magnetic field can be oriented in. In case of the MC electromagnet, one can sequentially generate magnetic fields homogeneous in up to 8 directions that can be described, due to the symmetry of the geometry, by just two different configurations of the current excitation.

Our aim is to improve the current geometry of the MC electromagnet in order to be better suited for measurements of the Kerr effect. The generated magnetic field should be strong and homogeneous enough in order to admit a magneto–optic effect. Unfortunately, these assumptions are contradictory and we have to balance them. From physical experience we know that the homogeneity of the magnetic field depends significantly on the shape of the pole heads. Hence, we aim at designing shapes of the pole heads in such a way that inhomogeneities of the magnetic field are minimized, but the field itself is still strong enough. From the mathematical point of view we consider the discretized shape optimization problem (P^h) with the following objective function:

$$\mathcal{J}^h(\mathbf{p}^h) := \frac{1}{2}\sum_{v=1}^{2}\left[\varphi^h(\mathbf{B}^{v,h}(\mathbf{p}^h,\mathbf{x})) + \rho\cdot\theta^{v,h}(\mathbf{B}^{v,h}(\mathbf{p}^h,\mathbf{x}))\right], \qquad (1)$$

where $\mathbf{B}^{v,h}(\mathbf{p}^h,\mathbf{x})$ denotes the magnetic flux density at a point $\mathbf{x}\in\mathbb{R}^3$ for the v–th configuration of the current excitation and for given design parameters $\mathbf{p}^h\in\Upsilon^h$ that describe the shape of the pole heads. This magnetic field is the solution to an underlying 3–dimensional linear magnetostatic problem which is discretized by the finite element method with a discretization parameter $h>0$. Further, in (1) the term φ^h measures the inhomogeneity of the magnetic field over the magnetization area Ω_{m}

$$\varphi^h(\mathbf{B}^{v,h}(\mathbf{p}^h,\mathbf{x})) := \int_{\Omega_{\mathrm{m}}}\left\|\mathbf{B}^{v,h}(\mathbf{p}^h,\mathbf{x}) - B^{\mathrm{avg},v}(\mathbf{B}^{v,h}(\mathbf{p}^h,\mathbf{x}))\cdot\mathbf{n}_{\mathrm{m}}^v\right\|^2 d\mathbf{x},$$

where $B^{\mathrm{avg},v}$ calculates the average magnetic flux density over Ω_{m} in the magnetization plane unit normal direction $\mathbf{n}_{\mathrm{m}}^v$. The second term in (1) penalizes the minimal average flux density over Ω_{m}

$$\theta^{v,h}(\mathbf{B}^{v,h}(\mathbf{p}^h,\mathbf{x})) := \left(\max\{0, B_{\min}^{\mathrm{avg},v} - B^{\mathrm{avg},v}(\mathbf{B}^{v,h}(\mathbf{p}^h,\mathbf{x}))\}\right)^2, \quad \rho := 10^6,$$

where $B_{\min}^{\mathrm{avg},v}$ is prescribed minimal average value of the magnetic flux density. Finally, the design parameters $\mathbf{p}^h$ are coordinates of the control nodes of a Bézier patch which determines the shape of the pole head.

3. NUMERICAL EXPERIMENTS WITH THE MULTILEVEL APPROACH

In the first three lines in Fig. 2, we compare the multilevel approach with the classical one for a 2–dimensional (2d) reduced problem of optimal shape design of the MC electromagnet. From the last column we can see that the multilevel approach is much faster than the classical one. Using the multilevel approach, the calculation took about 2 hours while it took almost 7 hours, when using the classical approach. Moreover, we apply the multilevel approach even more generally. We used the 2d optimized shape from the first line, prolong it into the third dimension by constant, and used at the fourth line in Fig. 2 as the initial design for the 3–dimensional (3d) shape optimization problem using 4 design variables. Then, the 3d optimized shape from the fourth line is prolonged to a one described by 12 design variables and used at the last fifth line in Fig. 2 as the initial guess. From the last line in Fig. 2, we can see that the whole calculation took almost 30 hours. We tried to compare this general multilevel approach with the classical one, but the calculation took more than 4 days and several re–meshings of the geometry had to be done. Unfortunately, in 4 days we were still not able to achieve the optimal solution, hence, the calculation was stopped.

4. CONCLUSION

We presented a new numerical technique for shape optimization and applied it to a problem of optimal shape design of an electromagnet that is used for measurements of magneto–optic Kerr effects. This approach has turned out to be much faster than the standard optimization approach. The key point to an efficient use of the multilevel approach is a proper refinement strategy between two successive levels. If we refine too much,

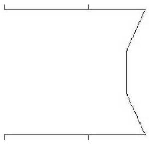
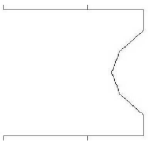
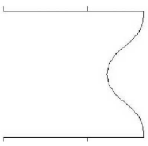
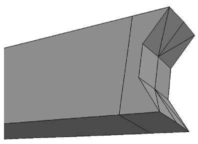
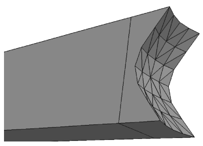
optimized designs	number of des. variables	number of unknowns	SQP iters.	CPU time
	2	1386	8	56s
	4	4705	47	27m 31s
		4970	53	30m 12s
	7	12272	72	1h 58m 55s
		12324	125	6h 44m 0s
	4	12086	47	7h 27m
	12	29541	93	29h 46m

Figure 2. Multilevel versus classical optimization approach

the nested Newton–like optimization algorithm takes many iterations. On the other hand, if the refinement is not remarkable, then the multilevel approach has to proceed at many levels. The multilevel technique is a step forward to adaptive optimization methods.

Let us note that the 2d calculated optimized pole head, see the third line in Fig. 2, was manufactured afterwards and the magnetic field was measured. The objective functional $\mathcal{J}$ calculated from the measured magnetic field decreased by the factor 4.5 in comparison to the initial design, see Fig. 1. Nevertheless, the magnitude of the magnetic field decreased from 1600 Gauss to 1000 Gauss. Choosing a proper compromise between the homogeneity and the magnitude of the magnetic field is a difficult task.

ACKNOWLEDGMENTS

This work has been supported by the Austrian Science Fund FWF within the SFB "Numerical and Symbolic Computing" under the grant SFB F013, by the Czech Ministry of Education under the research project CEZ: J17/98:272400019, and by the Grant Agency of the Czech Republic under the grant 105/99/1698.

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