Ortogonalita

$\bar{x} \perp \bar{y} \iff \boxed{\bar{x}\cdot\bar{y}=0}$

$\bar{x}\cdot\bar{y} \neq 0$ $\qquad x_1y_1+x_2y_2+\dots$

Př: Ortogonalizujte

$$f_1=\begin{pmatrix}1\\1\\-2\end{pmatrix},\quad \bar{f}_2=\begin{pmatrix}1\\0\\1\end{pmatrix},\quad \bar{f}_3=\begin{pmatrix}0\\-1\\1\end{pmatrix}$$

1. $\bar{e}_1=\bar{f}_1$

2. $\bar{e}_2=\bar{f}_2-\alpha\bar{e}_1 \perp \bar{e}_1$

$$\alpha=\frac{\bar{f}_2\cdot\bar{e}_1}{\bar{e}_1\cdot\bar{e}_1}=\frac{1\cdot1+0\cdot1+1\cdot(-2)}{1^2+1^2+(-2)^2}$$

$$\alpha=-\frac{1}{6}$$

$$\bar{e}_2=\begin{pmatrix}1\\0\\1\end{pmatrix}+\frac{1}{6}\cdot\begin{pmatrix}1\\1\\-2\end{pmatrix}=\begin{pmatrix}7/6\\1/6\\2/3\end{pmatrix}$$

3. $\bar{e}_3=\bar{f}_3-\beta\bar{e}_1-\gamma\bar{e}_2$

$$\beta=\frac{\bar{f}_3\cdot\bar{e}_1}{\bar{e}_1\cdot\bar{e}_1}=\frac{0\cdot1-1\cdot1+1\cdot(-2)}{6}$$

$$\beta=-1/2$$

$$\gamma=\frac{\bar{f}_3\cdot\bar{e}_2}{\bar{e}_2\cdot\bar{e}_2}=\frac{0\cdot\frac{7}{6}-1\cdot\frac{1}{6}+1\cdot\frac{2}{3}}{\left(\frac{7}{6}\right)^2+\left(\frac{1}{6}\right)^2+\left(\frac{2}{3}\right)^2}$$

$$=\frac{3/6}{\frac{49+1+16}{36}}=\dots=\frac{3}{11}$$

$= 4/9$

$$\bar{e}_3=\begin{pmatrix}0\\-1\\1\end{pmatrix}+\frac{1}{2}\begin{pmatrix}1\\1\\-2\end{pmatrix}-\frac{3}{11}\begin{pmatrix}7/6\\1/6\\2/3\end{pmatrix}$$

$= \dots$

Př: Vyp