Lineární zobrazení

$\mathcal{A}: U \to V$

$$\boxed{\mathcal{A}(\alpha_1 \vec{f}_1 + \alpha_2 \vec{f}_2 + \dots + \alpha_m \vec{f}_m) = \alpha_1 \mathcal{A}(\vec{f}_1) + \dots + \alpha_m \mathcal{A}(\vec{f}_m)}$$

Př: Rozhodněte, zda $\mathcal{A}: \mathbb{R}^3 \to \mathbb{R}^3$ je LZ

$$\mathcal{A}(x_1, x_2, x_3) := (x_1 - 2x_2, 2x_1 + x_3, x_1 + x_2 + x_3)^T$$

$$\boxed{\begin{aligned} &1)\ \mathcal{A}(\vec{x} + \vec{y}) \overset{?}{=} \mathcal{A}(\vec{x}) + \mathcal{A}(\vec{y}) \\ &2)\ \mathcal{A}(\alpha \vec{x}) = \alpha \mathcal{A}(\vec{x}) \end{aligned}}$$

$$= \begin{pmatrix} 1, & -2, & 0 \\ 2, & 0, & 1 \\ 1, & 1, & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

otom víme, že je LZ

Ano, je LZ.

Př: Dáno lin. zobr. $\mathcal{A}: \mathbb{R}^3 \to \mathbb{R}^2$

$$\mathcal{A}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} := \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \mathcal{A}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} := \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \quad \mathcal{A}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} := \begin{pmatrix} 2 \\ -1 \end{pmatrix}.$$

Vypočtěte $\mathcal{A}\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$.

1) Hledám $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$: $\alpha_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \dots + \alpha_3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & 1 \end{array}\right) -r_1 \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & -4 \\ 0 & 1 & 0 & 1 \end{array}\right) \begin{array}{l} \Rightarrow \alpha_3 = -4 \\ \Rightarrow \underline{\alpha_2 = 1} \end{array}$$

2) $\underline{\mathcal{A}\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}} = \alpha_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \alpha_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ 1. rov.: $\alpha_1 + 1 = 2$, $\underline{\alpha_1 = 1}$

$$= 1 \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 1 \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} - 4 \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$= \underline{\begin{pmatrix} -8 \\ 7 \end{pmatrix}}$$

kvadrat. funkce

Př: Dáno lin. zobr. $\mathcal{A}: P_2 \to \mathbb{R}^2$

$\mathcal{A}(1+x) := \begin{pmatrix}1\\1\end{pmatrix}$, $\mathcal{A}(1+x+x^2) := \begin{pmatrix}-1\\0\end{pmatrix}$, $\mathcal{A}(x) := \begin{pmatrix}2\\-1\end{pmatrix}$

Najděte alespoň jednu $p(x) \in P_2 : \mathcal{A}(p) = \begin{pmatrix}2\\3\end{pmatrix}$

1) Hledám $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$: $\alpha_1 \begin{pmatrix}1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}-1\\0\end{pmatrix} + \alpha_3 \begin{pmatrix}2\\-1\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 1 & 0 & -1 & 3 \end{array}\right) \begin{array}{c} \\ -r_1 \end{array} \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 1 & -3 & 1 \end{array}\right)$$

$\alpha_3 = t,\ t \in \mathbb{R}$

2. rov.: $\alpha_2 = 1 + 3t$

1. rov.: $\alpha_1 = 2 + (1+3t) - 2t$

$\alpha_1 = 3 + t$

2) $p(x) = (3+t)\cdot(1+x) + (1+3t)\cdot(1+x+x^2) + t\cdot x$

např. pro $t = 0$: $p(x) = 3\cdot(1+x) + (1+x+x^2) + 0\cdot x = 4 + 4x + x^2$

Př: $\mathcal{A}: \mathbb{R}^3 \to \mathbb{R}^2$ $\mathcal{A}\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix}$, $\mathcal{A}\begin{pmatrix}1\\1\\1\end{pmatrix} := \begin{pmatrix}-1\\1\end{pmatrix}$, $\mathcal{A}\begin{pmatrix}0\\1\\0\end{pmatrix} := \begin{pmatrix}2\\-1\end{pmatrix}$

Napište $\mathcal{H}(\mathcal{A})$, $h(\mathcal{A})$, $\mathcal{N}(\mathcal{A})$, $d(\mathcal{A})$

obor hodnot, hodnost, nulový prostor, defekt

1) $\mathcal{H}(\mathcal{A})$: $\begin{pmatrix} 1 & 2 \\ -1 & 1 \\ 2 & -1 \end{pmatrix} \begin{array}{c} \\ +r_1 \\ -2r_1 \end{array} \sim \begin{pmatrix} 1 & 2 \\ 0 & 3 \\ 0 & -5 \end{pmatrix} \begin{array}{c} \\ \\ +5/3\, r_2 \end{array}$

$\mathcal{H}(\mathcal{A}) = \left\langle \begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}-1\\1\end{pmatrix}, \begin{pmatrix}2\\-1\end{pmatrix} \right\rangle = \left\langle \begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}0\\3\end{pmatrix} \right\rangle$

$h(\mathcal{A}) := \dim \mathcal{H}(\mathcal{A}) = 2$

2) $\mathcal{N}(\mathcal{A}) = \{ \ldots \in \mathbb{R}^3 \ldots$

$$c)\ M(a) = \{ x \in \mathbb{R}^3 : a(\bar{x}) = \underline{\begin{pmatrix} 0 \\ 0 \end{pmatrix}} \}$$

$$\begin{array}{ccc} \alpha_1 & \alpha_2 & \alpha_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 2 & 1 & -1 & 0 \end{array}\right) -2r_1 \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 3 & -5 & 0 \end{array}\right)$$

$\alpha_3 = t$

2. ř.: $\alpha_2 = 5/3\, t$

1. ř.: $\alpha_1 = \alpha_2 - 2\alpha_3$
$= \frac{5}{3}t - 2t$
$= -1/3\, t$

$$\underline{\underline{M(a)}} = \left\{ -\frac{1}{3}t \cdot \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} + \frac{5}{3}t \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} : t \in \mathbb{R} \right\}$$

$$= \left\{ t \cdot \begin{pmatrix} 4/3 \\ 7/3 \\ 5/3 \end{pmatrix} : t \in \mathbb{R} \right\} = \underline{\underline{\left\langle \begin{pmatrix} 4/3 \\ 7/3 \\ 5/3 \end{pmatrix} \right\rangle}}$$

$$\underline{\underline{d(a)}} = \dim M(a) = \underline{\underline{1}}$$

$$\text{Zk.: } \underset{\substack{\parallel \\ 2}}{h(a)} + \underset{\substack{\parallel \\ 1}}{d(a)} = \dim \mathbb{R}^3 = 3$$

Př: Najděte matici lin. zobr. $a : \mathbb{R}^3 \to \mathbb{R}^2$ vzhledem ke kanonickým (standardní) bázím, $a(x_1, x_2, x_3) := (x_1 + x_2, x_2 - 2x_3)$

$$= \begin{pmatrix} 1x_1 + 1x_2 + 0x_3 \\ 0x_1 + 1x_2 - 2x_3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -2 \end{pmatrix}}} \bar{x}$$