Bilineární formy

$$\mathcal{B}(\bar{u},\bar{v}) = \bar{u}^T \cdot \mathbb{B} \cdot \bar{v} \quad \text{.. na } \mathbb{R}^n$$

$$\mathbb{B} = \underbrace{\tfrac{1}{2}(\mathbb{B}+\mathbb{B}^T)}_{=\mathbb{B}^S} + \underbrace{\tfrac{1}{2}(\mathbb{B}-\mathbb{B}^T)}_{=\mathbb{B}^A}$$

Př: Napište ke ~~Zapište~~ $\mathcal{B}(\bar{x},\bar{y}) := x_1y_1 - x_1y_3 + 2x_2y_2 + x_2y_3 - x_3y_1 + 2x_3y_2 + 2x_3y_3$ na $\mathbb{R}^3$

matici vzhledem ke standart. bázi

$$= x_1(1y_1 + 0y_2 - 1y_3) + x_2(0y_1 + 0y_2\ 1y_3) + x_3(-1y_1 + 2y_2 + 2y_3)$$

$$= (x_1, x_2, x_3) \cdot \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ -1 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 3/2 \\ -1 & 3/2 & 2 \end{pmatrix}}_{= \mathbb{B}^S} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1/2 \\ 0 & +1/2 & 0 \end{pmatrix}}_{= \mathbb{B}^A}$$

Kvadrat. formy

$$Q(\bar{v}) := \mathcal{B}(\bar{v},\bar{v})$$

$$= \bar{v}^T \cdot \underbrace{B^S}_{Q} \cdot \bar{v} + \underbrace{\bar{v}^T \cdot B^A \cdot \bar{v}}_{\substack{=0 \\ \text{vždy}}}$$

$$[\bar{v}]_E^T \, Q_E \cdot [\bar{v}]_E$$

$$\| \; E \to F$$

$$[\bar{v}]_E^T \, \underbrace{T^T \cdot Q_E \cdot T}_{Q_F} \, [\bar{v}]_E$$

Př: Klasifikujte $Q = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 3/2 \\ -1 & 3/2 & 2 \end{pmatrix} + r_1$

$$\sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 3/2 \\ 0 & 3/2 & 1 \end{pmatrix} \underset{+s_1}{} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 3/2 \\ 0 & 3/2 & 1 \end{pmatrix} \quad r_3 := 4r_3 - 3r_2$$

sym. ✓

$$\sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 3/2 \\ 0 & 0 & -1/2 \end{pmatrix} \sim \begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{2} & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$s_3 := 4s_3 - 3s_2$ diag. ✓

$$\left.\begin{matrix} 1, 2 > 0 \\ -2 < 0 \end{matrix}\right\} \Rightarrow \text{matice je } \underline{\text{indefinitní}}$$

Determinant, $\det A = |A|$

je antisymetrická multi-lineární forma taková, že $\det I = 1$

$$1.\ |A| \xrightarrow[\alpha \neq 0]{\vec{r}_i := \alpha \vec{r}_i + \beta \vec{r}_j} \frac{1}{\alpha} \cdot |B|$$

$$\downarrow$$

$$B$$

$$2.\ |A| \xrightarrow{\vec{r}_i \leftrightarrow \vec{r}_j} -|B|$$

$$3.\ |U| = u_{11} \cdot u_{22} \cdots u_{nn}$$

$$\begin{vmatrix} \ddots & & \\ & \ddots & \\ 0 & & \ddots \end{vmatrix}$$

Pr.:

$$\begin{array}{c} \begin{vmatrix} \boxed{1} & -2 & 2 & -2 \\ 2 & -2 & 1 & -2 \\ 1 & 2 & 1 & -2 \\ 2 & 0 & -1 & 0 \end{vmatrix} \end{array} \begin{array}{l} \\ -2\vec{r}_1 \\ -\vec{r}_1 \\ -2\vec{r}_1 \end{array}$$

$$= \begin{vmatrix} 1 & \cdot & \cdot & \cdot \\ 0 & \boxed{2} & -3 & 2 \\ 0 & 4 & -1 & 0 \\ 0 & 4 & -5 & 4 \end{vmatrix} \begin{array}{l} \\ \\ -2\vec{r}_2 \\ -2\vec{r}_2 \end{array}$$

$$= \begin{vmatrix} 1 & & & \\ & 2 & & \\ & 0 & \boxed{5} & -4 \\ & 0 & 1 & 0 \end{vmatrix} \quad \vec{r}_4 := 5\vec{r}_4 - \vec{r}_3$$

$$= \frac{1}{5} \begin{vmatrix} 1 & \cdot & \cdot & \cdot \\ & 2 & \cdot & \cdot \\ & & 5 & \cdot \\ & & 0 & 4 \end{vmatrix} = \frac{1}{\cancel{5}} \cdot 1 \cdot 2 \cdot \cancel{5} \cdot 4 = \underline{\underline{8}}$$