Lineární zobrazení

Př. $a : \mathbb{R}^2 \to \mathbb{R}$ Je a lineární?

$$a((x_1, x_2)) := x_1 + 2x_2$$

Ad 1) $\bar{x}, \bar{y} \in \mathbb{R}^2$

$$a(\bar{x}+\bar{y}) = a((x_1+y_1, x_2+y_2)) = (x_1+y_1)+2(x_2+y_2)$$

$$= (x_1+2x_2) + (y_1+2y_2)$$

$$= a(\bar{x}) + a(\bar{y})$$ ✓

Ad 2) $\bar{x} \in \mathbb{R}^2$ $\alpha \in \mathbb{R}$

$$a(\alpha\bar{x}) = a((\alpha x_1, \alpha x_2)) = (\alpha x_1) + 2(\alpha x_2)$$

$$= \alpha \cdot (x_1 + 2x_2)$$

$$= \alpha \cdot a(\bar{x})$$

Ano, a je lin. zobrazení.

Př. $a : \mathbb{R}^2 \to \mathbb{R}$

$$a(\bar{x}) := x_1 + x_2 - 1$$

Ad 2. $\bar{x} \in \mathbb{R}$, $\alpha \in \mathbb{R}$

$$a(\alpha\bar{x}) = a((\alpha x_1, \alpha x_2)) = (\alpha x_1) + (\alpha x_2) - 1$$

$$\neq \alpha(x_1 + x_2 - 1)$$

např. $\alpha = 2$, $\bar{x} = \bar{o}$: $a(2\cdot\bar{o}) = -1 \neq$

$2\cdot a(\bar{o}) = 2\cdot(0+0-1) = -2$

Př: $a : \mathbb{R} \to \mathbb{R}$ $\quad a(x) := \sin x$

např. $\sin(x+y) \neq \sin x + \sin y$

$$\begin{array}{c}\sin(x+y)\\ \parallel \\ \sin x\cos y + \cos x \sin y\end{array}$$

Není lin. zobrazení.

Př. $a : \mathcal{P}_2 \mapsto \mathcal{P}_1$ $\quad a(p(x)) := p'(x)$

$a(a_0 + a_1 x + a_2 x^2) = a_1 + 2a_2 x$

Ad 1) $p(x), q(x) \in \mathcal{P}_2$

$$a((p+q)(x)) = (p+q)'(x) = p'(x) + q'(x)$$
$$= a(p(x)) + a(q(x)) \checkmark$$

Ad 2) $p(x) \in \mathcal{P}_2$, $\alpha \in \mathbb{R}$

$$a((\alpha p)(x)) = (\alpha p)'(x) = \alpha\, p'(x)$$
$$= \alpha \cdot a(p(x)) \checkmark$$

Derivace funkce je lineární zobrazení z $\mathcal{F}$ do $\mathcal{F}$.

Př. $a : \mathcal{P}_2 \mapsto \mathbb{R}$ $\quad a(p(x)) := \int_0^1 p(x)\,dx$

Ad 1. $p, q \in \mathcal{P}_2$

$$a((p+q)(x)) = \int^1 [p(x) + q(x)]\,dx = \int^1 p(x)\,dx$$

$$+\int_0^1 q(x)\,dx$$

$$= a(p) + a(q) \checkmark$$

Ad 2. $\alpha \in \mathbb{R}$, $p(x) \in \mathcal{P}_2$

$$a((\alpha p)(x)) = \int_0^1 \alpha p(x)\,dx = \alpha \int_0^1 p(x)\,dx$$

$$= \alpha \cdot a(p(x)) \checkmark$$

Integrál je lineární zobrazení!

Př. Uvažujme matici $\mathbb{A} \in \mathbb{R}^{m \times n}$

$a : \mathbb{R}^n \to \mathbb{R}^m$ def. $\boxed{a(\bar{x}) := \mathbb{A} \cdot \bar{x}}$

Ad 1) $\bar{x}, \bar{y} \in \mathbb{R}^n$

$$a(\bar{x} + \bar{y}) = \mathbb{A} \cdot (\bar{x} + \bar{y}) = (\mathbb{A} \cdot \bar{x}) + (\mathbb{A} \cdot \bar{y})$$

$$= a(\bar{x}) + a(\bar{y}) \checkmark$$

Ad 2) $\alpha \in \mathbb{R}$, $\bar{x} \in \mathbb{R}^n$

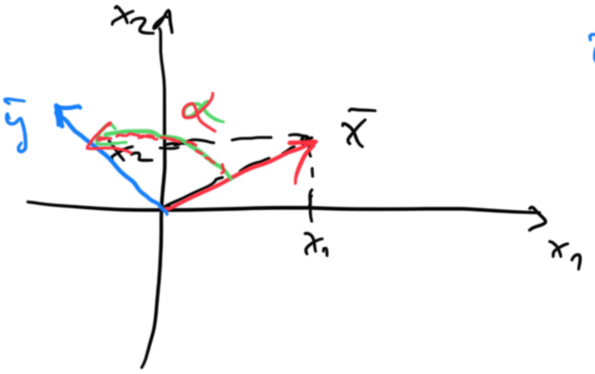
$$a(\alpha \cdot \bar{x}) = \mathbb{A} \cdot (\alpha \cdot \bar{x}) = \alpha \cdot (\mathbb{A} \cdot \bar{x})$$

$$= \alpha \cdot a(\bar{x}) \checkmark$$

Ano, násobení maticí realizuje lineární zobrazení!

lineární zobrazení.

Př. Rovinná rotace



$\bar y$ = rotace $\bar x$ o úhel α

$$\underline{y = y_1 + i y_2}$$
$$= e^{i\alpha}\cdot(x_1 + i x_2)$$
$$= (\cos\alpha + i\sin\alpha)\cdot(x_1 + i x_2)$$
$$= \underline{\cos\alpha\cdot x_1 - \sin\alpha\cdot x_2 + i\cdot(\sin\alpha\cdot x_1 + \cos\alpha\cdot x_2)}$$

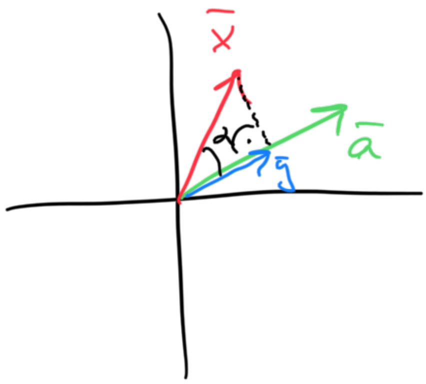
$$y_1 = \cos\alpha\cdot x_1 - \sin\alpha\cdot x_2$$
$$y_2 = \sin\alpha\cdot x_1 + \cos\alpha\cdot x_2$$

$$\bar y = \underbrace{\begin{bmatrix}\cos\alpha, & -\sin\alpha \\ \sin\alpha, & \cos\alpha\end{bmatrix}}_{\mathbb{A}}\cdot\bar x$$

↙

Ano, Rovinná rotace realizuje lineární zobrazení

Př. Ortogonální projekce na vektor $\bar a$



$$\cos\alpha = \frac{\|\bar y\|}{\|\bar x\|} = \frac{\bar x\cdot\bar a}{\|\bar x\|\cdot\|\bar a\|}$$

$$\bar y = \cos\alpha\cdot\|\bar x\|\cdot\frac{\bar a}{\|\bar a\|}$$

$$\bar y = \frac{(\bar x\cdot\bar a)}{\cancel{\|\bar x\|}\cdot\|\bar a\|}\cdot\cancel{\|\bar x\|}\cdot\frac{\bar a}{\|\bar a\|}$$

$$= \frac{\boxed{x_1}a_1 + \boxed{x_2}a_2}{\quad}\cdot\begin{pmatrix}a_1\\ a_2\end{pmatrix}$$

$$a_1^2 + a_2^2 \qquad \begin{pmatrix} a_2 \end{pmatrix}$$

$$= \frac{1}{a_1^2+a_2^2}\left[x_1 \cdot \begin{pmatrix} a_1 a_1 \\ a_1 a_2 \end{pmatrix} + x_2 \cdot \begin{pmatrix} a_2 a_1 \\ a_2 a_2 \end{pmatrix}\right.$$

$$\boxed{\bar y = \begin{bmatrix} \frac{a_1^2}{a_1^2+a_2^2} & \frac{a_1 a_2}{a_1^2+a_2^2} \\ \text{sy.} & \frac{a_2^2}{a_1^2+a_2^2} \end{bmatrix} \cdot \bar x}$$

... násobení matice krát vektor,

Důlež.: Rovinná rotace je lin. zobr.

$$\boxed{a(\underline{\alpha_1 \bar e_1 + \dots + \alpha_n \bar e_n}) \overset{\boxed{a \text{ lin.}}: U \to V}{=\!=} \alpha_1 a(\bar e_1) + \dots + \alpha_n a(\bar e_n)}$$

Je-li navíc $(\bar e_1, \dots, \bar e_n) =: E$ báze U, pak a je jednoznačně určeno vektory $a(\bar e_1), \dots, a(\bar e_n) \in V$

Př. Určete $a\left(\begin{pmatrix}1\\1\end{pmatrix}\right)$, kde $a: \mathbb{R}^2 \to \mathbb{R}$ je lin., když víme, že $a\left(\begin{pmatrix}1\\0\end{pmatrix}\right) := 1$, $a\left(\begin{pmatrix}0\\1\end{pmatrix}\right) := -1$.

kanonická báze $\mathbb{R}^2$

$$a\left(\begin{pmatrix}1\\1\end{pmatrix}\right) = a\left(\alpha_1 \begin{pmatrix}1\\0\end{pmatrix} + \alpha_2 \begin{pmatrix}0\\1\end{pmatrix}\right) =\!= \alpha_1 a\begin{pmatrix}1\\0\end{pmatrix} + \alpha_2 a\begin{pmatrix}0\\1\end{pmatrix}$$

$\underbrace{\quad}_{1} \qquad \underbrace{\quad}_{-1}$

1. Hledám $\alpha_1, \alpha_2 \in \mathbb{R}$:

$$\alpha_1 \begin{pmatrix}1\\0\end{pmatrix} + \alpha_2 \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix} \iff \underline{\underline{\alpha_1 = \alpha_2 = 1}}$$

2. $\underline{\underline{a\begin{pmatrix}1\\1\end{pmatrix}}} = 1 \cdot a\begin{pmatrix}1\\0\end{pmatrix} + 1 \cdot a\begin{pmatrix}0\\1\end{pmatrix} = 1 \cdot 1 + 1 \cdot (-1) = \underline{\underline{0}}$

Př. $a\begin{pmatrix}2\\3\end{pmatrix} = ?$, kde a je lin. def. obrazy báze $a\begin{pmatrix}1\\1\end{pmatrix} := \begin{pmatrix}1\\2\\3\end{pmatrix}$, $a\begin{pmatrix}1\\-1\end{pmatrix} := \begin{pmatrix}3\\2\\1\end{pmatrix}$

$$a\underline{\begin{pmatrix}2\\3\end{pmatrix}} = a\left(\underline{\boxed{\alpha_1}\begin{pmatrix}1\\1\end{pmatrix} + \boxed{\alpha_2}\begin{pmatrix}1\\-1\end{pmatrix}}\right) = \underline{\boxed{\alpha_1}\begin{pmatrix}1\\2\\3\end{pmatrix} + \boxed{\alpha_2}\begin{pmatrix}3\\2\\1\end{pmatrix}}$$

1. Hledám $\alpha_1, \alpha_2 \in \mathbb{R}$:

$$\alpha_1 \begin{pmatrix}1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & -1 & 3 \end{array}\right) \begin{matrix}\\ -r_1\end{matrix} \sim \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -2 & 1 \end{array}\right) \iff \begin{matrix} \underline{\underline{\alpha_1 = 5/2}} \\ \uparrow \\ \underline{\underline{\alpha_2 = -1/2}} \end{matrix}$$

2. $\underline{\underline{a\begin{pmatrix}2\\3\end{pmatrix}}} = \frac{5}{2} \cdot \begin{pmatrix}1\\2\\3\end{pmatrix} - \frac{1}{2} \cdot \begin{pmatrix}3\\2\\1\end{pmatrix} = \begin{pmatrix}2/2\\8/2\\14/2\end{pmatrix} = \underline{\begin{pmatrix}1\\4\\7\end{pmatrix}}$

Př. Vypočtěte $a(1-x^2)$, kde $a : P_2 \to \mathbb{R}$ je lineární a je určeno $a(1+x) := \frac{3}{2}$, $a(x+x^2) := \frac{5}{6}$, $a(1+x^2) := \frac{4}{3}$.

---

1. Hledám $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$:

$$\forall x \in \mathbb{R}: \alpha_1 (\underline{1} + \underset{\sim}{x}) + \alpha_2 (\underset{\sim}{x} + x^2) + \alpha_3 (\underline{1} + x^2) = 1 - x^2$$

$$\Updownarrow$$

$$\begin{array}{c|c} & \begin{array}{ccc} \alpha_1 & \alpha_2 & \alpha_3 \end{array} \\ \begin{array}{c} \underline{1}: \\ \underset{\sim}{x}: \\ x^2: \end{array} & \left(\begin{array}{ccc|c} \boxed{1} & 0 & 1 & 1 \\ ⓵ & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 \end{array}\right) \end{array} \sim_{-\check{r}_1} \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & \boxed{1} & -1 & -1 \\ 0 & ⓵ & 1 & -1 \end{array}\right)_{-\check{r}_2} \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 2 & 0 \end{array}\right.$$

3. rov.: $\underline{\alpha_3 = 0}$

2. rov.: $\underline{\alpha_2 = -1}$

1. rov.: $\underline{\underline{\alpha_1 = 1}}$

2. $\underline{\underline{\mathcal{A}(1-x^2)}} = 1 \cdot \frac{3}{2} - 1 \cdot \frac{5}{6} + 0 \cdot \frac{4}{3}$

$= \frac{9-5}{6} = \frac{4}{6} = \underline{\underline{\frac{2}{3}}}$

[Zk. $\int_0^1 (1-x^2)\,dx = \left[x - \frac{x^3}{3}\right]_0^1 = 1 - \frac{1}{3} = \frac{2}{3}$ ✓]

Př: Vypočítejte všechny $\bar{x} \in \mathbb{R}^2 : \mathcal{A}(\bar{x}) = 2$,

kde $\mathcal{A}$ je lin. zobraz. zadáno

$\mathcal{A}\begin{pmatrix} 1 \\ 0 \end{pmatrix} := 1$, $\mathcal{A}\begin{pmatrix} 0 \\ 1 \end{pmatrix} := -1$

$$\mathcal{A}(\bar{x}) = \mathcal{A}\left(\alpha_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = \boxed{\alpha_1 \cdot 1 + \alpha_2 \cdot (-1) = 2}$$

1. Hledám $\alpha_1, \alpha_2 \in \mathbb{R}$

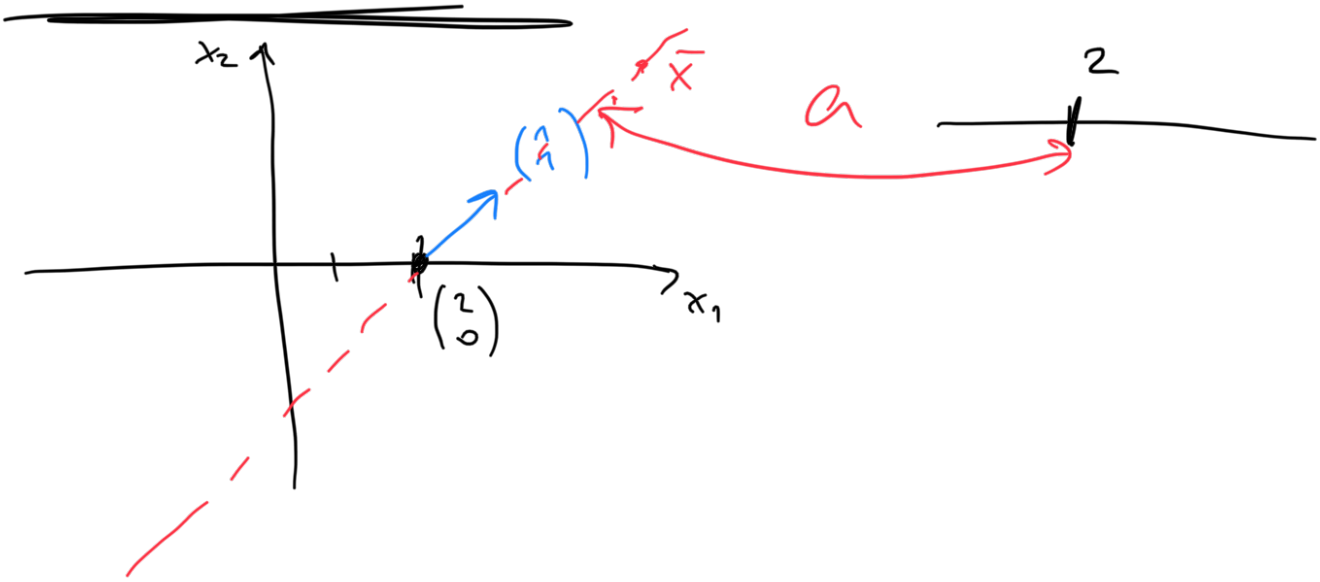
$$\boxed{\alpha_1 - \alpha_2 = 2}$$

Param. $\underline{\alpha_2 = t \in \mathbb{R}}$

$$\underline{\alpha_1 = 2+t}$$

2. $\bar{x} = \alpha_1 \cdot \begin{pmatrix}1\\0\end{pmatrix} + \alpha_2 \cdot \begin{pmatrix}0\\1\end{pmatrix} = (2+t)\cdot \begin{pmatrix}1\\0\end{pmatrix} + t\cdot\begin{pmatrix}0\\1\end{pmatrix}$

$$\bar{x} = \begin{pmatrix}2\\0\end{pmatrix} + t\cdot\begin{pmatrix}1\\1\end{pmatrix}, \quad t\in\mathbb{R}$$



Př: Vypočítejte všechny $p(x)\in \mathcal{P}_2 : a(p(x)) = 1$, kde $a : \mathcal{P}_2 \to \mathbb{R}$ je lin. zobr. určené obrazy báze $a(1+x) := \frac{3}{2}$, $a(1+x^2) := \frac{4}{3}$, $a(x+x^2) := \frac{5}{6}$

$$a(p(x)) = a(\alpha_1(1+x) + \alpha_2(1+x^2) + \alpha_3(x+x^2))$$

$$= \boxed{\alpha_1 \cdot \frac{3}{2} + \alpha_2\cdot\frac{4}{3} + \alpha_3\cdot\frac{5}{6} = 1}$$

1. Hledám $\alpha_1, \alpha_2, \alpha_3 \in\mathbb{R}$:

$\frac{3}{2}\alpha + \frac{4}{\ }\alpha + 5 \quad = 1$

$$2\,\alpha_1 + 3\,\alpha_2 + \frac{}{8}\,\alpha_3 = 1$$

Parametrizuji: $\alpha_3 = t \in \mathbb{R}$

$$\alpha_2 = s \in \mathbb{R}$$

Dosadím: $\frac{3}{2}\alpha_1 = 1 - \frac{4}{3}s - \frac{5}{6}t$

$$\alpha_1 = \frac{2}{3} - \frac{8}{9}s - \frac{5}{9}t$$

$$2.\ \underline{\underline{p(x) = \left(\frac{2}{3} - \frac{8}{9}s - \frac{5}{9}t\right)\cdot(1+x) + s\cdot(1+x^2) + t\cdot(x+x^2)}}$$

$$s, t \in \mathbb{R}$$

např.: $s = t = 0$: $p(x) = \frac{2}{3}(1+x)$

$$\int_0^1 \frac{2}{3}(1+x)\,dx = \frac{2}{3}\left[x + \frac{x^2}{2}\right]_0^1 = \frac{2}{3}\cdot\left(1+\frac{1}{2}\right) = \frac{2}{3}\cdot\frac{3}{2} = 1$$

✓

$\underline{N(A),\ \mathcal{H}(A),\ d(A),\ h(A)}$

Př.: Je dáno lin. zobr. $a(1,1) := \textcircled{1}$

$$a(1,-1) := \boxed{-1}$$

$n(a)$: Hledáme $\bar{x} \in \mathbb{R}^2 : a(\bar{x}) = \underline{0}$

$$1.\quad \alpha_1, \alpha_2 \in \mathbb{R} : \alpha_1 \cdot \textcircled{1} + \alpha_2 \cdot \boxed{-1} = 0$$

$$\underline{\alpha_2 = t \in \mathbb{R}}$$

$$\underline{\alpha_1 = t}$$

$$\underline{\bar{x}} = \alpha_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = t \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \underline{\underline{t \cdot \begin{pmatrix} 2 \\ 0 \end{pmatrix}}}$$

$$n(a) = \left\langle \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right\rangle$$

báze $\mathcal{N}(\mathcal{A})$ tj. $\underline{\dim \mathcal{N}(\mathcal{A}) = d(\mathcal{A}) = 1}$

$\underline{\mathcal{H}(\mathcal{A}) = \mathbb{R}}$: $\underset{\dim \mathcal{H}(\mathcal{A})}{\overset{\shortparallel}{h(\mathcal{A})}} = \dim(\mathbb{R}^2) - d(\mathcal{A}) = 2 - 1 = \underline{1}$ ✓

Př. $A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 0 \end{pmatrix}$, $\mathcal{A}(\bar{x}) := A \cdot \bar{x}$: $\mathbb{R}^2 \to \mathbb{R}^3$

$\mathcal{H}(\mathcal{A}) = \left\langle \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\rangle = S(A)$

Je to báze? Ano, je to ~~b~~báze $\underline{h(\mathcal{A}) = 2}$

$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 0 \end{pmatrix} \overset{}{+ř_1} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \end{pmatrix}$] 2 řádky > Jsou LN

$\mathcal{N}(\mathcal{A})$: $\boxed{d(A) = \dim(\mathbb{R}^2) - h(\mathcal{A})} = 2 - 2 = \underline{0}$

tj. $\underline{\mathcal{N}(\mathcal{A}) = \{\bar{o}\}}$

Matice lineárního zobrazení

$\mathcal{A} : \mathcal{U} \to \mathcal{V}$ .. lin. zobr.

$E = (\bar{e}_1, \bar{e}_2, \dots, \bar{e}_n)$ .. báze $\mathcal{U}$

$F = (\bar{f}_1, \dots, \bar{f}_m)$ .. báze $\mathcal{V}$

$\bar{u} \in \mathcal{U}$ $\qquad [\bar{u}]_E = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} = \bar{\alpha} \in \mathbb{R}^n$

$\overset{E}{\shortparallel}$ $\alpha_1 \bar{e}_1 + \alpha_2 \bar{e}_2 + \dots + \alpha_n \bar{e}_n$

$\mathcal{A}(\bar{u}) \overset{\curvearrowright}{=} \mathcal{A}(\alpha_1 \bar{e}_1 + \dots + \alpha_n \bar{e}_n) \overset{\mathcal{A}\ \text{lin.}}{=\!=} \alpha_1 \mathcal{A}(\bar{e}_1) + \dots + \alpha_n \mathcal{A}(\bar{e}$

$[\mathcal{A}(\bar{u})]_F = [\alpha_1 \mathcal{A}(\bar{e}_1) + \dots + \alpha_n \mathcal{A}(\bar{e}_n)]_F$

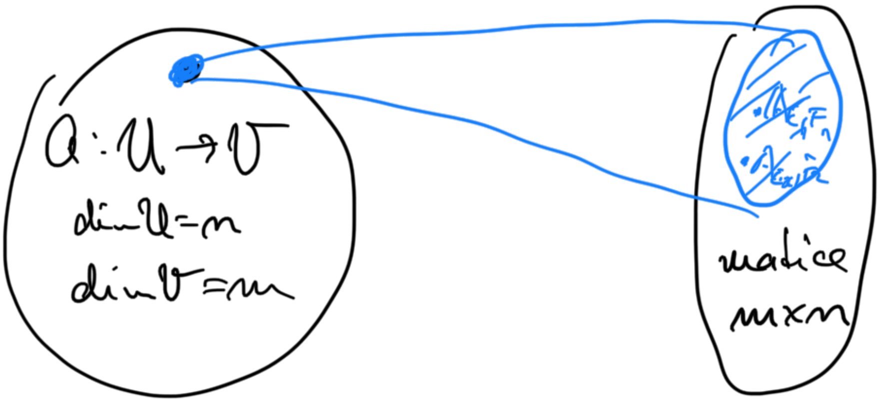
$\shortparallel$ $S : \bar{v} \mapsto [\bar{v}]_F$ je lineární

$$\| \qquad v \to \mathbb{R}^m$$

$$\alpha_1 \cdot \underbrace{[a(\bar{e}_1)]_F}_{\in \mathbb{R}^m} + \alpha_2 \cdot \underbrace{[a(\bar{e}_2)]_F}_{\in \mathbb{R}^m} + \dots + \alpha_n \underbrace{[a(\bar{e}_n)]_F}_{\in \mathbb{R}^m}$$

$$= \underbrace{\left([a(\bar{e}_1)]_F, [a(\bar{e}_2)]_F, \dots, [a(\bar{e}_n)]_F \right)}_{=: A_{E,F}} \cdot \overset{}{\underset{\underset{[\bar{u}]_E}{\|}}{\bar{\alpha}}}$$

matice lin. zobrazení a v bázích E, F.



Př: Mějme lin. zobrazení $a: \mathbb{R}^3 \to \mathbb{R}^2$

$$a\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} := \begin{pmatrix}x_1+x_2-x_3\\ x_2-2x_3\end{pmatrix}.$$

Vypočítejte A_{E_3,E_2} v kanonických bázích

$$E_3 := \left(\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right), E_2 := \left(\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right).$$

$$a(\bar{x}) = \begin{pmatrix}1\cdot x_1 + 1\cdot x_2 - 1\cdot x_3\\ 0\cdot x_1 + 1\cdot x_2 - 2\cdot x_3\end{pmatrix} = \underbrace{\begin{pmatrix}1 & 1 & -1\\ 0 & 1 & -2\end{pmatrix}}_{= A_{E_3,E_2}} \cdot \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

Př: —"— $A_{E_3,F} = ?$, kde $F := \left(\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\end{pmatrix}\right)$

$$\mathbb{A}_{E_3,F} = \left(\left[a\begin{pmatrix}1\\0\\0\end{pmatrix}\right]_F, \left[a\begin{pmatrix}0\\1\\0\end{pmatrix}\right]_F, \left[a\begin{pmatrix}0\\0\\1\end{pmatrix}\right]_F \right)$$

$$= \left(\left[\begin{pmatrix}1\\0\end{pmatrix}\right]_F, \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_F, \left[\begin{pmatrix}-1\\-2\end{pmatrix}\right]_F \right)$$

Hledám $\alpha_1, \alpha_2 \in \mathbb{R}$: $\alpha_1\begin{pmatrix}1\\1\end{pmatrix} + \alpha_2\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$

..

Hledám $\gamma_1, \gamma_2 \in \mathbb{R}$: $\gamma_1\begin{pmatrix}1\\1\end{pmatrix} + \gamma_2\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}-1\\-2\end{pmatrix}$

$$\left(\begin{matrix}1&1\\\boxed{1}&-1\end{matrix}\middle|\underbrace{\begin{matrix}1&1&-1\\0&1&-2\end{matrix}}_{\mathbb{A}_{E_3,E_2}}\right)\begin{matrix}\\-r_1\end{matrix} \sim \left(\begin{array}{cc|ccc}1&\boxed{1}&1&1&-1\\0&-2&-1&0&-1\end{array}\right)\sim$$

$$r_1 := 2r_1 + r_2$$

$$\sim\left(\begin{array}{cc|ccc}2&0&1&2&-3\\0&-2&-1&0&-1\end{array}\right)\begin{matrix}/\cdot 1/2\\ /\cdot(-1/2)\end{matrix}$$

$$\sim\left(\begin{matrix}1&0\\0&1\end{matrix}\middle|\underbrace{\begin{matrix}1/2&1&-3/2\\1/2&0&1/2\end{matrix}}_{\mathbb{A}_{E_3,F}}\right)$$

Př: $\mathbb{D}: \mathcal{P}_3 \to \mathcal{P}_2$ ..derivace $\quad E_3 = (1, x, x^2, x^3)$

1) Spočtěte $\mathbb{D}_{E_3,E_2} = ?$ $\quad E_2 = (1, x, x^2)$

2) Použijte $\mathbb{D}_{E_3,E_2}$ na zderivování $p_3(x) = 1-3x^2+2x^3$

---

1) $\mathbb{D}_{E_3,E_2} = \left([1']_{E_2}, [x']_{E_2}, [(x^2)']_{E_2}, [(x^3)']_{E_2} \right)$

$$= \left([0]_{E_2}, [1]_{E_2}, [2x]_{E_2}, [3x^2]_{E_2} \right)$$

$$= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

2) $[p_3]_{E_3} = [1 - 3x^2 + 2x^3]_{E_3} = \begin{pmatrix} 1 \\ 0 \\ -3 \\ 2 \end{pmatrix}$

$$[p_3'(x)]_{E_2} = D_{E_3, E_2} \cdot \begin{pmatrix} 1 \\ 0 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 & \textcircled{1} & 0 & 0 \\ 0 & 0 & \boxed{2} & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ \textcircled{0} \\ \boxed{-3} \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} \textcircled{0} \\ \boxed{2 \cdot (-3)} \\ 3 . 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ 6 \end{pmatrix}$$

$E_2 = (1, x, x^2)$

$$\underline{\underline{p_3'(x) = -6x + 6x^2}}$$