Nulový prostor matice $\mathbb{A}$, obor hodnot (sl. pr.) $\mathbb{B}$

Př.: Řešte
$$\boxed{\begin{array}{l} 1x_1 + 1x_2 - 1x_3 = 0 \\ 0 \quad\; 1x_2 + 1x_3 = 0 \end{array}}$$

Hledáme
$$\mathcal{N}\Bigg(\underbrace{\begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}}_{=\mathbb{A}}\Bigg) = \left\{\bar{x} \in \mathbb{R}^3 : \boxed{\mathbb{A}\cdot\bar{x} = \bar{o}}\right\}$$

Řešení, parametrizace:

$\underline{x_3 = t} \in \mathbb{R}$

2. rov.: $x_2 + t = 0 \iff \underline{x_2 = -t}$

1. rov.: $x_1 + (-t) - t = 0 \iff \underline{x_1 = 2t}$

$$\bar{x} \in \mathcal{N}(\mathbb{A}) = \left\{\bar{x} = t\cdot\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} : t \in \mathbb{R}\right\}$$

$$= \left\{\underbrace{\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}}_{\mathbb{B}}\cdot t : t \in \mathbb{R}\right\} = S(\mathbb{B}) = \mathcal{H}(\mathbb{B})$$

tj.:
$$\mathcal{N}(\underset{2\times 3}{\mathbb{A}}) = S(\underset{3\times 1}{\mathbb{B}})$$

Podprostory $\mathcal{F}$

Př. Je $\mathcal{P}_2 := \{a_0 + a_1x + a_2x^2 =: p_2(x) : a_0, a_1, a_2 \in \mathbb{R}\}$

je vektorový podprostor $\mathcal{F}$?

Ad 0. $p_2(x) = 0 \in \mathcal{P}_2$, tj. $\mathcal{P}_2 \neq \emptyset$. ✓

Ad 1. $\left.\begin{array}{l} p_2(x) = a_0 + a_1 x + a_2 x^2 \\ q_2(x) = b_0 + b_1 x + b_2 x^2 \end{array}\right\} \Rightarrow (p_2 + q_2)(x) \overset{?}{\in} \mathcal{P}_2$

$$(p_2 + q_2)(x) = a_0 + a_1 x + a_2 x^2 + b_0 + b_1 x + b_2 x^2$$
$$= \underbrace{(a_0 + b_0)}_{= c_0} + \underbrace{(a_1 + b_1)}_{c_1} x + \underbrace{(a_2 + b_2)}_{c_2} x^2 \quad ✓$$

Ad 2. $\left.\begin{array}{l} \alpha \in \mathbb{R} \\ p_2(x) = a_0 + a_1 x + a_2 x^2 \end{array}\right\} \Rightarrow (\alpha \cdot p)(x) \overset{?}{\in} \mathcal{P}_2$

$$(\alpha \cdot p_2)(x) := \alpha(a_0 + a_1 x + a_2 x^2) = \underbrace{(\alpha a_0)}_{c_0} + \underbrace{(\alpha a_1)}_{c_1} x + \underbrace{(\alpha a_2)}_{c_2} x^2 \quad ✓$$

Ano, $\mathcal{P}_2$ je vekt. podprostor $\mathcal{F}$.

Př. $V := \{ p_1(x) := a_0 + a_1 x \in \mathcal{P}_1 : a_0, a_1 \in \mathbb{R} \text{ a } \underline{a_0 + a_1 = 0} \}$

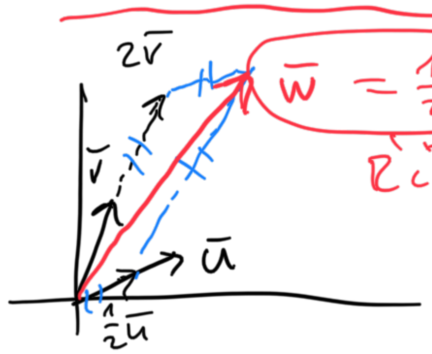
Ad 0. $0 = \underset{a_0}{0} + \underset{a_1}{0} x : \underset{a_0}{0} + \underset{a_1}{0} = 0$ ✓

Ad 1. $\left.\begin{array}{l} p_1(x) = a_0 + a_1 x : a_0 + a_1 = 0 \\ q_1(x) = b_0 + b_1 x : b_0 + b_1 = 0 \end{array}\right\} \overset{?}{\Rightarrow} (p_1 + q_1)(x) \in \mathcal{P}_2$

$$(p_1 + q_1)(x) = (a_0 + b_0) + (a_1 + b_1) x$$

$$(a_0 + b_0) + (a_1 + b_1)$$
$$= \underbrace{(a_0 + a_1)}_{= 0} + \underbrace{(b_0 + b_1)}_{= 0}$$

Lineární kombinace



$\bar{w} = \frac{1}{2}\bar{u} + 2\bar{v}$

Řekneme, že $\bar{w}$ je lin. kombinace $\bar{u}, \bar{v}$

$\bar{v}_1, \bar{v}_2, \dots, \bar{v}_n, \bar{v} \in V$

$\bar{v}$ je lin. kombinace $\bar{v}_1, \dots, \bar{v}_n$, pokud ex.

$\alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{R}$:

$$\boxed{\alpha_1 \bar{v}_1 + \alpha_2 \bar{v}_2 + \dots + \alpha_n \bar{v}_n = \bar{v}}$$

Př. Je $\bar{v} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ lin. kombinace vektorů $\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \bar{v}_1$ a $\begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} = \bar{v}_2$?

Hledám $\alpha_1, \alpha_2 \in \mathbb{R}$:

$$\alpha_1 \cdot \underbrace{\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}}_{\bar{v}_1} + \alpha_2 \cdot \underbrace{\begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}}_{\bar{v}_2} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

... soustava 3 rovnic o 2 neznámých

$$\left(\begin{array}{cc|c} \boxed{-1} & 2 & 1 \\ 2 & 0 & 2 \\ 1 & -2 & -1 \end{array}\right) \begin{array}{l} \\ +2r_1 \\ +r_1 \end{array} \sim \left(\begin{array}{cc|c} -1 & 2 & 1 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{array}\right)$$

$-\alpha_1 + 2 \cdot 1 = 1$

$\underline{\alpha_2 = 1}$

$\underline{\alpha_1 = 1}$

Ano, $\bar{v}$ je lin. kombinace $\bar{v}_1, \bar{v}_2$:

$1 \cdot \bar{v}_1 + 1 \cdot \bar{v}_2 = \bar{v}$.

$$\sim\left(\begin{array}{cc|c} \boxed{2} & 0 & 2\\ -1 & 2 & 1 \\ 1 & -2 & -1\end{array}\right) \begin{matrix} r_2 := 2r_2 + r_1 \\ r_3 := 2r_3 - r_1\end{matrix} \sim \left(\begin{array}{cc|c} 2&0&2\\0&4&4\\ \cancel{0}&\cancel{-4}&\cancel{-4}\end{array}\right) \begin{matrix} \Longleftrightarrow \alpha_1 = 1 \\ +r_2 \end{matrix}$$

$$\sim\left(\begin{array}{cc|c} 2&0&2\\0&0&1\\ \cancel{0}&\cancel{0}&\cancel{0}\end{array}\right)$$

Př. Je $p(x) = 1+x^2$ je lin. kombinací funkcí $p_1(x) = 1-x+2x^2$, $p_2(x) = 2-x+x^2$?

Hledám $\alpha_1, \alpha_2 \in \mathbb{R}$:

$$\forall x \in \mathbb{R}: \ \alpha_1\underbrace{(1-1x+2x^2)}_{p_1(x)} + \alpha_2\underbrace{(2-1x+1x^2)}_{p_2(x)} = 1+x^2$$

$$\Updownarrow$$

$1, x, x^2$ jsou LN

$$\forall x \in \mathbb{R}: (\alpha_1 + 2\alpha_2)\cdot 1 + (-\alpha_1 - \alpha_2)\cdot x + (2\alpha_1 + \alpha_2)x^2 = 1 + 0x + 1x^2$$

$$\Updownarrow$$

$$\boxed{\begin{array}{rl} 1: & \alpha_1 + 2\alpha_2 = 1 \\ x: & -\alpha_1 - \alpha_2 = 0 \\ x^2: & 2\alpha_1 + \alpha_2 = 1 \end{array}}$$

nemá řešení

$$\left(\begin{array}{cc|c} \boxed{1} & 2 & 1 \\ -1 & -1 & 0 \\ 2 & 1 & 1 \end{array}\right) \begin{matrix} \\ +r_1 \\ -2r_1 \end{matrix}$$

$$\sim\left(\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & \boxed{1} & 1 \\ 0 & -3 & -1 \end{array}\right) \begin{matrix} \\ \\ +3r_2 \end{matrix}$$

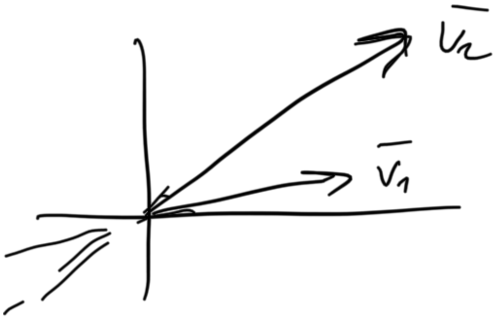
$$\sim\left(\begin{array}{cc|c} 1 & 2 & 1 \end{array}\right.$$

$$\left(\begin{array}{cc|c} 0 & 1 & 1 \\ 0 & 0 & 2 \end{array}\right)$$

$0 = 2$

p není lin. komb. p_1, p_2.

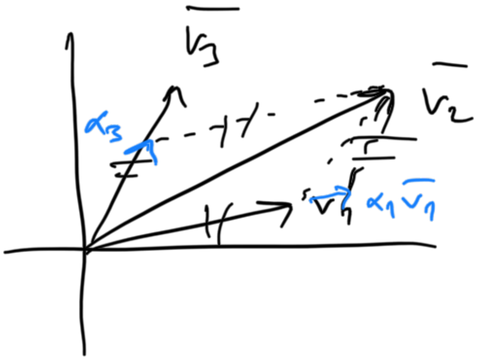
Lineární nezávislost



• $\bar v_1, \bar v_2$ jsou lin. nezávislé,

pokud $\bar v_2 \neq \alpha_1 \bar v_1$

ani $\bar v_1 \neq \beta_1 \bar v_2$



• $\bar v_1 \overset{?}{=} \alpha_2 \bar v_2 + \alpha_3 \bar v_3$

• $\bar v_2 \overset{?}{=} \beta_1 \bar v_1 + \beta_3 \bar v_3$

• $\bar v_3 \overset{?}{=} \gamma_1 \bar v_1 + \gamma_2 \bar v_2$

$$\boxed{\alpha_1 \bar v_1 + \alpha_2 \bar v_2 + \alpha_3 \bar v_3 = \bar 0} \Rightarrow \underline{\text{Jediné řeš.}}\ \alpha_1 = \alpha_2 = \alpha_3 = 0$$

pak $\bar v_1, \bar v_2, \bar v_3$ jsou lineárně nezávislé

Př. Jsou $\bar v_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \bar v_2 = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, \bar v_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ lin. nez.?

Hledám $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$:

$$\alpha_1 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right) \begin{array}{c} \\ -2r_1 \\ +r_1 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \Rightarrow$$

$$\alpha_3 = t \in \mathbb{R}$$

∞ řešení
$\Downarrow$
Jsou lin. záv.

2. rov.: $\alpha_2 = -t$

1. rov.: $\alpha_1 = +t$

Zk. Např. pro $t = 1$: $1 \cdot \bar{v}_1 - 1 \cdot \bar{v}_2 + 1 \cdot \bar{v}_3 = \bar{0}$ ✓

$$1 \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} - 1 \cdot \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1-1+0 \\ 2-3+1 \\ -1+0+1 \end{pmatrix} = \bar{0} \quad ✓$$

Př. Jsou $p_1(x) = 1 + x$, $p_2(x) = 1 - x$ lin. nez.?

Hledám $\alpha_1, \alpha_2 \in \mathbb{R}$:

$$\forall x \in \mathbb{R}: \alpha_1 \cdot (1 + x) + \alpha_2 (1 - x) = 0$$

$$\Updownarrow$$

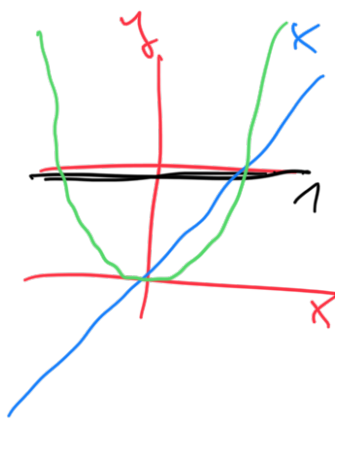
$$\forall x \in \mathbb{R}: \underbrace{(\alpha_1 + \alpha_2)}_{\beta_1} \cdot 1 + \underbrace{(\alpha_1 - \alpha_2)}_{\beta_2} \cdot x = 0$$

$\Updownarrow$ $1, x$ jsou LN

Jediné řešení je $\beta_1 = \beta_2 = 0$

tj.:

$$\boxed{\begin{aligned} \alpha_1 + \alpha_2 &= 0 \\ \alpha_1 - \alpha_2 &= 0 \end{aligned}}$$



Jediné řeš.

$\alpha_1 = 0$

$$\begin{pmatrix} 1 & 1 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 \end{pmatrix}$$

$\left(\begin{array}{cc|c} 1 & -1 & 0 \end{array}\right) -r_1 \sim \left(\begin{array}{cc|c} 0 & -2 & 0 \end{array}\right) \Leftrightarrow \underline{\underline{\alpha_2 = 0}}$

<u>Ano</u>, $p_1(x) = 1 + x$ a $p_2(x) = 1 - x$ jsou <u>lineárně nezávislé!</u>

<u>Báze vektorového prostoru</u>

Př. Najděte bázi $V := \{\bar{x} \in \mathbb{R}^2 : \boxed{x_1 + x_2 = 0}\}$

$= N(\underbrace{(1, 1)}_{A}) \overset{?}{=} S(\mathbb{B})$

$\mathbb{B} \cdot \bar{\alpha}$

$\underbrace{(b_1^s, \dots, b_n^s)}_{\text{báze}} \cdot \bar{\alpha} = 0$

~~Porovnej~~

$\boxed{x_1 + x_2 = 0}$

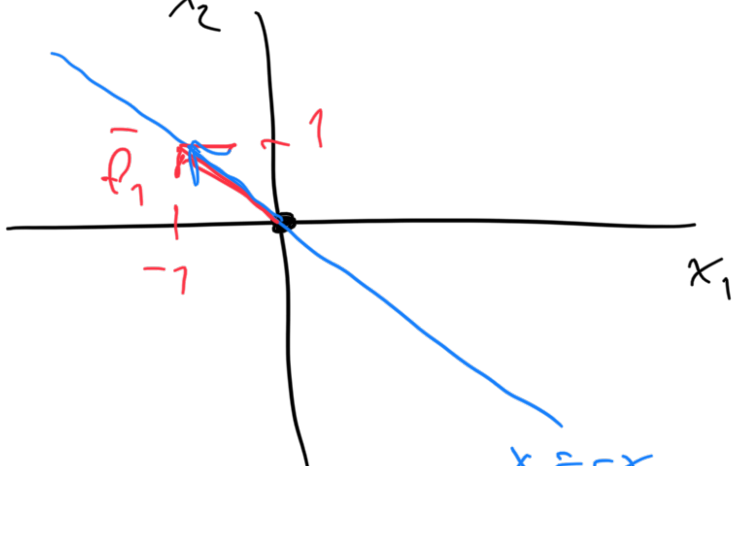
$x_2 = t \in \mathbb{R}$

$x_1 = -t$

$V = \left\{ \alpha \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} : \alpha \in \mathbb{R} \right\}$

báze $F := \left(\underbrace{\begin{pmatrix} -1 \\ 1 \end{pmatrix}}_{\bar{f}_1} \right)$ prostoru V

$\underline{\underline{\dim V = 1}}$



$a_2 \quad a_1$

Př: Najděte bázi $V := \{ p(x) = a_0 + a_1 x + a_2 x^2 \in \mathcal{P}_2 \mid a_0, a_1, a_2 \in \mathbb{R},\ \boxed{a_0 + a_1 + a_2 = 0,\ a_1 - a_2 = 0} \}$

$$\begin{array}{ccc} a_0 & a_1 & a_2 \end{array}$$
$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right)$$

$N(A)$ $\quad A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

$a_2 = t \in \mathbb{R}$

Z 2. rov.: $a_1 = t$

Z 1. rov.: $a_0 = -a_1 - a_2 = -2t$

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \bar{a} = t \cdot \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$$

$$V = \{ p(x) = \underbrace{-2t}_{a_0} + \underbrace{t}_{a_1} \cdot x + \underbrace{t}_{a_2} \cdot x^2 \in \mathcal{P}^2 : t \in \mathbb{R} \}$$

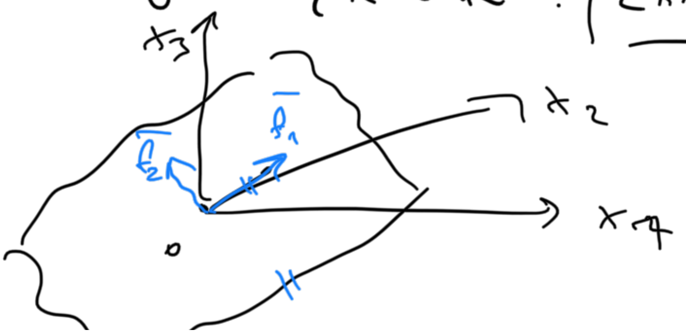
$$= \{ t \cdot (-2 + x + x^2) \in \mathcal{P}^2 : t \in \mathbb{R} \}$$

Báze V je (např.) $F = (f_1(x) := -2 + x + x^2)$

$\dim V = 1 \leftarrow$ jediný bázový vektor

Př: Najděte bázi a dimenzi

$$V := \{ \bar{x} \in \mathbb{R}^3 : \boxed{2x_1 - x_2 + x_3 = 0} \}$$



$$= N((2, -1, 1))$$

?

$$\left(\begin{array}{ccc|c} 2, & -1, & 1 & 0 \end{array}\right)$$

Parametrizace: $x_3 = t \in \mathbb{R}$

Dosadím do rovnice: $2x_1 - x_2 + t = 0$

$x_2 = s \in \mathbb{R}$

Dosadím do rovnice: $2x_1 - s + t = 0$

$x_1 = +\frac{1}{2}s - \frac{1}{2}t$

$$V = \left\{ \bar{x} = \begin{pmatrix} \frac{1}{2}s - \frac{1}{2}t \\ 1s + 0t \\ 0s + 1t \end{pmatrix} : s, t \in \mathbb{R} \right\}$$

$$= \left\{ \bar{x} = s \cdot \begin{pmatrix} 1/2 \\ 1 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} -1/2 \\ 0 \\ 1 \end{pmatrix} : s, t \in \mathbb{R} \right\}$$

$$= \left\{ \begin{pmatrix} 1/2, & -1/2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} s \\ t \end{pmatrix} : s, t \in \mathbb{R} \right\} = S\left(\begin{pmatrix} 1/2, & -1/2 \\ 1, & 0 \\ 0, & 1 \end{pmatrix}\right)$$

Báze je $F := \left(\bar{f}_1 := \begin{pmatrix} 1/2 \\ 1 \\ 0 \end{pmatrix}, \bar{f}_2 := \begin{pmatrix} -1/2 \\ 0 \\ 1 \end{pmatrix} \right)$

$\dim V = 2$

$1, x, x^2$

Př. Najděte bázi $V := \{ p(x) := a_0 + a_1 x + a_2 x^2 \in \mathcal{P}_2 : \boxed{a_0 + 2a_1 = a_2} \}$

$= N((1, 2, -1))$

$(1, 2, -1 \mid 0)$

Parametrizace: $a_2 = t \in \mathbb{R}$

$a_1 = s \in \mathbb{R}$

Dosadím: $a_0 + 2s - t = 0 \iff a_0 = -2s + t$

$$U = \{ (-2s + t) + s x + t x^2 \in \mathcal{P}_2 : s, t \in \mathbb{R} \}$$

$$= \{ s \cdot (\underline{-2 + x}) + t \cdot (\underline{1 + x^2}) : s, t \in \mathbb{R} \}$$

Báze $F := (f_1(x) := -2 + x,\ f_2 := 1 + x^2)$,

$\dim U = 2$.

Př. Spočtěte souřadnice $[1 + x + x^2]_F \in \mathbb{R}^3$ v bázi $F := (f_1(x) := 1 - x,\ f_2(x) := 1 + x^2,\ f_3(x) := x^2)$

Hledáme $\bar{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \in \mathbb{R}^3$:

$$\boxed{\forall x \in \mathbb{R} : \alpha_1 (1 - x) + \alpha_2 (1 + x^2) + \alpha_3 (x^2) = 1 + 1x + 1x^2}$$

$$\Updownarrow$$

$$\forall x \in \mathbb{R} : \underbrace{(\alpha_1 + \alpha_2 - 1)}_{\beta_1} \cdot 1 + \underbrace{(-\alpha_1 - 1)}_{\beta_2} \cdot x + \underbrace{(\alpha_2 + \alpha_3 - 1)}_{\beta_3} x^2 = 0$$

$$\Updownarrow \quad 1, x, x^2 \text{ jsou LN}$$

$$\beta_1 = \beta_2 = \beta_3 = 0$$

$$\begin{array}{c} \\ 1 : \\ x : \\ x^2 : \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{array} \right) \Leftrightarrow \underline{\alpha_1 = -1}$$

$-1 + \alpha_2 = 1$

$\underline{\alpha_2 = 2}$

$2 + \alpha_3 = 1$

$\underline{\alpha_3 = -1}$

Souřadnice $[1 + x + x^2]_F = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$.

Př. ...

vypočtěte souřadnice $[\begin{pmatrix}1\\2\end{pmatrix}]_F$, $[\begin{pmatrix}1\\-1\end{pmatrix}]_F$ v bázi $F := \left(\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}\right)$, $\begin{pmatrix}1\\2\end{pmatrix} = \bar{x}$, $\begin{pmatrix}1\\-1\end{pmatrix} = \bar{y}$

$$
\begin{array}{cc|cc} \bar{f}_1 & \bar{f}_2 & \bar{x} & \bar{y} \end{array}
$$

$$
\left(\begin{pmatrix}\boxed{1}\\1\end{pmatrix} \begin{pmatrix}1\\0\end{pmatrix} \middle| \begin{pmatrix}1\\2\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}\right) - \bar{r}_1 \sim \left(\begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ 0 & \boxed{-1} & 1 & -2 \end{array}\right) + \bar{r}_2
$$

$$
\sim \left(\begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & -1 & 1 & -2 \end{array}\right) / \cdot(-1)
$$

$$
\sim \left(I \middle| \underbrace{\begin{pmatrix}2\\-1\end{pmatrix}}_{[\bar{x}]_F} \underbrace{\begin{pmatrix}-1\\+2\end{pmatrix}}_{[\bar{y}]_F} \right)
$$

Souvislost báze, dimenze s řešením soustav lin. rovnic

Př.:

$$
\boxed{\begin{aligned} x_1 + x_2 &= -1 \\ x_1 - x_2 &= +1 \\ 2x_1 + 4x_2 &= -4 \end{aligned}}
$$

$$
\underbrace{\begin{array}{cc} x_1 & x_2 \\ \boxed{1} & 1 \\ 1 & -1 \\ 2 & 4 \end{array}}_{A}
\left|\;\underbrace{\begin{array}{c} \\ -1 \\ 1 \\ -4 \end{array}}_{\bar{b}}\right.
\begin{array}{l} \\ \\ -\bar{r}_1 \\ -2\bar{r}_1 \end{array}
\iff
\boxed{\bar{b} \in S(A)} \iff \exists \bar{x} : A \cdot \bar{x} = \bar{b}
$$

$$
A \cdot \bar{x} = \bar{b} \iff \boxed{x_1 \bar{a}_1^s + x_2 \bar{a}_2^s = \bar{b}}
$$

$$
x_1 \begin{pmatrix}1\\1\\2\end{pmatrix} + x_2 \begin{pmatrix}1\\-1\\4\end{pmatrix} = \begin{pmatrix}-1\\1\\4\end{pmatrix}
$$

$$\sim\left(\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & -2 & 2 \\ \cancel{0} & \cancel{2} & \cancel{-2} \end{array}\right) + r_2 \qquad \begin{array}{l} x_1 = 0 \\ x_2 = -1 \end{array}$$

Tj. $\begin{pmatrix} -1 \\ 1 \\ -4 \end{pmatrix} = 0 \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} - 1 \cdot \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \in S(A)$

Soustava lin. rovnic $A \cdot \bar{x} = \bar{b}$

- má alespoň 1 řešení, pokud $\bar{b} \in S(A)$
- nemá řešení, pokud $\underline{\bar{b} \notin S(A)}$.

$A \cdot \bar{x} = \bar{b}$, $\bar{b} \in S(A)$

$$\left.\begin{array}{l} A \cdot \bar{x}_p = \bar{b} \\ A \cdot \bar{x}_H = \bar{o} \end{array}\right\} \Rightarrow \boxed{\bar{x} = \overline{\bar{x}_p + N(A)}}$$

$\bar{x}_H$

$N(A)$

Počet parametrů = $\dim N(A)$

$= \{ t_1 \bar{n}_1 + \dots + t_d \cdot \bar{n}_d + \bar{x}_p :$ $\bar{n}_1, \dots, \bar{n}_d$ je báze $N(A)$

Zobecněná Gauss-Jordanova metoda

Př: $\boxed{x_1 + x_2 - 2x_3 = 3}$

$$\left(\begin{array}{ccc|c} \boxed{1} & \boxed{1 \;\; -2} & & \boxed{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

I_1 $\qquad$ $= F$

$(A \mid \bar{b}) \sim (I, F \mid \tilde{x}_p)$ $\qquad$ $A \cdot \bar{x} = \bar{o}$ $\Updownarrow$

$(I, F) \cdot \bar{x}_H = \bar{o}$

$$\mathbb{N} = \begin{pmatrix} -F \\ I_2 \end{pmatrix} = \begin{pmatrix} -1 & +2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\bar{x}_p$

báze $N(A)$

$$(I, F) \cdot \bar{x}_H = \bar{o}$$

$$\begin{pmatrix} -F \\ I_2 \end{pmatrix}$$

$$\boxed{I \cdot (-F) + F \cdot I_2 = \mathbb{O}}$$

$$(1,1,-2) \cdot \begin{pmatrix} -F \\ I_2 \end{pmatrix} \overset{?}{=} (0, 0)$$

$$(1,1,-2) \cdot \begin{pmatrix} -1 & +2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} = \left((1,1,-2) \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, (1,1,-2) \cdot \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right)$$

$\in N(A)$ $\in N(A)$ $= (0 \;,\; 0)$ ✓ ✓

$$\left(\boxed{1}, \underbrace{\boxed{1, -2}}_{F} \Big| \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \right)$$

$\bar{x}_p$

$$\mathbb{N} = \begin{pmatrix} -1 & +2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$-F$

Řešení je $\boxed{\bar{x} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}}$

Př.:

$$x_1 - x_2 + x_3 = 1$$

$$x_1 + x_2 - x_3 = 0$$

$$\left(\begin{array}{ccc|c} \boxed{1} & -1 & 1 & 1 \\ 1 & 1 & -1 & 0 \end{array} \right) \begin{array}{c} \\ -r_1 \end{array} \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 2 & -2 & -1 \end{array} \right)$$

$\tilde{r}_1 := 2\tilde{r}_1 + \tilde{r}_2$

$$\sim \left(\begin{array}{ccc|c} 2 & 0 & 0 & 1 \\ 0 & 2 & -2 & -1 \end{array}\right) \begin{array}{c} 1/2 \\ 1/2 \end{array} \sim \left(I_2 \; \underbrace{\begin{pmatrix} 0 \\ -2 \end{pmatrix}}_{F} \;\middle|\; \underbrace{\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}}_{\bar{x}_p} \right)$$

$$N(A) = \begin{pmatrix} -0 \\ +2 \\ \hline 1 \end{pmatrix}$$

$$\bar{x} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, \quad t \in \mathbb{R}$$