Vektorové prostory

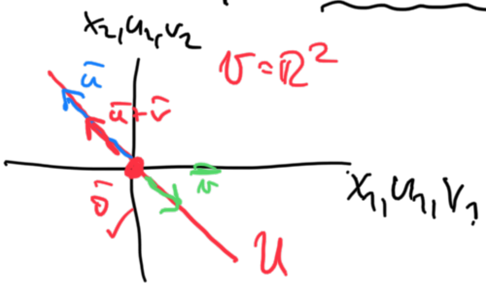
Př. Rozhodněte, zda $U := \{\bar{x} \in \mathbb{R}^2 : x_1 + x_2 = 0\}$ je vektorovým podprostorem $\mathbb{R}^2 = (V)$ ✓

0. $U \subset V$ ✓

$\bar{x} = (0,0)$ ✓ $: x_1 + x_2 = 0 + 0 = 0$ ✓ $\Rightarrow U \neq \emptyset := \{\}$

1. Ptám se, zda platí: $\bar{u}, \bar{v} \in U \overset{?}{\Rightarrow} \bar{u} + \bar{v} \in U$

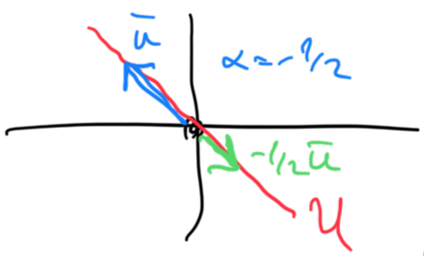
$$\left.\begin{array}{l} \bar{u} \in U \text{ tj. } u_1 + u_2 = 0 \\ \bar{v} \in U \text{ tj. } v_1 + v_2 = 0 \end{array}\right\} \overset{?}{\Longrightarrow} \bar{u} + \bar{v} \in U \text{ tj. } \boxed{(\bar{u}+\bar{v})_1 + (\bar{u}+\bar{v})_2 \overset{?}{=} 0}$$



$$(\bar{u}+\bar{v})_1 + (\bar{u}+\bar{v})_2 = (u_1 + v_1) + (u_2 + v_2) = u_1 + u_2 + v_1 + v_2 = 0 + 0 = 0$$

2. Ptám se, zda platí: $\alpha \in \mathbb{R}, \bar{u} \in U \Rightarrow \alpha \cdot \bar{u} \in U$ ✓

$$\left.\begin{array}{l} \alpha \in \mathbb{R} \\ \bar{u} \in U \text{ tj. } u_1 + u_2 = 0 \end{array}\right\} \Rightarrow (\alpha \cdot \bar{u})_1 + (\alpha \cdot \bar{u})_2 \overset{?}{=} 0$$



$$(\alpha \cdot \bar{u})_1 + (\alpha \cdot \bar{u})_2 = (\alpha u_1) + (\alpha u_2) = \alpha(u_1 + u_2) = \alpha \cdot 0 = 0 \;✓$$

Ano, U je vektorový podprostor V.

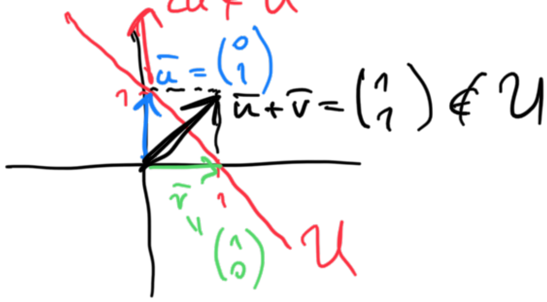
Př. Je $U := \{\bar{x} \in \mathbb{R}^2 : x_1 + x_2 = 1\}$ je podprostorem $\mathbb{R}^2$

... je podprostor $\mathbb{R}$ ...

Podprostor je vekt. prostorový prostor, který musí obsahovat nulový vektor $\bar{0} = (0,0)$!

$0 + 0 = 0 \neq 1$

<u>Ne</u>, U není podprostorem V.



$\boxed{\mathbb{R}^2} := \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} : x_1, x_2 \in \mathbb{R} \right\}$ $\qquad \mathbb{C}^2 := \left\{ \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} : z_i \in \mathbb{C}\right.$

$\boxed{\mathbb{R}^3} := \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} : x_1, x_2, x_3 \in \mathbb{R} \right\}$

$\boxed{P_n} := \left\{ p(x) = a_0 + a_1 x + \dots + a_n x^n : \mathbb{R} \to \mathbb{R} \mid a_0, a_1, \dots, a_n \in \mathbb{R} \right.$