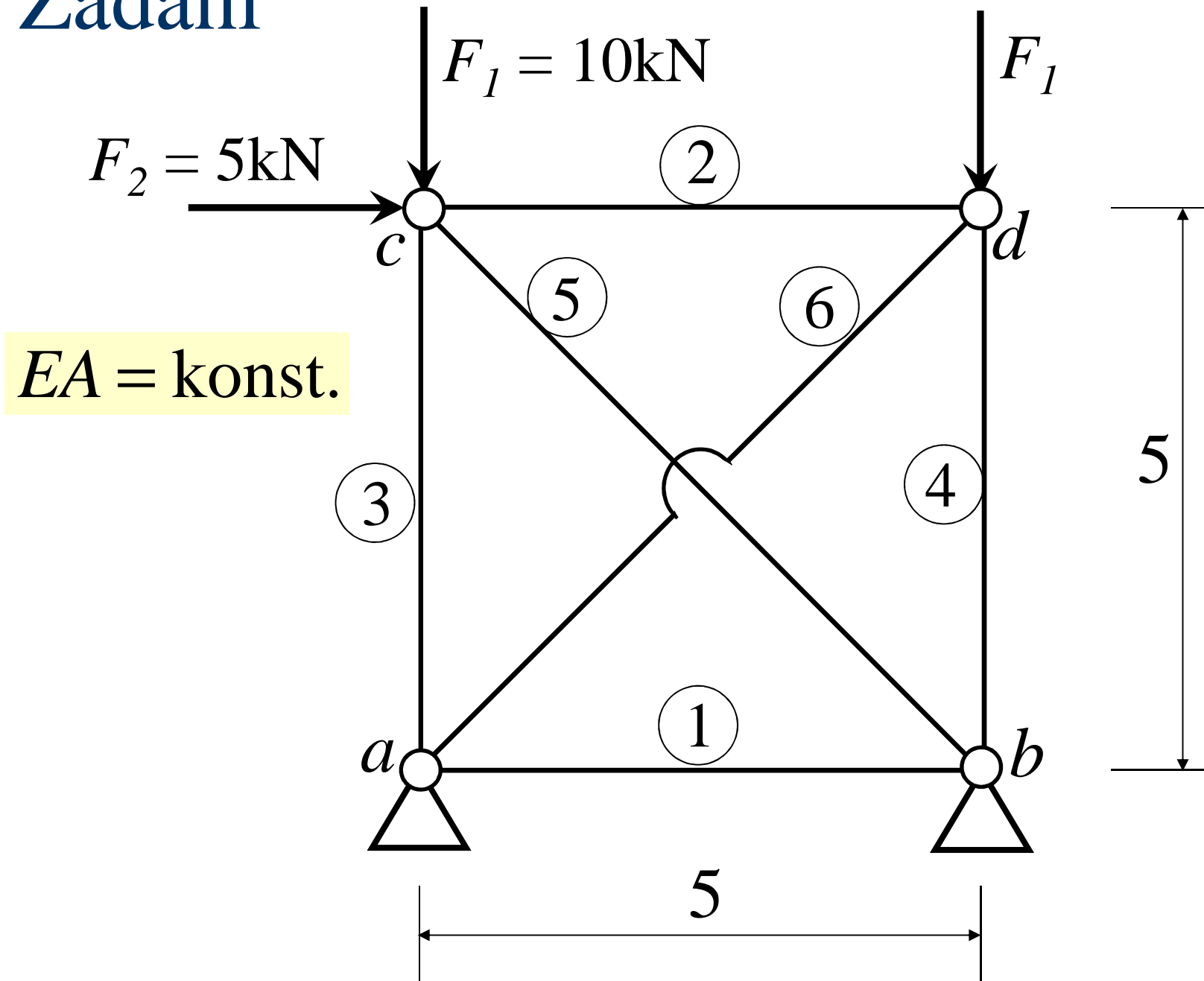


# Statically indeterminate plane frame structure

- solution by the force method

# Zadání



| Prut     | $l$ [m] | $N_0$   | $N_1$ | $N_2$  | $N_0N_1l$ | $N_0N_2l$ | $N_1N_1l$ | $N_1N_2l$ | $N_2N_2l$ |
|----------|---------|---------|-------|--------|-----------|-----------|-----------|-----------|-----------|
| 1        | 5,000   | 5,000   | 1,000 | -0,707 | 25,000    | -17,678   | 5,000     | -3,536    | 2,500     |
| 2        | 5,000   | 0,000   | 0,000 | -0,707 | 0,000     | 0,000     | 0,000     | 0,000     | 2,500     |
| 3        | 5,000   | -5,000  | 0,000 | -0,707 | 0,000     | 17,678    | 0,000     | 0,000     | 2,500     |
| 4        | 5,000   | -10,000 | 0,000 | -0,707 | 0,000     | 35,355    | 0,000     | 0,000     | 2,500     |
| 5        | 7,071   | -7,071  | 0,000 | 1,000  | 0,000     | -50,000   | 0,000     | 0,000     | 7,071     |
| $\Sigma$ |         |         |       |        | 25,000    | -14,645   | 5,000     | -3,536    | 17,071    |

$$\delta_{10} = \sum_{i=1}^5 \frac{N_{1,i}N_{0,i}l_i}{EA} = \frac{25}{EA}$$

$$\delta_{20} = \sum_{i=1}^5 \frac{N_{2,i}N_{0,i}l_i}{EA} = \frac{-14,645}{EA}$$

$$\delta_{11} = \sum_{i=1}^5 \frac{N_{1,i}N_{1,i}l_i}{EA} = \frac{5}{EA}$$

$$\delta_{12} = \delta_{21} = \sum_{i=1}^5 \frac{N_{1,i}N_{2,i}l_i}{EA} = -\frac{3,536}{EA}$$

$$\delta_{22} = \sum_{i=1}^5 \frac{N_{2,i}N_{2,i}l_i}{EA} = \frac{17,071}{EA}$$

$$\delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 = 0$$

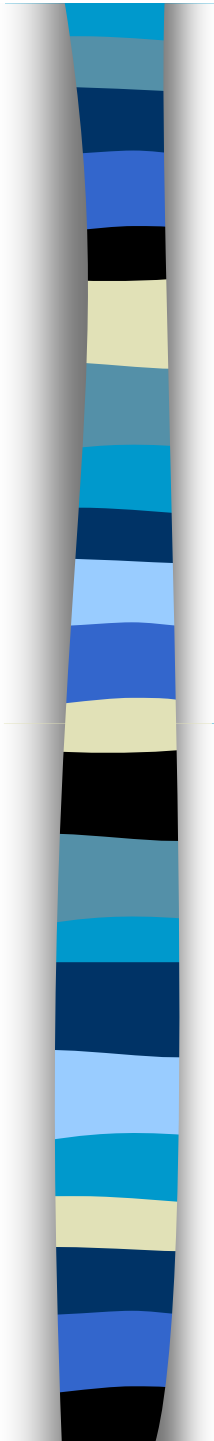
$$\delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 = 0$$

$$25 + 5X_1 - 3,536X_2 = 0$$

$$-14,645 - 3,536X_1 + 17,071X_2 = 0$$

$$X_1 = R_{bx} = -5,15 \text{ kN}$$

$$X_2 = N_6 = -0,21 \text{ kN}$$



$$R_{ax} = 0,15 \text{ kN } (\rightarrow)$$

$$R_{bx} = 5,15 \text{ kN } (\leftarrow)$$

$$R_{az} = 5 \text{ kN } (\uparrow)$$

$$R_{bz} = 15 \text{ kN } (\uparrow)$$

$$N_i = N_{i,0} + N_{i,1}X_1 + N_{i,2}X_2$$

| Prut | $N$ [kN] | tah/tlak |
|------|----------|----------|
| 1    | 0,00     | -        |
| 2    | 0,15     | tah      |
| 3    | -4,85    | tlak     |
| 4    | -9,85    | tlak     |
| 5    | -7,28    | tlak     |
| 6    | -0,21    | tlak     |

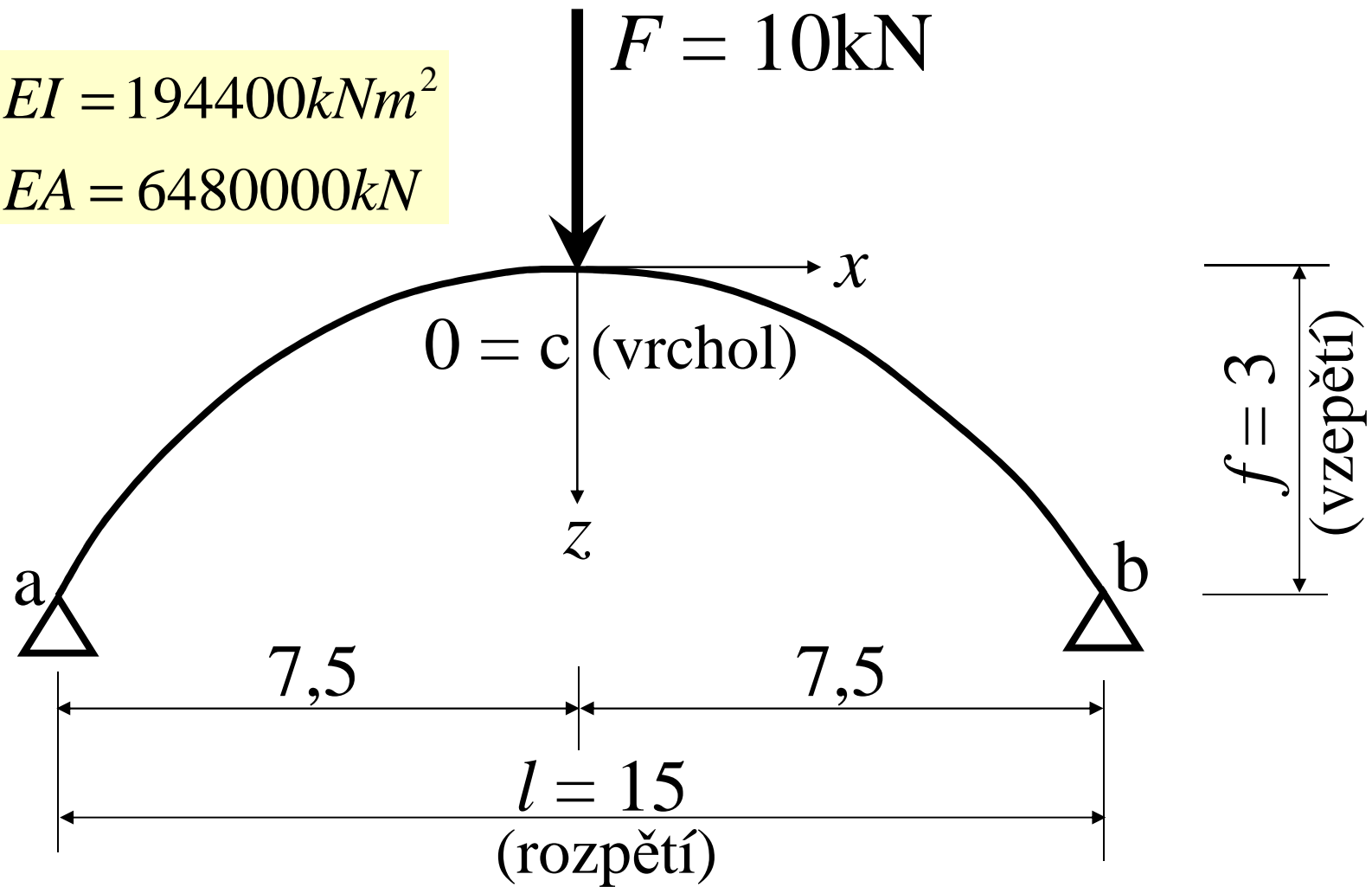
# Statically indeterminate planar arch

- solution by force method
- numerical solution

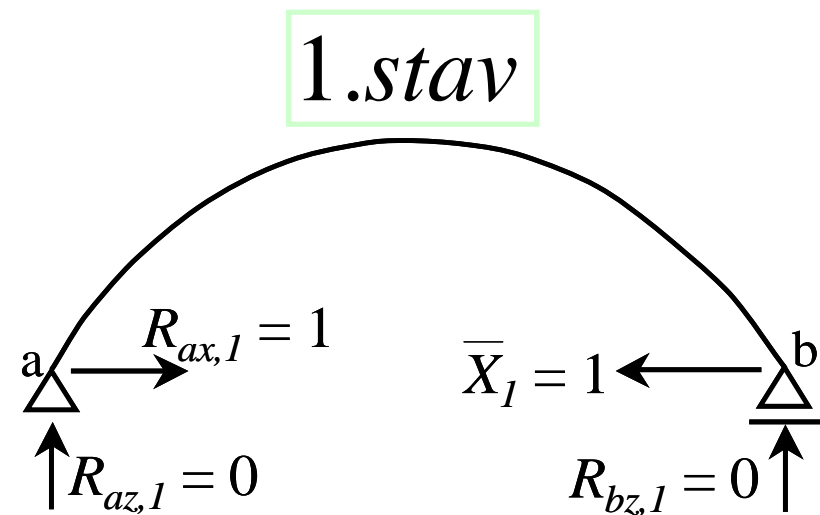
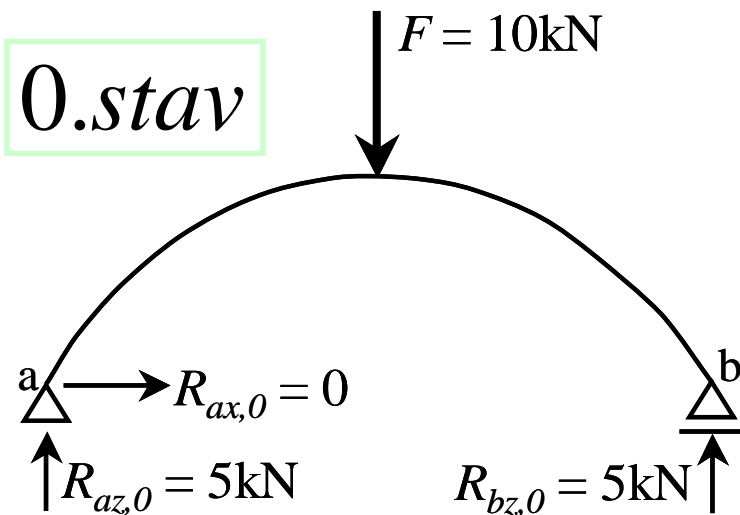
# Zadání

$$EI = 194400 \text{ kNm}^2$$

$$EA = 6480000 \text{ kN}$$



# Zatěžovací stavy



$$V = S \cdot \cos \psi - H \cdot \sin \psi$$

$$N = H \cdot \cos \psi + S \cdot \sin \psi$$



# Analýza střednice

- rovnice paraboly

$$z = k \cdot x^2 = 0,0533x^2$$

$$k = \frac{z_a}{x_a^2} = \frac{z_b}{x_b^2} = \frac{3}{7,5^2} = 0,0533m^{-1}$$

- sklon střednice  $\psi$

$$tg \psi = \frac{dz}{dx} = 2 \cdot k \cdot x = 0,1067x$$



# Numerické řešení

| i | x     | z    | tgψ   | sinψ  | cosψ | M <sub>0</sub> | N <sub>0</sub> | M <sub>1</sub> | N <sub>1</sub> | M <sub>1</sub> <sup>2</sup> /cos | N <sub>1</sub> <sup>2</sup> /cos | M <sub>0</sub> M <sub>1</sub> /cos | N <sub>0</sub> N <sub>1</sub> /cos |
|---|-------|------|-------|-------|------|----------------|----------------|----------------|----------------|----------------------------------|----------------------------------|------------------------------------|------------------------------------|
| 0 | -7,50 | 3,00 | -0,80 | -0,62 | 0,78 | 0,00           | -3,12          | 0,00           | -0,78          | 0,00                             | 0,78                             | 0,00                               | 3,12                               |
| 1 | -5,00 | 1,33 | -0,53 | -0,47 | 0,88 | 12,50          | -2,35          | -1,67          | -0,88          | 3,15                             | 0,88                             | -23,61                             | 2,35                               |
| 2 | -2,50 | 0,33 | -0,27 | -0,26 | 0,97 | 25,00          | -1,29          | -2,67          | -0,97          | 7,36                             | 0,97                             | -69,00                             | 1,29                               |
| 3 | 0,00  | 0,00 | 0,00  | 0,00  | 1,00 | 37,50          | 0,00           | -3,00          | -1,00          | 9,00                             | 1,00                             | -112,50                            | 0,00                               |
| 4 | 2,50  | 0,33 | 0,27  | 0,26  | 0,97 | 25,00          | -1,29          | -2,67          | -0,97          | 7,36                             | 0,97                             | -69,00                             | 1,29                               |
| 5 | 5,00  | 1,33 | 0,53  | 0,47  | 0,88 | 12,50          | -2,35          | -1,67          | -0,88          | 3,15                             | 0,88                             | -23,61                             | 2,35                               |
| 6 | 7,50  | 3,00 | 0,80  | 0,62  | 0,78 | 0,00           | -3,12          | 0,00           | -0,78          | 0,00                             | 0,78                             | 0,00                               | 3,12                               |
|   |       |      |       |       |      |                |                |                |                | <b>30,02</b>                     | <b>6,26</b>                      | <b>-297,71</b>                     | <b>13,53</b>                       |

$$\delta_{10} = \sum_{i=0}^6 \frac{M_{1,i}M_{0,i}}{EI \cdot \cos \psi_i} + \sum_{i=0}^6 \frac{N_{1,i}N_{0,i}}{EA \cdot \cos \psi_i} = \frac{-297,71}{EI} + \frac{13,53}{EA} = -0,00153$$

$$\delta_{11} = \sum_{i=0}^6 \frac{M_{1,i}M_{1,i}}{EI \cdot \cos \psi_i} + \sum_{i=0}^6 \frac{N_{1,i}N_{1,i}}{EA \cdot \cos \psi_i} = \frac{30,02}{EI} + \frac{6,26}{EA} = 0,00016$$

$$X_1 = R_{bx} = -\frac{\delta_{10}}{\delta_{11}} = \underline{\underline{9,84kN(\leftarrow)}}$$