

Př: Nalezněte řešení rovnice

a) $4(x+4) + 6(x-1) = 12 + 3(x+4)$

$$4x + 16 + 6x - 6 = 12 + 3x + 12$$

$$7x - 14 = 0$$

 $ax+b=0$ Lineární rovnice

$$7x = 14$$

$$x = 2$$

{2}

zk: $L = 4 \cdot (2+4) + 6 \cdot (2-1) = 24 + 6 = 30$

$$= 24 + 6 = 30$$

$$P = 12 + 3(2+4) = 12 + 18 = 30 \quad L=P$$

c) $|x-3| = 9$ N.B.: 3

$$\begin{array}{c} - \\ 3 \\ + \end{array}$$

$$-x+3=9$$

$$-x=6$$

$$x=-6$$

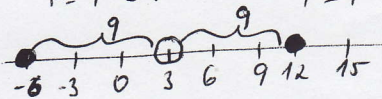
$$x-3=9$$

$$x=12$$

{-6, 12}

zk: $x=-6: L = |-6-3| = 9$
 $P = 9 \quad L=P$

$x=12: L = |12-3| = 9$
 $P = 9 \quad L=P$

Př: Nalezněte řešení kvadratické rovnice $ax^2+bx+c=0$.

a) $2x^2 + 11x + 12 = 0$

$$D = b^2 - 4ac = 11^2 - 4 \cdot 2 \cdot 12 = 121 - 96 = 25$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-11 \pm \sqrt{25}}{2 \cdot 2} = \frac{-11 \pm 5}{4}$$

$$x_1 = -\frac{3}{2}, x_2 = -4$$

zk: $2 \cdot (-\frac{3}{2})^2 + 11 \cdot (-\frac{3}{2}) + 12 = \frac{9}{2} - \frac{33}{2} + \frac{24}{2} = 0$

$$2 \cdot (-4)^2 + 11 \cdot (-4) + 12 = 32 - 44 + 12 = 0$$

Součin kořenových činitelů:
 $2 \cdot (x + \frac{3}{2}) \cdot (x + 4) = 2x^2 + 11x + 12$

b) $5x^2 - 2x + 11 = 0$

$$D = 4 - 4 \cdot 5 \cdot 11 = 4 - 220 = -216 < 0 \Rightarrow \text{nema' řešení v } \mathbb{R}$$

c) v $\mathbb{C}$ řešte $x^2 - 2x + 5 = 0$

$$D = 4 - 4 \cdot 5 = -16 < 0$$

$$x_{1,2} = \frac{-b \pm i\sqrt{|D|}}{2a} = \frac{2 \pm i\sqrt{16}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$$

$$x_1 = 1+2i, x_2 = 1-2i$$

$$x^2 - 2x + 5 = (x-1-2i)(x-1+2i)$$

Př: Nalezněte řešení racionální rovnice

a) $\sqrt{x-5} = 2$

$$x-5=4$$

$$x=9$$

zk: $L = \sqrt{9-5} = 2$

$$P=2 \quad L=P$$

b) $4\sqrt{3x+1} - 3\sqrt{5x-1} = 2$

$$16(3x+1) - 2 \cdot 4\sqrt{3x+1} \cdot 3\sqrt{5x-1} + 9(5x-1) = 4$$

$$-24\sqrt{3x+1}\sqrt{5x-1} = 4 - 48x - 16 - 45x + 9 \quad | :(-3)$$

$$8\sqrt{3x+1}\sqrt{5x-1} = 31x+1 \quad |^2$$

$$64(3x+1)(5x-1) = 31^2x^2 + 62x + 1$$

$$x^2 - 66x + 65 = 0$$

$$x_1 = 1 \quad x_2 = 65$$

{1, 65}

zk: $4\sqrt{3 \cdot 1 + 1} - 3\sqrt{5 \cdot 1 - 1} = 2; 4\sqrt{196} - 3\sqrt{324} = 2$

d) $\sqrt{x+5} + \sqrt{x-1} = 1$

$$(x+5) + 2\sqrt{x+5}\sqrt{x-1} + (x-1) = 1$$

$$2\sqrt{x+5}\sqrt{x-1} = 2x-3 \quad |^2$$

$$4(x+5)(x-1) = 4x^2 + 12x + 9$$

$$4x^2 + 5x - x - 5 = 4x^2 + 12x + 9$$

$$4x - 29 = 0$$

$$x = \frac{29}{4}$$

nemá řešení

zk: $L = \sqrt{\frac{29}{4}+5} + \sqrt{\frac{29}{4}-1} = \sqrt{\frac{49}{4}} + \sqrt{\frac{25}{4}} = \frac{7}{2} + \frac{5}{2} = 6$

$$P=1 \quad L \neq P$$

 $\emptyset$

e) $\sqrt{5-\sqrt{x+1}+\sqrt{2x^2+x+3}} = 1$

$$5 - \sqrt{x+1} + \sqrt{2x^2+x+3} = 1$$

$$-\sqrt{x+1} + \sqrt{2x^2+x+3} = -4 \quad |^2$$

$$x+1 + \sqrt{2x^2+x+3} = 16$$

$$\sqrt{2x^2+x+3} = -x+15 \quad |^2$$

$$2x^2+x+3 = x^2-30x+225$$

$$x^2+31x-222=0$$

$$x_1 = 6 \quad x_2 = -37$$

zk: $x_1=6: L = \sqrt{5-\sqrt{7}+\sqrt{72+9}} = \sqrt{5-\sqrt{16}} = 1$

$$P=1 \quad L=P$$

$x_2=-37: L = \sqrt{5-\sqrt{-36}+\sqrt{2 \cdot 37^2-34}} = \sqrt{5-\sqrt{16}} = 1$

$$P=1 \quad L=P$$

{6, -37}