

Základní integrály

$$[1.] \int 0 \, dx = C$$

$$[2.] \int 1 \, dx = x + C$$

$$[3.] \int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad \text{pro } n \in \mathbb{Q}, n \neq -1, x > 0$$

$$[4.] \int \frac{1}{x} \, dx = \ln|x| + C \quad \text{pro } x \neq 0$$

$$[5.] \int \sin x \, dx = -\cos x + C$$

$$[6.] \int \cos x \, dx = \sin x + C$$

$$[7.] \int \frac{1}{\cos^2 x} \, dx = \operatorname{tg} x + C \quad \text{pro } x \neq (2k+1)\frac{\pi}{2}, k \in \mathbb{Z}$$

$$[8.] \int \frac{1}{\sin^2 x} \, dx = -\operatorname{cotg} x + C \quad \text{pro } x \neq k\pi, k \in \mathbb{Z}$$

$$[9.] \int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + C \quad \text{pro } x \in (-1,1)$$

$$[10.] \int \frac{1}{1+x^2} \, dx = \operatorname{arctg} x + C$$

$$[11.] \int a^x \, dx = \frac{a^x}{\ln a} + C \quad \text{pro } a > 0, a \neq 1$$

$$[12.] \int e^x \, dx = e^x + C$$

$$[13.] \int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + C$$

$$[14.] \int \frac{dx}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C$$

$$[15.] \int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C \quad \text{pro } x \in (-a, a)$$

$$[16.] \int f(ax+b) \, dx = \frac{1}{a} F(ax+b) + C$$

Metoda per partes

$$\int u(x) \cdot v'(x) \, dx = u(x) \cdot v(x) - \int u'(x) \cdot v(x) \, dx$$

Univerzální goniometrická substituce:

$$\operatorname{tg} \frac{x}{2} = t \quad x = 2 \operatorname{arctg} t \quad dx = \frac{2}{1+t^2} dt \quad \sin x = \frac{2t}{1+t^2} \quad \cos x = \frac{1-t^2}{1+t^2}$$

Aplikace určitého integrálu

$$S = \int_a^b |f(x) - g(x)| dx$$

$$\ell = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

$$V = \pi \int_a^b |f^2(x) - g^2(x)| dx$$

$$S = \int_{t_1}^{t_2} |y \cdot \dot{x}| dt, \quad t \in \langle t_1, t_2 \rangle$$

$$\ell = \int_{t_1}^{t_2} \sqrt{(\dot{x})^2 + (\dot{y})^2} dt, \quad t \in \langle t_1, t_2 \rangle$$

$$V = \pi \int_{t_1}^{t_2} y^2 \cdot |\dot{x}| dt, \quad t \in \langle t_1, t_2 \rangle$$

Vázané extrémny

$$\Phi(x, y, \lambda) = F(x, y) + \lambda \cdot G(x, y)$$

$$J = \begin{vmatrix} \Phi'_{xx}(x_0, y_0, \lambda) & \Phi'_{yx}(x_0, y_0, \lambda) \\ \Phi'_{xy}(x_0, y_0, \lambda) & \Phi'_{yy}(x_0, y_0, \lambda) \end{vmatrix}$$

Přehled základních goniometrických vzorců

$$\sin^2 x + \cos^2 x = 1$$

$$\sin 2x = 2 \sin x \cos x,$$

$$\sin^2 x = \frac{1}{2} (1 - \cos 2x),$$

$$\operatorname{tg} x \cdot \operatorname{cotg} x = 1,$$

$$\cos 2x = \cos^2 x - \sin^2 x,$$

$$\cos^2 x = \frac{1}{2} (1 + \cos 2x).$$