FULL SPECTRA AS A TOOL FOR ANALYSIS OF A SHAFT ROTATING IN JOURNAL BEARINGS

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Abstract: (In English) The topic of the paper deals with full spectrum evaluation. The full spectrum plays a key role in the processing of complex signals. The real part of this kind of signals is a displacement in the X-direction while the imaginary part is a displacement in the perpendicular direction, as we say in the Y-direction. A good example is a shaft rotating in the journal bearing. The shaft displacements can be detected by proximity probes. The spectrum analysis results in full spectra. This analysis technique is demonstrated on the example of the fluid induced shaft.

Key words: main English key words (according to their importance)

1. Introduction

Full spectrum plots have recently received a great importance in rotors and journal bearings diagnostics thanks to research work that was done at Bently Rotor Dynamics Research Corporation and Bently Nevada Corporation. Related work done in Korea and the People's Republic of China must be mentioned [1, 2]. This article discusses the benefits of full spectra and how to use full spectrum plots in a machinery diagnostic analysis for fluid-induced instabilities.

In contrast to the frequency spectrum of a simple time signal the full spectrum is a tool for processing a two-coordinate signal. The one-coordinate signal gives information about motion along a straight line as for instance acceleration signal while the two-coordinate signal describes the motion of a point in a plane. The topic of this paper is focused on the motion of shafts in journal bearings. Measurement instrumentation is shown in figure 1. Proximity probes are a non-contacting device, which measures the displacement motion and position of an observed shaft surface relative to the probe mounting location. Typically, proximity probes used for rotating machinery measurements operate on the eddy current principle, and measure shaft displacement motion and position relative to the machine bearings or housing. In addition to the shaft displacements a voltage pulse for each turn of the shaft, called the Keyphasor® signal is
used primarily to measure shaft rotative speed and serves as a reference for measuring vibration phase lag angle. It is an essential element in measuring rotor slow roll bow or runout information.

The simplest signal can be constructed in a complex plane from two vectors, $\mathbf{A}$ and $\mathbf{B}$, rotating in opposite direction at the same frequency $\omega$. The sum of these vectors is a vector parallel to the real axis only if both the vectors are the same and the initial phases are opposite. The resulting vector is a model of a real harmonic signal (sine or cosine function). In the case that these additional conditions are not fulfilled the sum of both vectors describes an ellipse, called an elementary orbit. The principle of the orbit construction is shown in figure 2. As the plane is complex, the vector $\mathbf{A} + \mathbf{B}$ is a complex quantity. The real part of this vector is a time signal $x(t)$ while the imaginary part is a time signal $y(t)$. The vector end point is a complex function $x(t) + jy(t)$, where $j$ is the complex unity.

It is well known that the Fourier transform of the harmonic signal is complex conjugate symmetric function of the frequency $\omega$. This is a reason that the frequency spectrum is plotted only for the positive value of the frequency. If the time domain signal is a complex signal then the frequency domain function is non-symmetric and the plot of the magnitude of the complex number against the frequency is called a full spectrum. This spectrum contains both the positive and negative frequencies.

2. **Orbit and Full Spectrum Measurements**

To study motion of the shaft in a journal bearing the Rotorkit device, product of Bently Nevada, was used. The proximity probes and the Keyphasor sensor belong to the instrumentation of Rotorkit (see figure 3). The centre shaft motion can be analysed only in the plane that is perpendicular to the shaft axis.

The RPM profile determined the shaft operation condition. The run-up is the first stage of the test continuing after a delay to the second stage that is a coast-down. The RPM as a time function is shown in figure 4. The corresponding displacement in direction X is shown in figure 5. After crossing the threshold of RPM at the run-up the shaft steady state oscillation suddenly occurs. When the shaft RPM decreases under the certain value of RPM the oscillation stops. $\mu$m

![Figure 2: Model of orbit construction](image1)

![Figure 3: Bently Nevada Rotorkit](image2)

![Figure 4: RPM time history](image3)

![Figure 5: X-axis displacement time history](image4)
To determine the relationship between the shaft centre vibration frequency and the shaft rotation speed the frequency analysis of the mentioned centre motion is performed. A full multispectrum of the signal $x(t) + jy(t)$ composed from full spectra for RPM as the third axis is shown in figure 6. This form of the 3D multispectrum is designated as a waterfall plot. The multispectrum frequency axis is in Hz. The orders 1 and 2 of rotational speed form a line of spectrum peaks determining the instantaneous RPM.

The analysis in the term of speed shaft and order gives better and clearer information about the shaft behaviour than the time or frequency in Hz. After resampling both the signals $x(t)$ and $y(t)$ to the sampling frequency that is proportional to the rotational frequency the shaft displacement in the both directions is a function of revolution. The real and imaginary part of the complex resampled signal is shown in figure 7. 10 revolutions of the rotor shaft correspond to less than 5 runs, called precession, along the elementary orbit. The shapes of the orbit for the different value of RPM are shown in figure 8 and 9.

The frequency of the dominating component in the full multispectrum in figure 10 is equal to 0.475 ord. This subharmonic component in relation to the shaft rotational frequency is corresponding to shaft precession. The contour plot of the multispectrum is shown in figure 11. As the peaks of the subharmonic component is parallel to the RPM axis, the frequency in Hz of the excited vibration is a multiple of the rotational speed and the frequency in order is independent on the shaft rotational speed.
3. **Mathematical model**

Let the rotor angular velocity be designated by \( \Omega \). It is assumed that the stator is fixed while rotor is rotating at the mentioned angular velocity. The rotor drags the fluid between two cylinders into motion and acts as a pump. It is easy to understand that the fluid velocity is varying across the gap as a consequence of the fluid viscosity. If the fluid average circumferential angular velocity is designated by \( v_{avg} \) then the fluid circumferential velocity ratio \( \lambda \) is given by the formula
The value of this ratio is slightly less than 0.5 due to the profile of the fluid velocity in the space between the shaft and journal, see figure 12.

\[ \lambda = \frac{v_{avg}}{\Omega} \]  

(1)

The internal spring, damping and tangential forces are acting on the rotor. The external forces refer to forces that are applied to the rotor, such as unbalance, impacts and preloads in the form of constant radial forces. All these external forces are considered as an input for the mathematical model based on the concept developed by Bently Rotor Dynamics Research Corporation [3].

As it is shown in figure 12 the fluid pressure wedge is the actual source of the fluid film stiffness in a journal bearing and maintains the rotor in equilibrium. These bearing forces can be modeled as a rotating spring and damper system at the angular frequency \( \lambda \Omega \). Fluid forces acting on the rotor in coordinates rotating at the same angular frequency as the spring and damper system are given by the formula

\[ F_{rot} = K r_{rot} + D \dot{r}_{rot}, \]  

(2)

where the parameters, \( K \) and \( D \), are specifying proportionality of stiffness and damping to the rotor centre-line displacement vector \( r_{rot} \) and velocity vector \( \dot{r}_{rot} \), respectively. The spring force acts opposite to the displacement vector. The sources of this force are shaft stiffness, fluid bearing film, bearings, seals and foundation. The damping force acts opposite to the velocity vector. The sources of this force are viscosity of fluid in bearing and seals.

To model the rotor system, the fluid forces have to be expressed in the stationary coordinate system, in which the rotor centre-line displacement and velocity vectors are designated by \( r \) and \( \dot{r} \), respectively. Conversion the complex rotating vector \( r_{rot} \) to the stationary coordinate system can be done by multiplication this vector by \( \exp(j \lambda \Omega t) \), which is the same as multiplying the vector in the stationary coordinates by \( \exp(-j \lambda \Omega t) \), see figure 13. The relationship between the mentioned vectors in rotating and stationary coordinates are given by the formulas

\[ r_{rot} = r \exp(-j \lambda \Omega t) \]  

\[ \dot{r}_{rot} = (\dot{r}_{rot} - j \lambda \Omega r) \exp(-j \lambda \Omega t) \]  

(3)

Substitution into the fluid force equation results in the following formula

\[ F = K r + D \dot{r} - jD \lambda \Omega r, \]  

(4)

where the complex term \( jD \lambda \Omega r \) has the meaning of the force acting in the perpendicular direction to the vector \( r \) and this force is called tangential. As the rotor angular velocity increases, this force can become very strong and can cause instability of the rotor behaviour.

As it was mentioned the rotor is under influence of the external forces, for instance produced by unbalance mass or simply by gravity. This external perturbation force, resulting from unbalance, is assumed to be rotating at the angular velocity \( \omega \), which is considered to be completely independent of the rotor angular velocity \( \Omega \) to
obtain general solution. The unbalance force, which is produced by unbalance mass \( m \) mounted at a radius \( r_u \), acts in the radial direction and has a phase \( \delta \) at time \( t = 0 \)

\[
F_{\text{Perturbation}} = mr_u \omega^2 \exp(j(\omega t + \delta)), \quad (5)
\]

The equation of motion for a rotor operating in a small, localized region in the journal bearing is as follows

\[
M \ddot{r} = -K r - D \dot{r} + jD \lambda \Omega \ r + mr_u \omega^2 \exp(j(\omega t + \delta)), \quad (6)
\]

where \( M \) is the rotor mass. After rearranging an ordinary, linear differential equation with constant coefficients is obtained

\[
M \ddot{r} + D \dot{r} + (K - jD \lambda \Omega) r = mr_u \omega^2 \exp(j(\omega t + \delta)), \quad (7)
\]

The solution of this type of equation is well known. It is assumed as a response vector with amplitude \( A \) and phase \( \alpha \) and rotating at the angular velocity \( \omega \)

\[
r = A \exp(j(\omega t + \alpha)), \quad \dot{r} = j \omega A \exp(j(\omega t + \alpha)), \quad \ddot{r} = -\omega^2 A \exp(j(\omega t + \alpha)). \quad (8)
\]

The amplitude and phase of the rotor shaft centre-line is given by the following formula

\[
r = A \exp(j\alpha) = \frac{mr_u \omega^2 \exp(j\delta)}{\left( K - M \omega^2 \right) + jD(\omega - \lambda \Omega)}. \quad (9)
\]

The perturbation frequency is the same as the rotor angular velocity \( \omega = \Omega \) for the experimental data. The general non-synchronous model is converted to the special synchronous model

\[
r = A \exp(j\alpha) = \frac{mr_u \Omega^2 \exp(j\delta)}{\left( K - M \Omega^2 \right) + jD\Omega(1 - \lambda)}. \quad (10)
\]

This solution describes the shaft motion, which is shown in figure 8, with the relatively small amplitude. The frequency of the dominating component in the frequency spectra is equal to the shaft rotational frequency.

The shaft/fluid wedge bearing/system can be demonstrated as a servomechanism working in the closed loop, which is shown in figure 14. The direct and quadrature dynamic stiffness is introduced according to the acting force direction. The imaginary variable \( j \omega \) is replaced by a complex variable \( s \)

\[
K_{\text{Direct}}(s) = K + Ds + Ms^2 \\
K_{\text{Quadrature}}(s) = -j \lambda Ds
\]

and the equation of motion takes the form

\[
r = \left( F_{\text{Perturbation}} - K_{\text{Quadrature}}(j\omega) r \right) / K_{\text{Direct}}(j\omega), \quad (12)
\]

The individual transfer function \( 1 / K_{\text{Direct}}(s) \) (direct dynamic compliance) is stable while the feedback path in the closed-loop system acts as a positive feedback and introduces instability for the closed-loop system. The gain of the positive feedback depends on the angular velocity \( \Omega \). The closed-loop system is stable for the low rotor rotational speed. There is a margin for the stable behavior. If the gain of the positive feedback crosses over a limit value then the closed-loop becomes unstable. The properties of the unstable behavior can be analyzed using the servomechanism in figure 14.
The stability of the closed-loop dynamic system is depending on the open-loop transfer function for $s = j\omega$

$$G_o(j\omega) = \frac{K_{\text{Quadrature}}(j\omega)}{K_{\text{Direct}}(j\omega)} = \frac{-\lambda \Omega D}{\omega D + j(K - M\omega^2)}.$$  \hspace{1cm} (13)

According to the Nyquist stability criterion, a margin of stability, generally referred as gain and phase margins [4], is resulting from the condition

$$G_o(j\omega_{\text{Crit}}) = -1.$$ \hspace{1cm} (14)

An angular frequency, at which a system can oscillate without damping, is designated by $\omega_{\text{Crit}}$. Substitution (14) into (13) results in formulas for the oscillating frequency

$$\omega_{\text{Crit}}^2 = \frac{K}{M} \quad \text{and} \quad \omega_{\text{Crit}} = \lambda \Omega.$$ \hspace{1cm} (15)

It can be concluded that the frequency of the rotor subharmonic oscillation is the same as the fluid average circumferential angular velocity. The measurement results in the value of the parameter $\lambda = 0.475$. This result confirms the introductory assumption about the fluid forces acting on the rotor.

If the system was linear then the unstable rotor vibration would spiral out to infinity. The rotor shaft lateral oscillations are limited by the journal bearing surface. A new balance of nonlinear forces comes into action and forms a steady-state limit cycle of the rotor orbital motion. The stiffness and damping coefficients are non-linear function of the eccentricity ratio, especially when the shaft is approaching to the journal wall.

As it is experimentally verified the frequency spectrum of the fluid induced oscillation contains the single dominating component as it would be a solution of the linear differential equation without damping. The proportionality between $\omega$ and $\Omega$ is maintained for a wide range of $\Omega$. If the rotational speed of the Rotorkit exceeds the value of 3000 RPM the whip vibration appears. The frequency of the whip oscillation is equal to a constant value.

A fluid-induced instability, commonly referred to as oil whirl, is the special resonance condition with the frequency that is proportional to the rotational speed. The shaft precession is self-excited by fluid induced instability and it is called whirl vibration. The whirl vibration is always forward precession and starts at the rotor rotational frequency that is called Bently and Muszynska threshold. The orbit shape is nearly circular for whirl vibration [5,6]. It can be noted that the increase of $\Omega$ for a given $\omega_{\text{Crit}}$ can be reached by decrease of $\lambda$.

The multispectrum as a waterfall plot, corresponding to the full multispectrum in figure 9, is shown in figure 10. As the dominating peaks in this contour plot form a line perpendicular to the frequency axis the multispectrum can be taken as an experimental verification of the fact that the value of $\lambda$ is constant and independent on the shaft rotational speed.
4. Conclusion
The paper describes a powerful analytical tool for rotor system diagnostics. The new term is a full spectrum. The full spectrum is a good tool for studying rotor instability in journal bearings. The paper demonstrates whirl vibration and the independence of the ratio relating the precession speed to the shaft rotational speed on the shaft absolute rotational speed.

The mathematical model of the shaft motion in the journal bearing gives explanation of the self-excited vibration.

5. References

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