Vibration damping of the cantilever beam with the use of the parametric excitation

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Motivation

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History

Simulation result (obtained with an analog computer) of a self-excited system exhibiting vibration suppression near the parametric combination resonance frequency \( \eta_0 = \Omega_2 - \Omega_1 \). From [Tondl, 1998]

New papers

Horst Ecker: Parametri excitation in engineering systems, 20th International Congress of Mechanical Engineering, November 15-20, 2009, Gramado, RS, Brazil


Petermeier’s and Ecker’s paper uses the beam model of the Timoshenko type.
Parametric resonances

Principle Parametric Resonances at frequencies

\[ \omega_{j/n}^{Pr} = \frac{2\Omega_j}{n}, \quad n = 1, 2, \ldots \]

Combination Parametric Resonances at frequencies

\[ \omega_{j\pm k/n}^{Cr} = \frac{|\Omega_j - \Omega_k|}{n}, \quad j, k = 1, 2, \ldots, \quad n = 1, 2, \ldots \]

\( \Omega_j \) and \( \Omega_k \) are the \( j \)-th and \( k \)-th natural frequency of the linear system (\( \mu = 0 \)).

Time-varying stiffness

\[ K_{n+1} = K_\delta(1 + 2\mu \cos(\omega_0 t)) \]

\( \mu, \omega_0 \) are parameters

Problem of stability
Damped Mathieu equation

\[
\frac{d^2 y}{dt^2} + 2 \xi \omega \frac{dy}{dt} + \omega^2 (1 + 2 \mu \cos (\omega_0 t)) y = 0
\]

Principle Parametric Resonances at frequencies

\[
\omega_{1/2}^{Pr} = \frac{2\omega}{n}, \quad n = 1, 2, \ldots
\]

\(\omega\) is a natural frequency of the linear system.

Linear part of the equation

\[
\frac{d^2 y}{dt^2} + 2 \xi \omega \frac{dy}{dt} + \omega^2 y = -2\mu y \omega^2 \cos (\omega_0 t)
\]

Non-stationary feedback

Stability regions

\[
\frac{\omega_0}{2\omega} = \frac{1}{2} \quad \frac{\omega_0}{2\omega} = 1
\]

\[
\omega_0 = \omega \quad \omega_0 = 2\omega
\]
Lumped parameter model of the cantilever beam

Euler-Bernoulli beam

Potential energy \( V \) of the deflected beam is as follows

\[
V = \sum_{n=0}^{N-1} \frac{1}{2} K_n (\Delta \delta_n)^2 = \frac{1}{2 \Delta L^2} \sum_{n=0}^{N-1} K_n (y_{n+1} - 2y_n + y_{n-1})^2.
\]

Lagrange's equations of motion

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{y}_n} \right) - \frac{\partial T}{\partial y_n} + \frac{\partial V}{\partial y_n} = 0, \quad n = 1, 2, ..., N.
\]

Variable stiffness affects only 3 of \( N \) equations

\[
\begin{align*}
 n-1: & \quad \ldots + 6K_\delta \Delta L^2 y_{n-1} + \ldots = -K_\delta \Delta L^2 \delta(t) \\
n: & \quad \ldots + 6K_\delta \Delta L^2 y_n + \ldots = 2K_\delta \Delta L^2 \delta(t) \\
n+1: & \quad \ldots + 6K_\delta \Delta L^2 y_{n+1} + \ldots = -K_\delta \Delta L^2 \delta(t)
\end{align*}
\]

where

\[
\delta(t) = 2\mu(y_{n-1} - 2y_n + y_{n-1}) \cos(\omega_0 t) = \frac{2\mu}{2}[(y_{n+1} - y_n) - (y_n - y_{n-1})] \cos(\omega_0 t).
\]

Periodically varying stiffness is replaced by a periodic external forces whose amplitude is proportional to the deflections of 3 elements.

Lumped parameter model

\[
M \ddot{y} + C \dot{y} + Ky = F_e(t), \quad C = \alpha M + \beta K,
\]

\[
F_e = K_\delta \Delta L^2 \delta(t)[\ldots, 0, -1, +2, -1, 0, \ldots]^T.
\]

\[
K = \frac{K_\delta}{\Delta L^2} \begin{bmatrix}
7 & \cdots & \cdots & \cdots & \cdots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
1 & \cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
\]

\[
M = \begin{bmatrix}
B & A & A & \vdots & \vdots \\
A & B & A & \vdots & \vdots \\
A & B & A & \vdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \ddots \\
A & B/2 & \vdots & \vdots & \vdots \\
\end{bmatrix}
\]

\[
A = \frac{\Delta m}{4} \left( 1 - \frac{1}{3} \left( 1 + \left( \frac{h}{\Delta L} \right)^2 \right) \right),
\]

\[
B = \frac{\Delta m}{2} \left( 1 + \frac{1}{3} \left( 1 + \left( \frac{h}{\Delta L} \right)^2 \right) \right).
\]
Matlab-Simulink model

Matlab-Simulink lumped parameter model of the cantilever beam for arbitrary number of elements

Linear time-invariant (LTI) system

Index $k$  | 1   | 2   | 3    | 4    | 5    |
---        |-----|-----|------|------|------|
[Hz]       | 16.14 | 103.04 | 291.9 | 563.9 | 839.4 |
[rad/s]    | 101.4 | 647.4 | 1834.1 | 3543.1 | 5274.4 |

Central difference of the second order

$\approx (\Delta L)^2 \frac{\partial y^2}{\partial z^2}$
Location of the patch piezoactuator at the clamp of the beam

Effect of the patch piezoactuator position on the damping of vibrations

Simulink model

Alternative linear piezoactuator
Effect of the excitation amplitude on the decay rate of the free end

Initial beam deformation

Decay of the free end of the beam without parametric damping

Effect of the $\mu$ on the time of decay, at which the vibrations are reduced by 40 dB (100 times)

$$\omega_0 = \omega_2^{-1/2} = \omega_2 - \omega_1 = 546 \text{ rad/s}$$

Decay of the free end of the beam with parametric damping

Effect of the time varying stiffness

Envelope in dB

$$\text{dB} = 20 \times \log \left( \frac{y(5)}{\log y(5)_{t=0}} \right)$$
Effect of the random disturbing force on damping

Frequency spectrum of the excitation and response at the beam free end for $\omega_{2-1/1}$

$$\omega_0 = \omega_{2-1/1} = \omega_2 - \omega_1$$

$$f_0 = f_{2-1/1} = f_2 - f_1 = 86.9 \text{ Hz}$$

Splitting of the dominant spectrum peak into two smaller peaks

$f_0$ … antiresonance frequency

Splitting of the dominant spectrum peak into two smaller peaks
Conclusion

- The objective of this paper was to demonstrate that parametric damping reduces vibration. Amplitude changes in stiffness were chosen by the simulation approach.
- The parametric excitation is one of the tools to increase the efficiency of vibration damping. The paper examined the Principle and Combination Parametric Resonance frequencies and their effect on the decay rate for the cantilever beam.
- It was found that the greatest effect on vibration damping has the difference frequency between the first and second resonance frequency of the cantilever beam. This frequency difference is a Combination Parametric Resonance frequency of the first order. The optimum size of half the amplitude of excitation for this frequency was also determined.
- Other parametric resonance frequencies are without effect on the vibration damping. The frequency spectra clearly explain why increases damping of the parametrically damped systems. The dominating peak in the spectrum splits into two adjacent peaks and their magnitude is reduced.
Thank you for attention

http://homel.vsb.cz/~tum52