Lecture on Angular Vibration Measurements Based on Phase Demodulation

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Outline

• Motivation
• Principle of phase demodulation using Hilbert transform
• Gear angular vibration measurements
• Transmission error (TE) measurements
• Measurements of the car engine rotational speed uniformity
• Software tools for phase demodulation
Motivation

Angular vibration as the source of the machine vibration and noise
Angular and Linear Vibration Excitation

$F_T$ force acting to the wheel at the pitch point

$F_S$ force acting at the wheel support bearing

$|F_S| = |F_T|

Forces $F_T$ and $F_S$ result in torque

Force $F_S$ excites gearcase vibration
Gear Angular Vibration

- Angular vibration
- Double differentiation
- Angular acceleration
- Linear acceleration on the gearbox housing
Source of car shaking while running at idle speed

- Crankshaft angular vibration
- Engine linear vibration
- Car body linear vibration
Variation of the Angular Acceleration
Variation in 3D Surface Plot
Transducers and signal processing methods
Transducers for Angular Vibration Measurements

• Tangentially mounted accelerometers
• Laser Torsional Vibration Meter (Doppler effect)
• **Incremental rotary encoders** (several hundreds of pulses per revolution)
How to Process Impulse Signals

• Time interval length measurements
  – Sample number & Interpolation
  – High frequency oscillator (10 GHz) & Impulse counter (Signal analyzer Rotec)

• Phase demodulation
Principle of the Hilbert transform
Analytic Signal Property

Real harmonic signal \( x(t) \)\n
\( X_N \quad \xrightarrow{\text{-f}} \quad X \quad \xrightarrow{\text{+f}} \quad X_P \)

Complex analytic signal \( Z = 2X_P \)

\( \omega_P = 2\pi f \)

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Analytic Signal in a Helix Shape

\[ \omega_\Phi = 2\pi f \]

\[ Z = 2X_\Phi \]
Evaluation of Analytic Signal

\[ X = X_p + X_N \]

\[ Z = 2X_p \]

Evaluation of the Hilbert transform using …

- Fast Fourier Transform (FFT)
- Digital filters

Time signal \[ + j \] Hilbert transform = Analytic signal
Evaluation of the Hilbert Transform using FFT

\[ X(j\omega) = FFT\{x(k)\} \]

\[ X(j\omega) \rightarrow Y(j\omega) \]

\[ y(k) = IFFT\{Y(j\omega)\} \]

\[ Y_N = jX_N \]

\[ Y_P = -jX_P \]
Evaluation of Analytic Signal using Digital Filter

Impulse response

$$g_{HT}(n) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} G_{HT}(e^{j\omega}) e^{j\omega n} d\omega = \begin{cases} 0, & n = 2k \\ 2/\pi n, & n = 2k + 1 \end{cases}$$

Frequency response function

$$G_{HT}(e^{j\omega}) = \begin{cases} -j, & +\pi > \omega > 0 \\ j, & -\pi < \omega < 0 \end{cases}$$
Hilbert Transformer

160-order FIR filter
Impulse response $n = -80, \ldots, 80$

Frequency response function
Principle of phase demodulation
Phase Modulation

Real phase modulated signal

\[ x(t) = A \cos(\omega P t + \Delta \phi_M(t)) \]

Modulation signal

Phase

Analytic signal

Carrying component

Sideband components

Revolution

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Phase Unwrapping and Linear Trend Removing

Discontinuities removing \( (2f \leq f_{\text{sampl}} \Rightarrow |\Delta \varphi| \leq \pi) \)

\[ \Delta \varphi < -\pi \Rightarrow \varphi + 2\pi \rightarrow \varphi, \quad \Delta \varphi > +\pi \Rightarrow \varphi - 2\pi \rightarrow \varphi \]
An alternative procedure

Phase ................. \[ \varphi(t) = \arctan \left( \frac{y(t)}{x(t)} \right) \]

Angular frequency … \[ \omega(t) = \frac{d\varphi(t)}{dt} = \frac{dx(t)}{dt} \frac{y(t)}{x^2(t) + y^2(t)} - x(t) \frac{dy(t)}{dt} \]

Phase ................. \[ \varphi(t) = \int_{0}^{t} \omega(\tau) d\tau \]

Envelope .............. \[ e(t) = \sqrt{x^2(t) + y^2(t)} \]
Gear Angular Vibration Measurements

Solving the gearbox noise problem at the very source
Transmission error measurements

Emitted gearbox noise level is proportional to the transmission error level.

Decreasing TE by 10 dB results in decreasing the noise level by 7 dB.
**Measurement Principle**

TE – Transmission error

\[ TE(\text{rad}) = \Theta_2 - \frac{n_2}{n_1} \Theta_1 \]

\[ TE(m) = \left( \Theta_2 - \frac{n_2}{n_1} \Theta_1 \right) r_2 \]

\( n_1, n_2 \) .... Teeth number

\( \Theta_1, \Theta_2 \) .... Angle of rotation [rad]

\( r_2 \) .... Wheel radius

\( E_1, E_2 \) .... Incremental rotary encoders
Instrumentation

9/2 channels PULSE Order Analysis

Heidehain encoders of the ERN 460-500 type (less than 300 €)
Encoder Accuracy

Circle part

Heidehain encoders of the ERN 460-500 type (500 pulses per revolution)
Measurement Arrangement

4/2 channels PULSSE
Order Analysis
&
Special software

Heidehain encoders
of the ERN 460-500 type

Car gearbox

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Using the Fourier to evaluate the Hilbert transform
Effect of Phase Modulation on Pulse Frequency Spectrum

Enhanced Spectrum, 21-Tooth Gear

Enhanced Spectrum, 44-Tooth Gear

RMS dB/ref 1 V

Order [-]

Pinion 21 T

Wheel 44 T

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Pinion Angular Vibration

Unwrapped phase

Phase variation

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Phase Modulation Frequency Spectrum

Autospectrum: Pinion 21T: Enhanced
Time (Impulsy500)

RMS dB/ref 1

Order [-]

Pinion 21 T

Autospectrum: Wheel 44T: Enhanced
Time (Impulsy500)

RMS dB/ref 1 deg

Order [-]

Wheel 44 T

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Comb Filter 1 - Frequency Response

$|H(j f/f_0)|$

Pass Band

5 harmonics of the toothmeshing frequency with the limited number of sidebands

$f_0$ ...toothmeshing frequency
Angular Vibration of the 21-Tooth Gear in Deg (after Comb Filtration)

Toothmeshing frequency harmonics with 3 sideband components

Time History : Pinion 21T : Enhanced Time(Impulsy500)
Angular Vibration of the 44-Tooth Gear in Deg (after Comb Filtration)

Toothmeshing frequency harmonics with 6 sideband components

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Comb Filter 2 - Frequency Response

$|H(j\,f/f_0)|$

Pass Band 0

Only harmonics of the toothmeshing frequency without sidebands

$f_0$ ...toothmeshing frequency
Phase Delay Between Signals

Original delay

Zero delay

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Transmission Error
(average per a tooth pitch rotation)

500 RPM, +40 Nm  500 RPM, +80 Nm

Tooth pitch rotation

TE [micron]
Truck Gearbox
Transmission Error

2R

Tooth pitch rotation

micron

2N

Tooth pitch rotation

micron

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Using the FIR filter to evaluate the Hilbert transform
Measured Impulse Signals

Impulse signals

Frequency spectra

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Filtered Impulse Signals

Filtered impulse signals

Time capture analyzer: Time: Real (Expanded Time(Encoder1)); Time 2: Real (Expanded Time(Encoder2))

Frequency spectra of filtered signals

Autospectrum 1: Time capture analyzer: Time: Real (Expanded Time(Encoder1)); Time 2: Real (Expanded Time(Encoder2))
Phase Difference

Unwrapped phase of impulse signals

FIR Filters : Time Capture Analyzer : Time: Real (Expanded Time(Encoder1)); Time: Real (Expanded Time(Encoder2))

Phase difference (Signal1 – Signal2 * 27/44)

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Phase spectrum

Autospectrum 2: Time Capture Analyzer: Difference (FIR Filters: Unwrapped Phase (Time: Real (Expanded Time(Encoder1))) - FIR Filters: Unwrapped Phase (Time 2: Real (Expanded Time(Encoder2))))

RMS dB/ref 1 deg

Frequency [Hz]
Time Domain Signal

IFFT of phase spectrum

Time History : Time Capture Analyzer : Time 1: Real (Difference (FIR Filters: Unwrapped Phase (Time: Real (Expanded Time(Encoder1))) - FIR Filters: Unwrapped Phase (Time 2: Real (Expanded Time(Encoder2))))))

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Transmission Error Time History

**Pinion**

![Transmission Error Time History](image1)

**Wheel**

![Transmission Error Time History](image2)

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Averaged Transmission Error

**Pinion**

Time History : Time Capture Analyzer : Resampling:
1: Averaged (Time 1: Real (Difference (FIR Filters:
Unwrapped Phase (Time: Real (Expanded
Time(Encoder1)))) - FIR Filters: Unwrapped Phase
(Time 2: Real (Expanded Time(Encoder2))))))

**Wheel**

Time History : Time Capture Analyzer : Resampling:
Averaged (Time 1: Real (Difference (FIR Filters:
Unwrapped Phase (Time: Real (Expanded
Time(Encoder1)))) - FIR Filters: Unwrapped Phase
(Time 2: Real (Expanded Time(Encoder2))))))
Results of the gear design improvements

Effect of the design improvements on the gearbox noise
Effect of Contact Ratio on the Average Toothmesh Acceleration Signal

\[ \varepsilon_\gamma \approx \varepsilon_\beta \]

\( \varepsilon_\gamma \) \quad \text{profile contact ratio}

\( \varepsilon_\alpha \) \quad \text{face contact ratio}

\( \varepsilon_\beta \) \quad \text{total contact ratio = profile contact ratio + face contact ratio}

Truck Gearbox

(\( \varepsilon_\beta \approx 1.0 \))

[Graphs showing Tooth Pitch Rotation and Acceleration for different contact ratios: LCR and HCR]
Effect of Contact Ratio on the Noise Level in dB

Truck gearbox noise level at the distance of 1m

<table>
<thead>
<tr>
<th>Speed</th>
<th>3R</th>
<th>3N</th>
<th>4R</th>
<th>4N</th>
<th>5R</th>
<th>5N</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>LCR</td>
<td>92,0</td>
<td>92,9</td>
<td>95,0</td>
<td>95,4</td>
<td>95,0</td>
<td>96,5</td>
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<tr>
<td>HCR</td>
<td>90,0</td>
<td>91,8</td>
<td>90,4</td>
<td>89,7</td>
<td>88,2</td>
<td>90,3</td>
</tr>
</tbody>
</table>
Effect of Tooth Surface Modification

Gear train S

- Hluk v dB
  - 88-92
  - 84-88
  - 80-84
  - 76-80
  - 72-76
  - 68-72
  - 64-68
  - 60-64

- Torque Nm

Gear train T2

- Hluk v dB
  - 88-92
  - 84-88
  - 80-84
  - 76-80
  - 72-76
  - 68-72
  - 64-68
  - 60-64

- Torque Nm

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Measurements of a car engine rotational speed variation

Solving the problem of a car with random burst shaking while its engine is running in idle

Car body vibrations correlate with changes in engine rotational speed
Engine rotation uniformity at idle speed

Average RPM during 250 consecutive double revolutions

800 RPM = 13.3 Hz
Measurements of a Car Engine Rotational Speed and Acceleration

4/2 channels PULSE Order Analysis

Impulse signals

crankshaft

tacho

&

(Divider)

camshaft
Source of an Impulse Signal

- **teeth**
- **fly wheel**
- **gap**
- **transducer**
- **segment**
- **gap 180°**
- **ignition**
- **Tooth No.**
  - 58
  - 61
  - 74
  - 88
  - 104
  - 118
  - 1
  - 14
  - 28
  - 44
  - 58
  - 61
  - 74
- **tacho**
- **crankshaft**
- **2nd revolution**
- **1st revolution**
- **a complete revolution**
- **camshaft**
- **gap**

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Impulse Signal

Impulse signal for engine control unit

Addition of missing impulses

60 – 2 = 58 impulses per revolution
Angular Variation

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Engine rotation uniformity at idle speed

Instantaneous RPM during the 2-revolution engine cycle

800 RPM = 13.3 Hz
Differentiation in the Frequency Domain

- Angle: \( \varphi(t), \Phi(j\omega) \)
- Velocity: \( \omega = d\varphi/dt, \Omega = j\omega\Phi \)
- Acceleration: \( \varepsilon = d\omega/dt, E = j\omega\Omega \)

\[ \varphi(t), \Phi(j\omega) \]

\[ \omega = d\varphi/dt, \Omega = j\omega\Phi \]

\[ \varepsilon = d\omega/dt, E = j\omega\Omega \]
Engine Crankshaft Angular Velocity and Acceleration

Angular velocity

Angular acceleration

6 ord limit

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Angular acceleration variation during two engine revolutions

4-cylinder / 4-stroke engine

combustion cycle

compression cycle
Effect of sinusoidal signal distortion on its frequency spectrum

1.5 ord = 6.6 Hz
Crankshaft angular acceleration frequency spectrum

6.6 Hz  13.3 Hz  26.6 Hz
Linear acceleration frequency spectra

Absorber effect

Engine

6.6 Hz Human body extra sensitive

Car body

6.6 - 13.3 - 26.6 Hz
Ride comfort

RMS of Acceleration

Frequency [Hz]

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Results

Original absorber

Improved absorber

0.5 ord = 6.6 Hz

1 ord = 13.3 Hz

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Software Tools for Transmission Error Evaluation
Automation Program for PULSE, the BK Signal Analyser
Signal Analyser
Conclusion

• The lecture is focused on the problem of the angular vibration measurements using phase demodulation

• The shaft angular vibration excite the housing linear vibration and consequently machine noise

• The theory is illustrated by experimental data.
Thank you for your attention