VSB TECHNICAL UNIVERSITY OF OSTRAVA | FACULTY OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE | DEPARTMENT OF COMPUTER SCIENCE

Machine Learning

Outlier Analysis

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Outlier Analysis

Outlier analysis

Informal definition (Hawkins)

An outlier is an observation which deviates so much from the other observations as to arouse suspicions that it was generated by a different mechanism.

Outlier applications

- Data cleaning
  - An outlier represent a noise data.
- Credit card fraud
  - Credit card activity outside the usual pattern may represent an fraud.
- Network intrusion detection
  - Unusual records in traffic that do not follow the regular patterns.

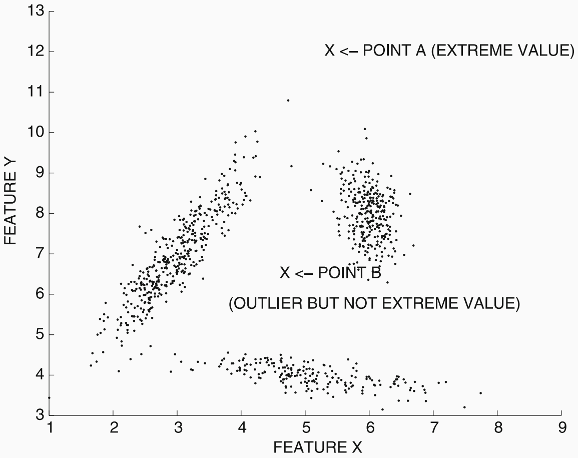
Outlier analysis - Classification types

- Real-valued outlier score
  - A real value that represents the outlierness of a data point.
  - May be based on probability, distance measurement, etc.
- Binary label
  - Strict assignment of a outlier flag.
  - Contain less information than real-value score.
  - May be based on real-values with threshold.

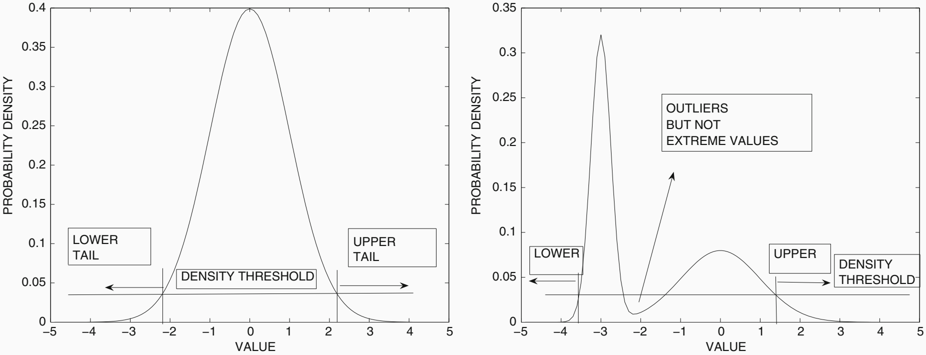
Outlier analysis - Model types

- Extreme values
- Clustering models
- Distance-based models
- Density based models
- Probabilistic models
- Information-theoretic models

Outlier analysis - Extreme values - Example

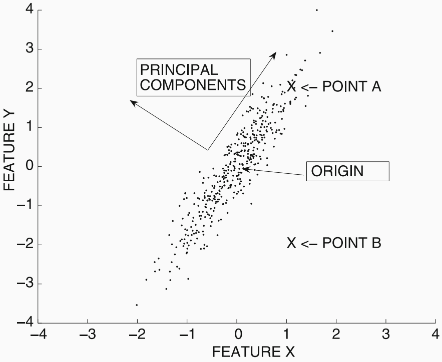


Outlier analysis - Extreme values - Univariate

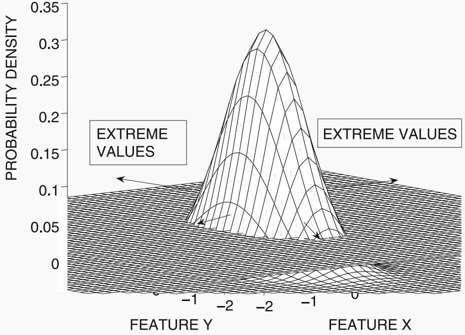


(a) Symmetric distribution (b) Asymmetric distribution

Outlier analysis - Extreme values - Multivariate

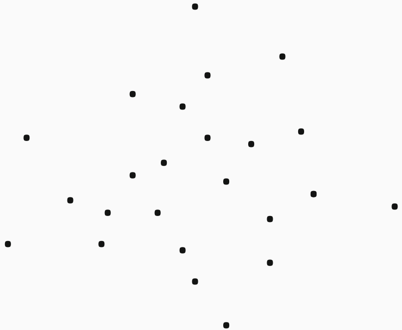


(a) Multivariate extreme values

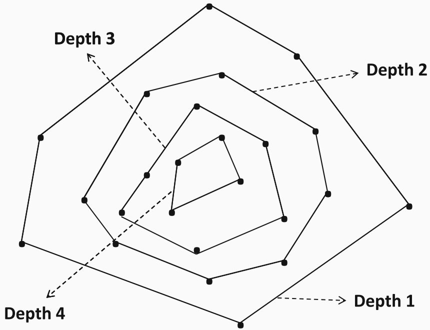


(b) Multivariate extreme values (probabilistic interpretation)

Outlier analysis - Extreme values - Depth-based

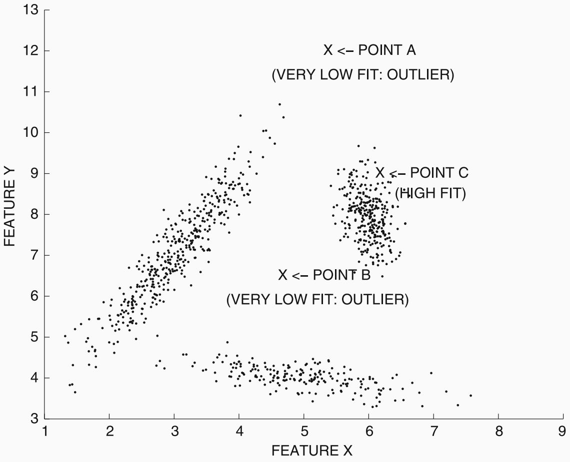


(a)



(b)

Outlier analysis - Probability values - Likelihood

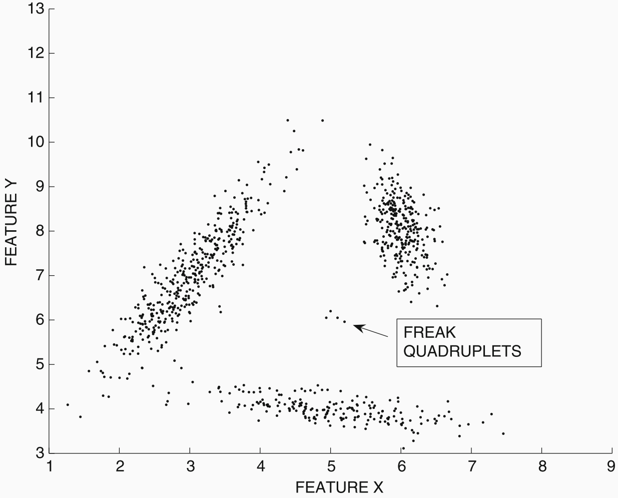


Outlier analysis - Distance-based detection

The distance-based outlier score of an object O is its distance to its k-th nearest neighbor.

- The definition is based on the user defined k.
- The $k > 1$ helps to removes a group of outliers.
- Outlier detection methods use finer granularity than clustering methods.
- The ambient noise has lover k-nearest neighbor distance than truly isolated anomaly.
- The better granularity brings higher computational complexity.
- The speed-up techniques are used:
    - Index structures
    - Pruning tricks.

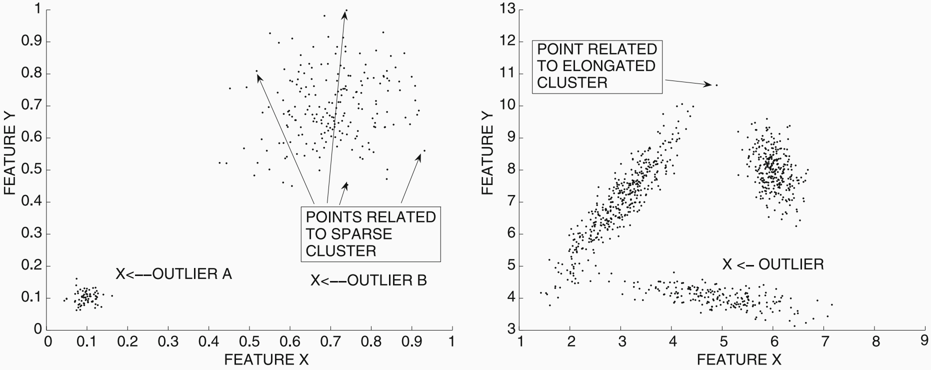
Outlier analysis - Distance-based detection



Outlier analysis - Distance-based detection - Pruning methods

- Sampling methods
    - A sample S of size s is sampled from data.
    - Distances between all pairs from D and S are computed.
    - The complexity is $O(n \cdot s) \ll O(n^2)$.
    - The top r ranked outlier in sample S is determined.
    - The score of the rth ranked outlier is the lover bound L over the the whole dataset.
    - The upper bound $V^k(X)$ for each point $X \in D - S$ is known from distances of pairs.
    - When $V^k(X)$ is not larger than the lower bound L the point X may be excluded from the testing.
    - Large number of points is removed due to this condition.
    - The remaining points $R \subseteq D - S$ is tested by outlier measure.
    - The proper ordering of R and $D - S$ may significantly improve the speed of the algorithm.

Outlier analysis - Distance-based detection - Local distance correction



(a) Varying cluster density (b) Varying cluster shape

Outlier analysis - Distance-based detection - Local distance correction

- Local outlier factor (LOF)
  - Adjusting the local variations in cluster density by normalization of distanced with the average point-specific distances in a data locality.
  - This approach solves the varying cluster density situation.
  - For a given point X the $V^k(X)$ is its distance to its k-nearest neighbor.
  - The $L_k(X)$ is the set of points within the k-nearest neighbor distance of X.
  - The number of points in $L_k(X)$ is k or more.
  - The reachability distance is defined as

$$R_k(X, Y) = \max\{Dist(X, Y), V^k(Y)\}$$

  - The average reachability distance of X with respect to $L_k(X)$ is then

$$AR_k(X) = MEAN_{Y \in L_k(X)} R_k(X, Y)$$

  - The Local Outlier Factor LOF_k is then

$$LOF_k(X) = MEAN_{Y \in L_k(X)} \frac{AR_k(X)}{AR_k(Y)}$$

Outlier analysis - Distance-based detection - Local distance correction

- Instance-Specific Mahalanobis distance
  - The goal is to deal with the varying cluster shape.
  - A k-local neighborhood $L_k(X)$ with respect to the cluster shape have to be defined.
  - $L_k(X)$ is constructed with the single-linkage agglomerative approach around the point X.
  - A mean $\mu_k(X)$ and the covariance matrix $\Sigma_k(X)$ are computed.
  - The distance $LMaha_k(X)$ then represent the outlier score.

$$LMaha_k(X) = Maha(X, \mu_k(X), \Sigma_k(X))$$

Outlier analysis - Density based methods

- The idea is similar to the density-based clustering.
- The main difference is that only the non-dense regions are detected.
- The points in sparse regions are reported as outliers.
- Histogram-based technique
  - Popular method for univariate data.
  - Represents the statistical distribution of points.
  - Difficult to adapt to varying density in different data locality.
  - Difficult to adapt this method to higher dimensions.
- Grid-based techniques
  - The space is partitioned into p equi-width ranges.
  - The sparse regions with the density less than τ are reported as outliers.
  - It is difficult to select proper p.
  - The τ may be defined using univariate extreme value analysis.
  - Outlier groups may not be reported because the cluster shapes are not recognized.

Outlier analysis - Density based methods

- Kernel-based density estimation
  - Similarly to Histogram- or Grid-based methods a local density is detected.
  - The density in each point is computed as smoothed values of a kernel functions associated with each data point.

$$f(X) = \frac{1}{n} \sum_{i=1}^{n} K_h(X - X_i)$$

  - The h is a parameter of a function.
  - typical choice is the Gausian kernel with the width h.

$$K_h(X - X_i) = \left(\frac{1}{\sqrt{2\pi}h} \right)^d \cdot e^{-\|X - X_i\|^2 / (2h^2)}$$

Questions?