Table of the Laplace Transform Properties[1]

$$\mathcal{L}(f(t)) = \int_0^\infty f(t)e^{-pt}\,\mathrm{d}\,t = F(p)$$

I. linearity	$\mathcal{L}(\sum_{k=1}^{n} c_k f_k(t)) = \sum_{k=1}^{n} c_k F_k(p)$
II. time scaling	$\mathcal{L}(f(\lambda t)) = \dfrac{1}{\lambda} F\left(\dfrac{p}{\lambda}\right), \ \ \lambda > 0$
III. Laplace domain shifting	$\mathcal{L}(e^{at} f(t)) = F(p-a)$
IV. derivative by a parameter	$\dfrac{\partial f(t,\lambda)}{\partial \lambda} = \dfrac{\partial F(p,\lambda)}{\partial \lambda}$, kde $\mathcal{L}(f(t,\lambda)) = F(p,\lambda)$
V. time shifting	$\mathcal{L}(f(t-\tau)\eta(t-\tau)) = e^{-\tau p} F(p)$ for every $\tau > 0$
VI. time domain derivative	$\mathcal{L}(f^{(1)}(t)) = pF(p) - f(0_+)$, $\mathcal{L}(f^{(2)}(t)) = p^2 F(p) - pf(0_+) - f^{(1)}(0_+)$, $\mathcal{L}(f^{(n)}(t)) =$ $= p^n F(p) - p^{n-1} f(0_+) - \cdots - f^{(n-1)}(0_+)$
VII. Laplace domain derivative	$\mathcal{L}(-tf(t)) = F'(p)$
VIII. time domain integration	$\mathcal{L}\left(\int_0^t f(\tau)\,\mathrm{d}\,\tau\right) = \dfrac{F(p)}{p}$
IX. Laplace domain integration	$\mathcal{L}\left(\frac{f(t)}{t}\right) = \int_p^\infty F(z)\,\mathrm{d}\,z =$ $= \lim_{\mathrm{Re}\, q \to \infty} \int_p^q F(z)\,\mathrm{d}\,z$

[1]by doc. RNDr. Marek Lampart, Ph.D., November 4, 2020

Table of Laplace Transforms of Common Functions

Time domain	Laplace domain	Region of convergence
1	$\frac{1}{p}$	$\mathrm{Re}(p) > 0$
e^{at}	$\frac{1}{p-a}$	$\mathrm{Re}(p) > \mathrm{Re}(a)$
$\sin(\omega t)$	$\frac{\omega}{p^2+\omega^2}$	$\mathrm{Re}(p) > 0$
$\cos(\omega t)$	$\frac{p}{p^2+\omega^2}$	$\mathrm{Re}(p) > 0$
$\sinh(\omega t)$	$\frac{\omega}{p^2-\omega^2}$	$\mathrm{Re}(p) > \lvert\omega\rvert$
$\cosh(\omega t)$	$\frac{p}{p^2-\omega^2}$	$\mathrm{Re}(p) > \lvert\omega\rvert$
$e^{at}\sin(\omega t)$	$\frac{\omega}{(p-a)^2+\omega^2}$	$\mathrm{Re}(p) > a$
$e^{at}\cos(\omega t)$	$\frac{p-a}{(p-a)^2+\omega^2}$	$\mathrm{Re}(p) > a$
$t^n,\ n \in \mathbb{N}$	$\frac{n!}{p^{n+1}}$	$\mathrm{Re}(p) > 0$
$t^n e^{at},\ n \in \mathbb{N}$	$\frac{n!}{(p-a)^{n+1}}$	$\mathrm{Re}(p) > \mathrm{Re}(a)$
$t\sin(\omega t)$	$\frac{2p\omega}{(p^2+\omega^2)^2}$	$\mathrm{Re}(p) > 0$
$t\cos(\omega t)$	$\frac{p^2-\omega^2}{(p^2+\omega^2)^2}$	$\mathrm{Re}(p) > 0$