Network Analysis Methods 2

Network Science

Multilayer Social Networks

Link (edge) prediction

2023/24

Outline

- Problem statement
- Single-layer network case
- Multi-layer case
- Layer Associativity

Edge Prediction

- The problem of edge prediction can be stated as a classification problem.
- Two classes: existing x missing link (edge). AUCS expample.

Pair	Degree product	Comm. neigh.	Edge (t_0)	**Edge ($\mathbf{t_1}$)**
Mark, Mat	6	1	no	**yes**
Mark, Bin	6	1	no	**no**
Mark, Sere	4	0	no	**no**
…	…	…	…	

Features to Use

- Each pair of actors can be described using some features:
  - the product of the number of neighbors of the two nodes
  - the number of common neighbors.
- A crucial task to achieve accurate predictions is the choice of good features.
  - Only structural features are taken into account.

Methods

- $score(a,b) = \ |N(a)| \times \ |N(b)|, |N(a)| + \ |N(b)|$
- $score(a,b) \ = \ |N(a) \ \cap \ N(b)|$
- $score(a,b) = \ \left|\frac{N(a)\cap N(b)}{N(a)\cup N(b)}\right|$
- $score(a,b) = \sum_{c\in N(a)\cap N(b)} \frac{1}{log\ |N(c)|}$

The Multilayer Case

- In multiple layers, the problem of edge prediction can be extended in two aspects:

  1. We do not just predict the occurrence of a new edge but a more complex event including information about the layers, for example, the specific lay er where the edge is going to appear.

  2. Using information about the relationships between the layers to define new features.

Common Actors

- The problem of edge prediction in multilayer networks can be reformulated so that:
  - we try to predict not only the pair of authors between whom an edge is going to appear…
  - …but also the layer where this is happening.
- In this case, if we have two layers *A* and *B*, we would compute the number of common neighbors on layer *A* and the number of common neighbors on layer *B*, and given a pair of actors (a_i, a_j), we would use the first feature to compute the likelihood that an edge will appear (or disappear) on the first layer, and the second feature for the second layer.

How to think about that…

- Beyond the basic approach, we can exploit relationships between different layers to build new features.

- The main idea is that if a connection on layer *A* is often associated with a connection on layer B, then we can have a **new feature** to predict the appearance of an edge on *B* whose value is proportional to the likelihood of the association.

Layer Associativity

- There is a hypothesis that the presence (or absence) of an edge between two actors on a specific layer may change the probability that those actors will be connected to another layer.
- We can compute association rules between sets of layers to learn more about their mutual relationships.
- We do not want to compute all possible correlations, but only those that are frequent and statistically significant.

Pair to Pair Connectivity

- Let us consider pairs of actors. For each pair with at least one edge connecting them, we list all the layers where they are connected.

Pair	Layers
U102,U139	lunch
U106,U118	coauthor, leisure, lunch, work
U106,U123	Facebook, work
U106,U041	leisure, lunch, work
U110,U072	lunch, work

Edge Frequency

- Most frequent layers, ranked by the frequency of connected pairs of nodes with an edge in each of the indicated layers

Frequency	Layer
0.548	work
0.545	lunch
0.351	Facebook
0.276	work, lunch
0.248	leisure
0.171	lunch, leisure

Association Rules

Rank	Rule	Support	Lift	Confidence
c1	leisure, coauthor $\rightarrow$ work	0.025	1.64	0.9
c2	coauthor $\rightarrow$ work	0.051	1.56	0.857
c3	lunch, coauthor $\rightarrow$ work	0.03	1.544	0.846
c4	work, leisure $\rightarrow$ lunch	0.112	1.525	0.832

Support

- The support of a set of layers *X*, notated *s(X)*, is the proportion of connected pairs of actors directly connected on all those layers.

Confidence

- Given an association rule X→Y, its confidence is defined as:

$$c(X,Y) = \frac{S(X) \cup S(Y)}{s(X)}$$

Lift

- Given an association rule $X \rightarrow Y$, lift is defined as:

$$l(X, Y) = \frac{S(X) \cup S(Y)}{s(X) \cdot s(Y)}$$

References

- Dickison, M. E., Magnani, M., Rossi, L. (2016). *Multilayer social networks.* Cambridge University Press. http://multilayer.it.uu.se.

- Bianconi, G. (2018). *Multilayer networks: structure and function*. Oxford university press.