

Network Analysis Methods 2

Network Science

Multilayer Social Networks

Basics and Degree-based Measures

2023/2024

References

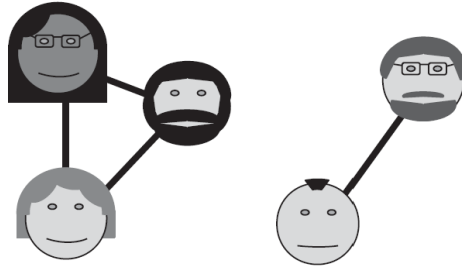
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Problem...

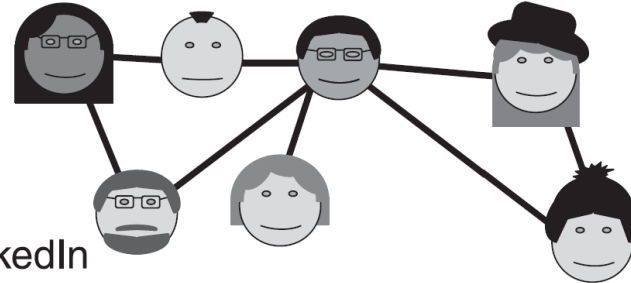
- Dealing with multiple social network is part of our daily experience...
 - We live in many networks...
- We continuously juggle our networks...
- We bridge them to move valuable information from one network to another...

Example

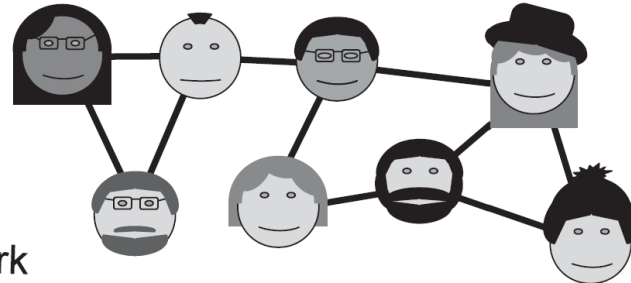
Facebook



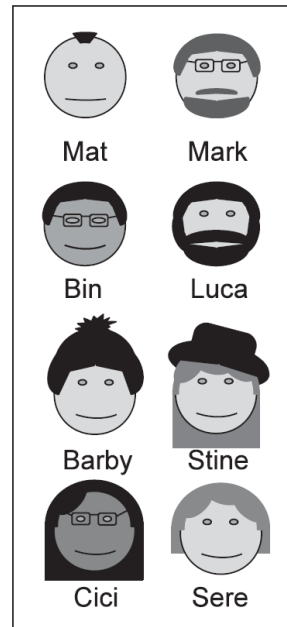
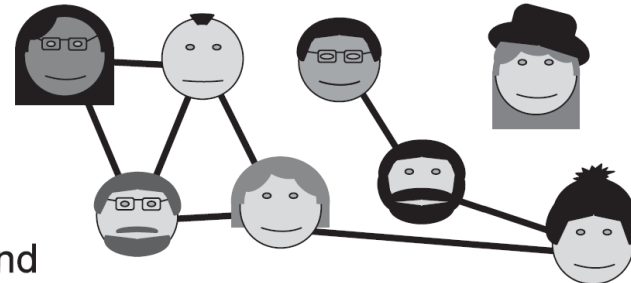
LinkedIn



work



friend



Definition (1/2)

- **Multilayer Network:** Given a set of actors A and a set of layers L , a multilayer network is defined as a quadruple $M = (A, L, V, E)$ where (V, E) is a graph and $V \subseteq A \times L$.
- **Actor:** An entity that can have relationships with other actors.
- **Layer:** The same actor can be represented in different layers, where each layer represents a type of actors or a type of edge between actors.

Definition (2/2)

- **Node:** A specific actor on a specific layer.
- **Edge:** A relationship between two nodes.
- **Multilayer Network:** A social network represented as a set of layers, where nodes in different layers refer to a global set of actors and edges can also connect nodes in the same or different layers.
- **Single-layer Network:** A social network represented as a network with only one layer.

Tasks

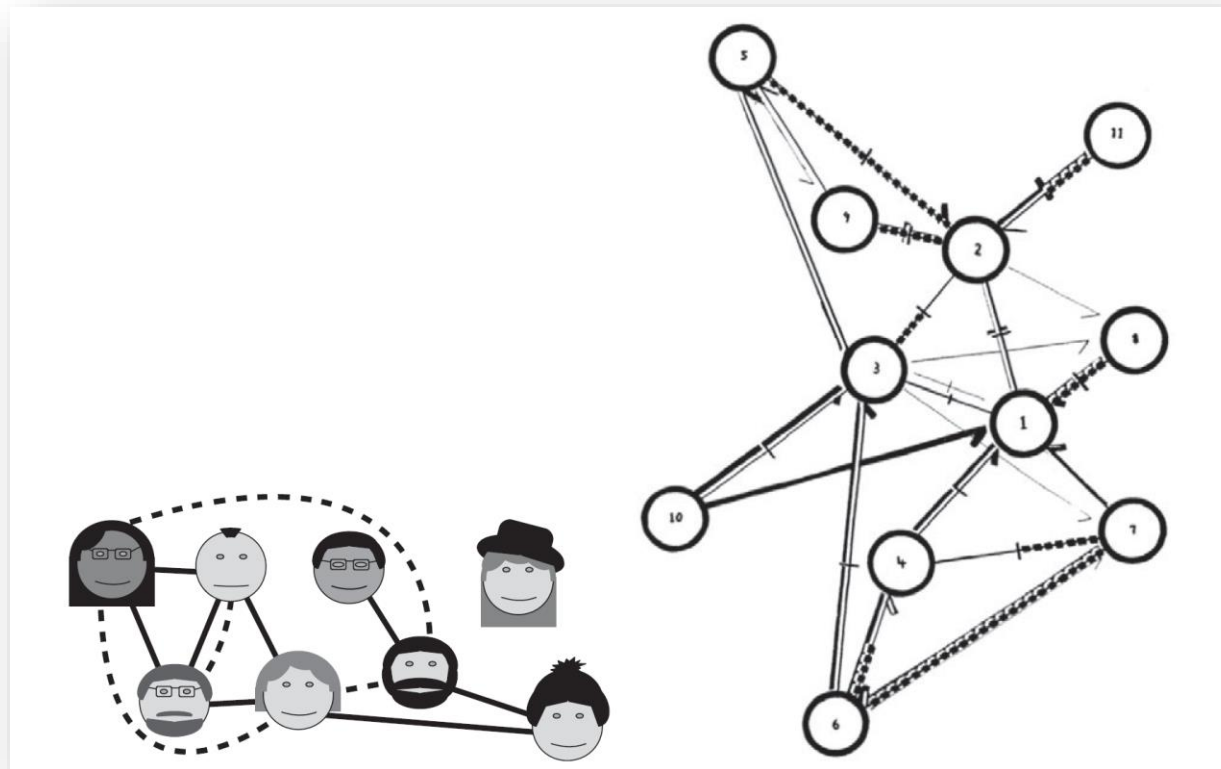
- **Models and measures**
 - Actor measures
 - Layer measures
- Mining multilayers networks
 - Visualizing multilayer networks
 - Community detection
 - Edge patterns
- Dynamical processes
 - Formation of multilayer networks
 - Information and behavior diffusion

Related Models

- Multiplex networks
- Multimode and multilevel networks
- Heterogeneous information networks
- Networks of networks
- Temporal networks
- Exponential random graph models

Multiplex Networks

- A common set of actors is connected through multiple types of edges. These different kinds of relational ties are typically known as *multiplex ties* in the SNA literature.



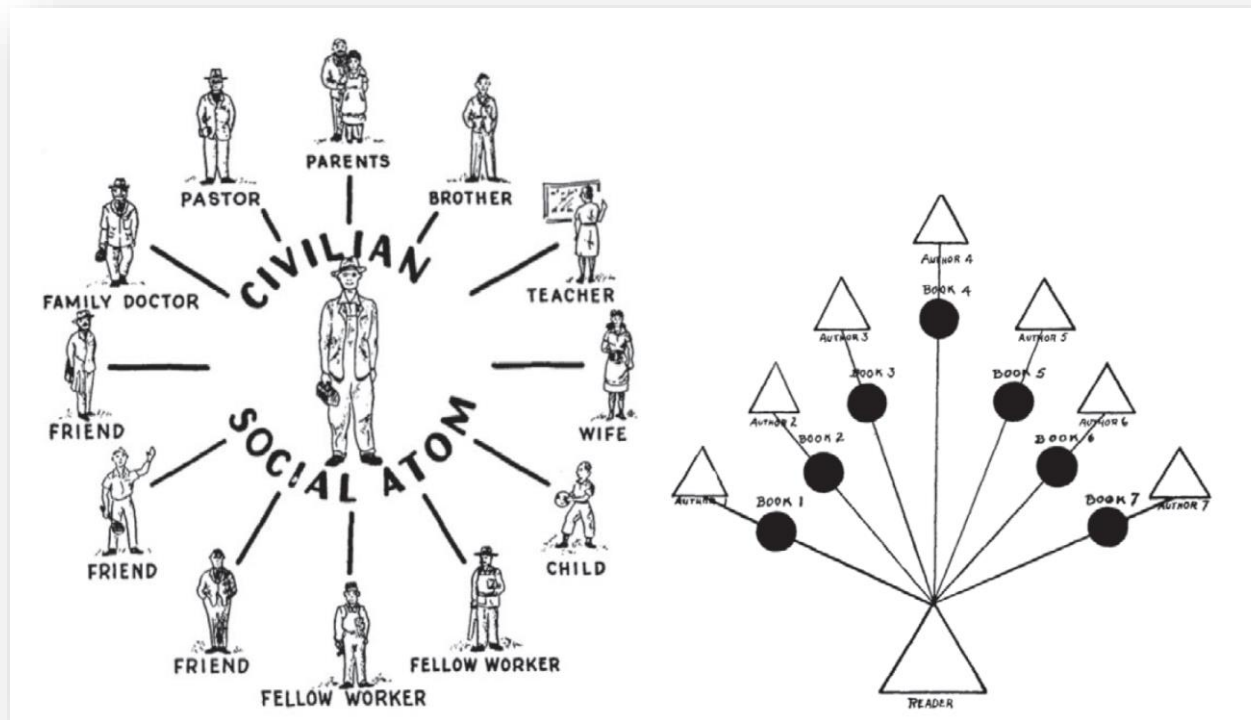
Representing Multiplex Social Networks

	<i>Cici</i>	<i>Mat</i>	<i>Mark</i>	<i>Sere</i>	<i>Bin</i>	<i>Luca</i>	<i>Stine</i>	<i>Baby</i>
<i>Cici</i>	0	1	1	0	0	0	0	0
<i>Mat</i>	1	0	1	1	0	0	0	0
<i>Mark</i>	1	1	0	1	0	0	0	0
<i>Sere</i>	0	1	1	0	0	0	0	1
<i>Bin</i>	0	0	0	0	0	1	0	0
<i>Luca</i>	0	0	0	0	1	0	0	1
<i>Stine</i>	0	0	0	0	0	0	0	0
<i>Baby</i>	0	0	0	1	0	1	0	0

	<i>Cici</i>	<i>Mat</i>	<i>Mark</i>	<i>Sere</i>	<i>Bin</i>	<i>Luca</i>	<i>Stine</i>	<i>Baby</i>
<i>Cici</i>	0	0	0	1	0	1	0	0
<i>Mat</i>	0	0	1	0	0	0	0	0
<i>Mark</i>	0	1	0	0	0	0	0	0
<i>Sere</i>	1	0	0	0	0	1	0	0
<i>Bin</i>	0	0	0	0	0	0	0	0
<i>Luca</i>	1	0	0	1	0	0	0	0
<i>Stine</i>	0	0	0	0	0	0	0	0
<i>Baby</i>	0	0	0	0	0	0	0	0

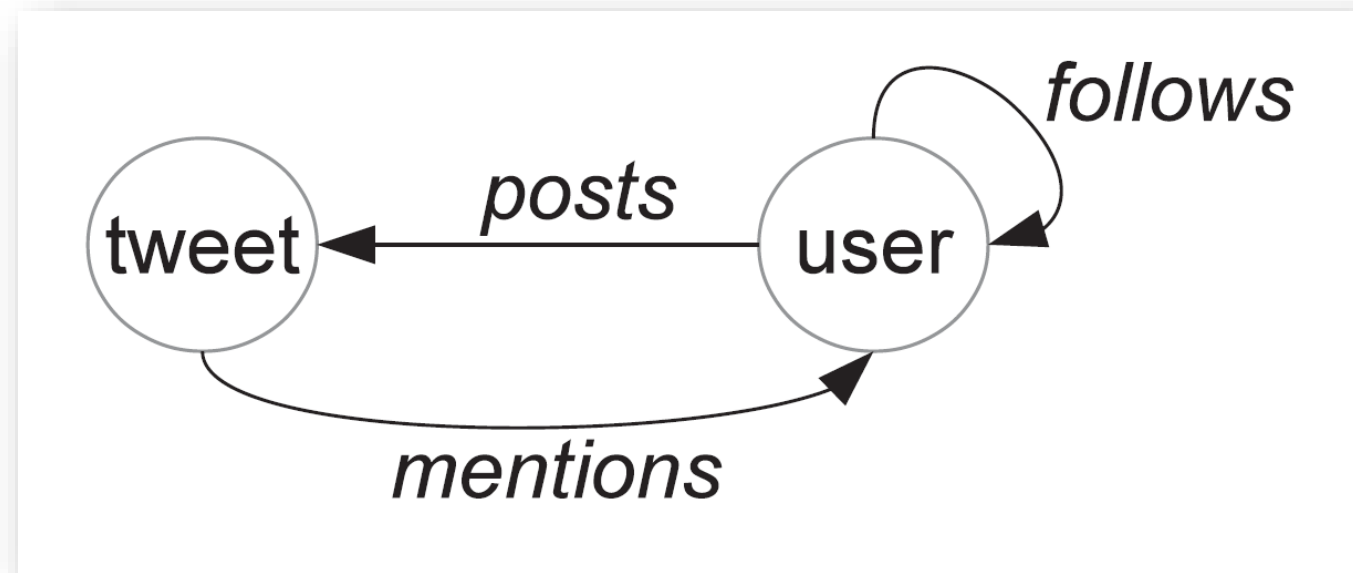
Multimode and Multilevel Networks

- Multiple types of actors inside the same network. A network where two types of actors can be identified is a two-mode (also called bipartite, or in some cases *affiliation*) network, where there are two types of nodes and edges connect pairs of nodes of different types.



Heterogeneous Information Networks

- Graph-based model allowing different types of nodes and edges (data mining purpose). This schema can be used to formulate patterns.
- For example, the pattern *user1 - user2 - tweet* indicates a user following another user who posted a tweet. These patterns can be used to express information extraction queries, for example, retrieve all the followers of users who posted a specific tweet.



Other Models

- Multilayer(ed) Models
- Network of Networks
- Temporal Networks
- Exponential Random Graph Models
- The common aspects of all these models are as follows:
 - the fact that actors are organized into different layers and
 - the fact that nodes in different layers can correspond to the same actor.

Temporal Networks

- Time is a kind of continuous attribute, making it quite distinct from discrete layers – even if some methods for temporal networks are defined on discretized versions of the data.
- As different points in time can be considered as different layers, in theory a social network observed at different times can be analyzed using multilayer methods.

Measuring (four approaches)

- Flattening
 - Projection to non-layered network
- Single layer SNA
 - Layers analyzed separately
- Intralayer = interlayer
 - Cross-layers
- Intralayer \leftrightarrow interlayer
 - Common usage

Flattening and Projection

- A *Basic (Unweighted) Flattening* process consists in a layer with one node for each actor and an edge between two nodes if an edge among two nodes corresponding to those actors exists somewhere in the multilayer network.
- A *Weighted Flattening* process consists in adding a weight to each edge proportional to the number of edges between the actors corresponding to those nodes.

Degree Based Measures

- **Degree Centrality:** Let $a \in A$ be an actor on $L \subseteq \mathbf{L}$ a set of layers and $M = (A, \mathbf{L}, V, E)$ a multilayer network. The degree centrality of a on L is defined as

$$\text{degree}(a, L) = |\{(a, l), (a', l')\} \in E \text{ s.t. } l, l' \in L|.$$

- **Degree deviation** is defined as the standard deviation of degrees of a over the layers.

$$\sqrt{\frac{\sum_{l \in L} \left(\text{degree}(a, \{l\}) - \frac{\text{degree}(a, L)}{|L|} \right)^2}{|L|}}$$

Neighborhood Based Measures

- **Neighbors:** Let $a \in A$ be an actor an $L \subseteq \mathbf{L}$ a set of layers and $M = (A, \mathbf{L}, V, E)$ a multilayer network. The neighbors of a on layers L are defined as

$$\mathit{neighbors}(a, L) = \{a' \in A \mid \{(a, l), (a', l')\} \in E \text{ and } l, l' \in L\}$$

- **Neighborhood Centrality:** The neighborhood of a on layers L is defined as

$$\mathit{neighborhood}(a, L) = |\mathit{neighbors}(a, L)|$$

- **Connective Redundancy:** The connective redundancy of a on layers L is defined as

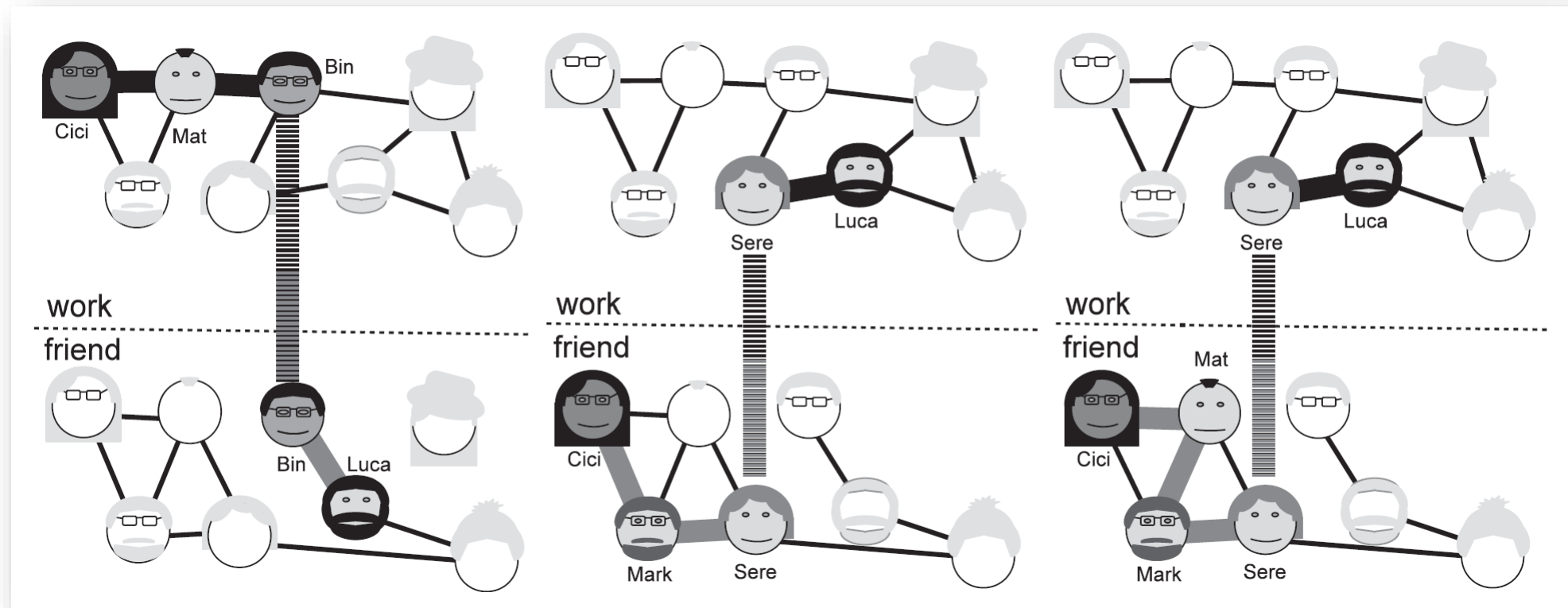
$$\mathit{connective\ redundancy}(a, L) = 1 - \frac{\mathit{neighborhood}(a, L)}{\mathit{degree}(a, L)}$$

- **Exclusive Neighborhood:** The exclusive neighborhood of a on layers L is defined as

$$\mathit{xneighborhood}(a, L) = |\mathit{neighbors}(a, L) \setminus \mathit{neighbors}(a, \mathbf{L} \setminus L)|$$

Distance Based Measures

- A more general concept of *Multilayer Distance* that makes a difference between edges on different layers.



Seminar Assignment

- Select one multi-layer network from the source below and compute all the measures given in the presentation for each network actor/node.
- <http://multilayer.it.uu.se/datasets.html>
- <https://manliodedomenico.com/data.php>

References

- Dickison, M. E., Magnani, M., Rossi, L. (2016). *Multilayer social networks*. Cambridge University Press. <http://multilayer.it.uu.se>.
- Porter, M-A. (2014) *Multilayer Networks (tutorial)*. <http://www.slideshare.net/masonporter/multilayer-tutorialnetsci2014slightlyupdated>.
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