DATA ANALYSIS II

Multilayer Social Networks I
Basics and Degree Based Measures
2021/2022

References

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- Bianconi, G. (2018). *Multilayer networks: structure and function.* Oxford university press. https://www.maths.nottingham.ac.uk/plp/pmzsc/tnetworks/Threshold_GBIANCONI.pdf

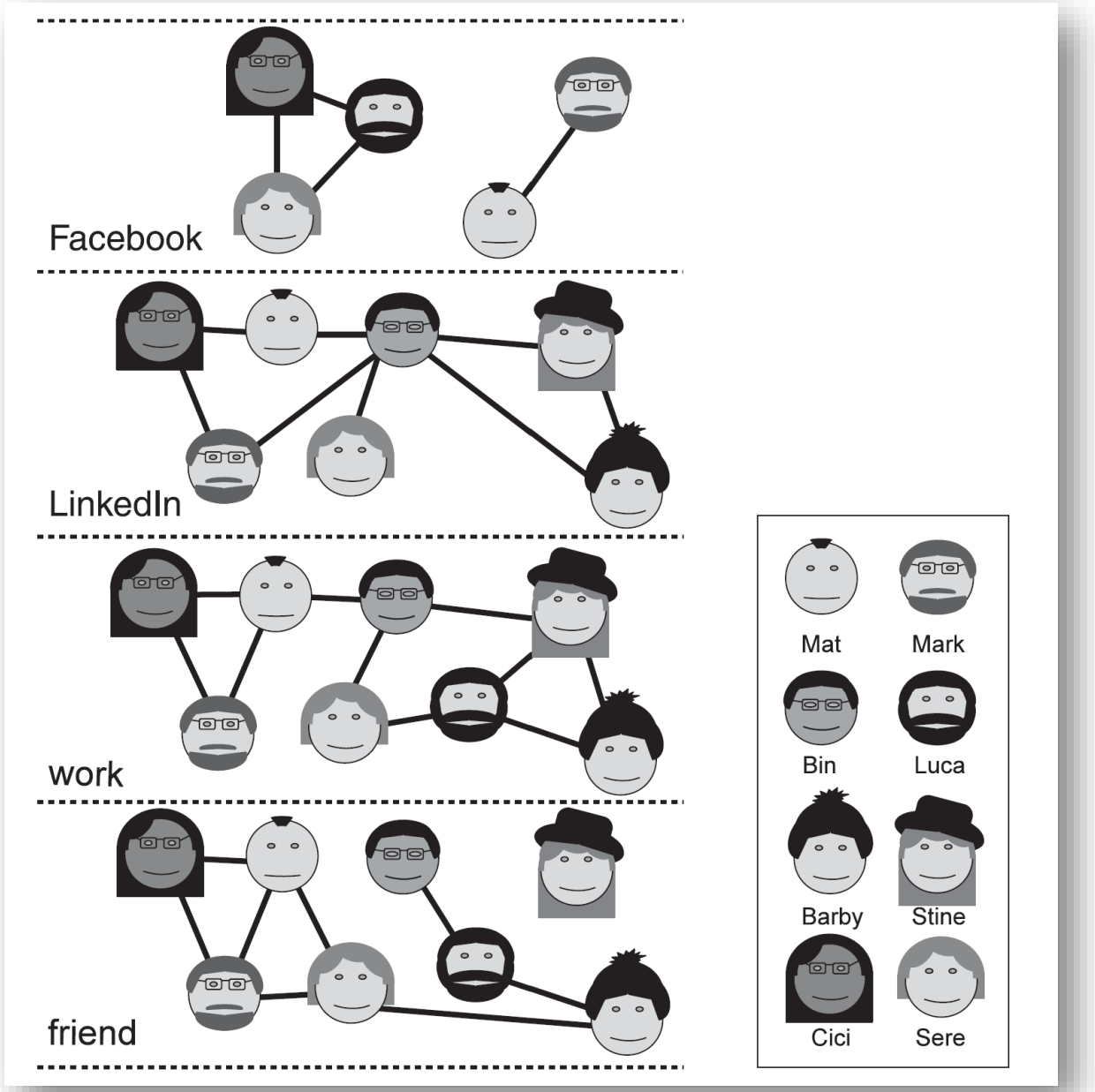
- Porter, M-A. (2014) *Multilayer Networks (tutorial).* http://www.slideshare.net/masonporter/multilayer-tutorialnetsci2014slightlyupdated.

- Kivelä, M., Arenas, A., Barthelemy, M., Gleeson, J. P., Moreno, Y., Porter, M. A. (2014). *Multilayer networks.* Journal of complex networks, 2(3), 203-271. http://people.maths.ox.ac.uk/porterm/papers/multilayer_review-published.pdf.

Problem…

- Dealing with multiple social network is part of our daily experience…
  - We live in many networks…
- We continuously juggle our networks…
- We bridge them to move valuable information from one network to another…

Example


Facebook
LinkedIn
work
friend
Mat
Mark
Bin
Luca
Barby
Stine
Cici
Sere

Definition (1/2)

- **Multilayer Network:** Given a set of actors A and a set of layers L, a multilayer network is define as a quadruple $M = (A, \boldsymbol{L}, V, E)$ where (V, E) is a graph and $V \subseteq A \times \boldsymbol{L}$.

- **Actor:** An entity that can have relationships with other actors.

- **Layer:** The same actor can be represented in different layers, where each layer represents a type of actors or a type of edge between actors.

Definition (2/2)

- **Node:** A specific actor on a specific layer.
- **Edge:** A relationship between two nodes.
- **Multilayer Network:** A social network represented as a set of layers, where nodes in different layers refer to a global set of actors and edges can also connect nodes in the same or different layers.
- **Single-layer Network:** A social network represented as a network with only one layer.

Tasks

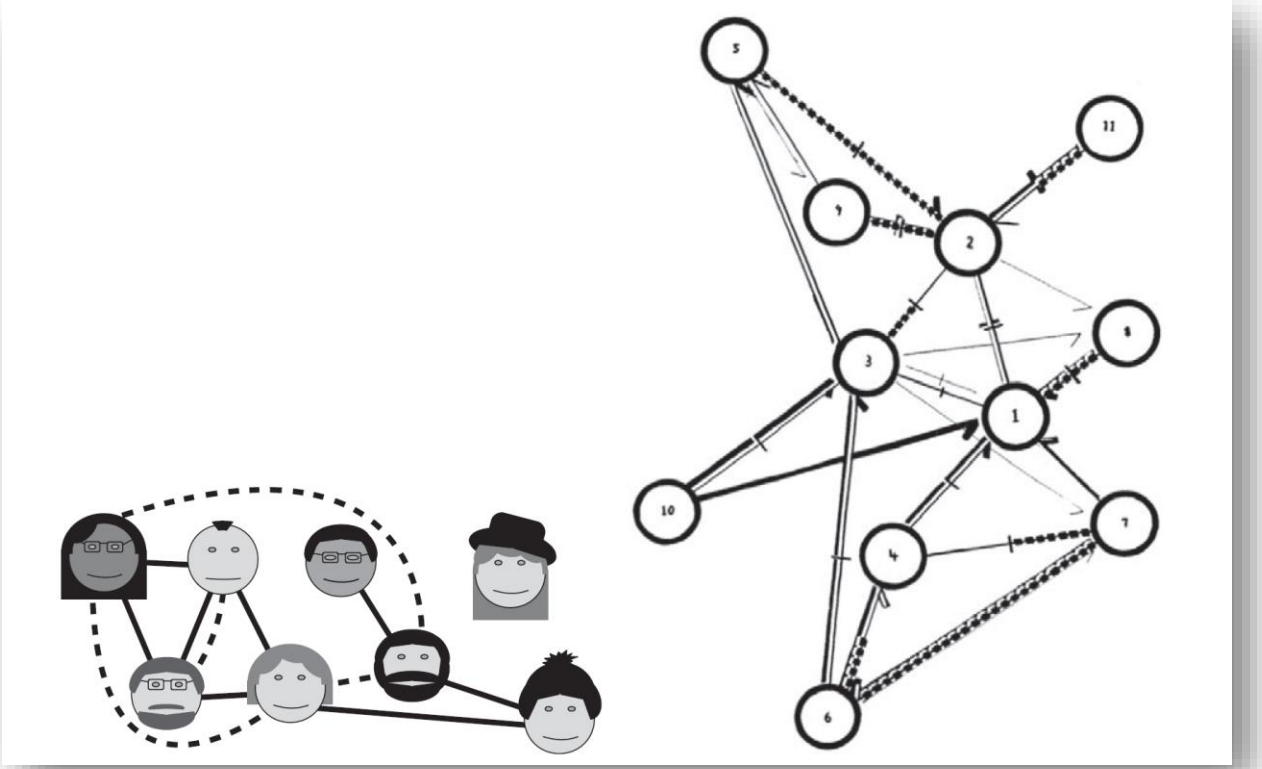
- **Models and measures**
  - **Actor measures**
  - Layer measures
- Mining multilayers networks
  - Visualizing multilayer networks
  - Community detection
  - Edge patterns
- Dynamical processes
  - Formation of multilayer networks
  - Information and behavior diffusion

Related Models

- Multiplex networks
- Multimode and multilevel networks
- Heterogeneous information networks
- Networks of networks
- Temporal networks
- Exponential random graph models

Multiplex Networks

- A common set of actors is connected through multiple types of edges. These different kinds of relational ties are typically known as *multiplex ties* in the SNA literature.



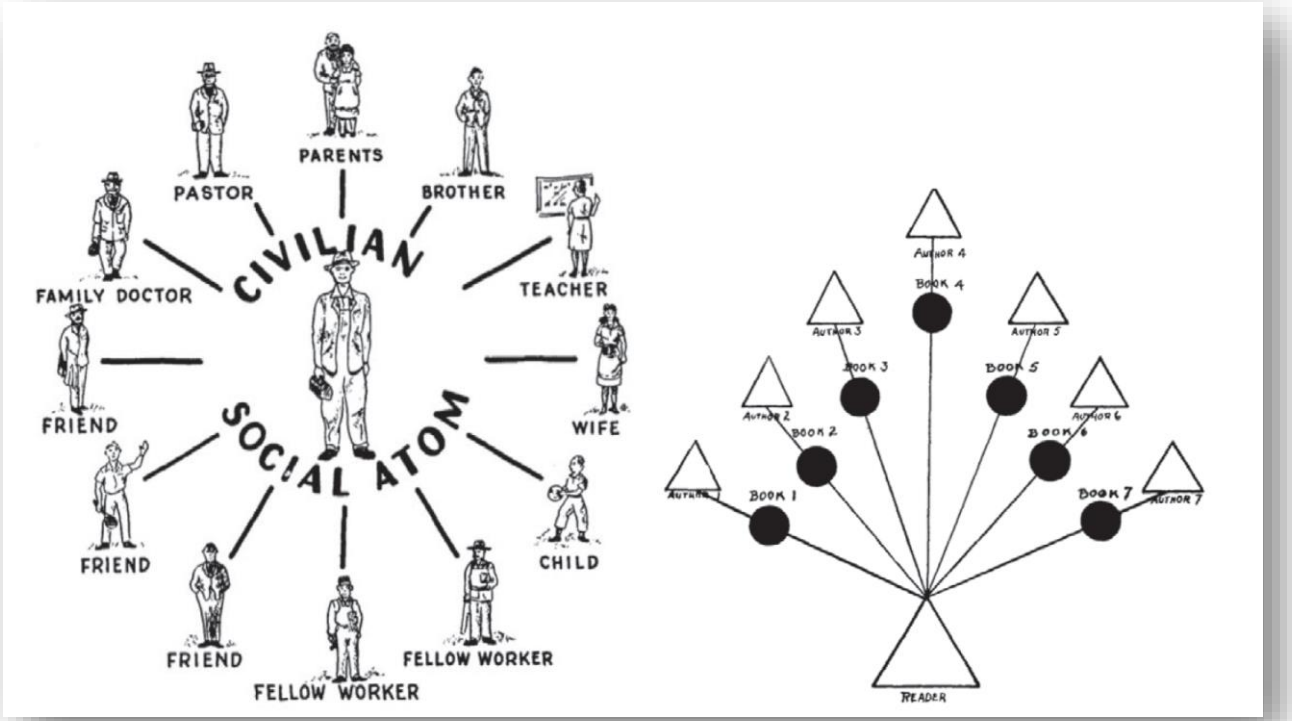
Representing Multiplex Social Networks

	Cici	*Mat*	*Mark*	*Sere*	*Bin*	*Luca*	*Stine*	*Baby*
Cici	0	1	1	0	0	0	0	0
Mat	1	0	1	1	0	0	0	0
Mark	1	1	0	1	0	0	0	0
Sere	0	1	1	0	0	0	0	1
Bin	0	0	0	0	0	1	0	0
Luca	0	0	0	0	1	0	0	1
Stine	0	0	0	0	0	0	0	0
Baby	0	0	0	1	0	1	0	0

	Cici	*Mat*	*Mark*	*Sere*	*Bin*	*Luca*	*Stine*	*Baby*
Cici	0	0	0	1	0	1	0	0
Mat	0	0	1	0	0	0	0	0
Mark	0	1	0	0	0	0	0	0
Sere	1	0	0	0	0	1	0	0
Bin	0	0	0	0	0	0	0	0
Luca	1	0	0	1	0	0	0	0
Stine	0	0	0	0	0	0	0	0
Baby	0	0	0	0	0	0	0	0

Multimode and Multilevel Networks

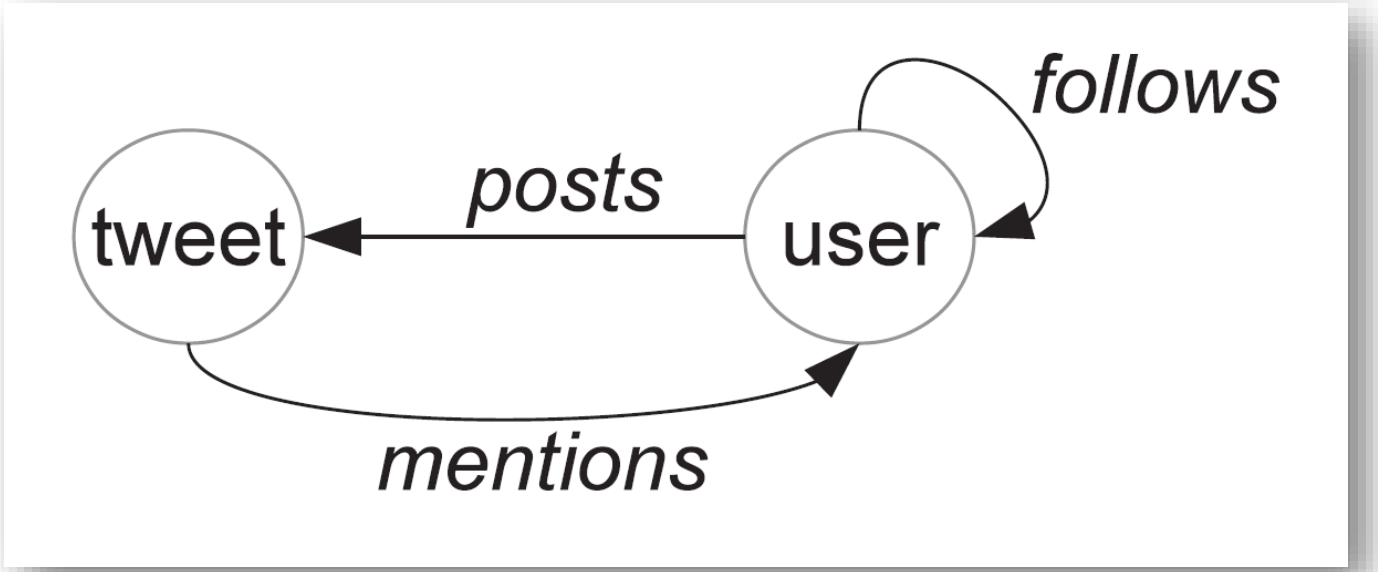
- Multiple types of actors inside the same network. A network where two types of actors can be identified is a two-mode (also called bipartite, or in some cases *affiliation*) network, where there are two types of nodes and edges connect pairs of nodes of different types.



Heterogeneous Information Networks

- Graph-based model allowing different types of nodes and edges (data mining purpose). This schema can be used to formulate patterns.

- For example, the pattern *user1 - user2 - tweet* indicates a user following another user who posted a tweet. These patterns can be used to express information extraction queries, for example, retrieve all the followers of users who posted a specific tweet.



Other Models

- Multilayer(ed) Models
- Network of Networks
- Temporal Networks
- Exponential Random Graph Models
- The common aspects of all these models are as follows:
  - the fact that actors are organized into different layers and
  - the fact that nodes in different layers can correspond to the same actor.

Temporal Networks

- Time is a kind of continuous attribute, making it quite distinct from discrete layers – even if some methods for temporal networks are defined on discretized versions of the data.

- As different points in time can be considered as different layers, in theory a social network observed at different times can be analyzed using multilayer methods.

Measuring (four approaches)

- Flattening
  - Projection to non-layered network
- Single layer SNA
  - Layers analyzed separately
- Intralayer = interlayer
  - Cross-layers
- Intralayer <> interlayer
  - Common usage

Flattening and Projection

- A ***Basic (Unweighted) Flattening*** process consists in a layer with one node for each actor and an edge between two nodes if an edge among two nodes corresponding to those actors exists somewhere in the multilayer network.

- A ***Weighted Flattening*** process consists in adding a weight to each edge proportional to the number of edges between the actors corresponding to those nodes.

Degree Based Measures

- **Degree Centrality:** Let $a \in A$ be an actor an $L \subseteq \boldsymbol{L}$ a set of layers and $M = (A, \boldsymbol{L}, V, E)$ a multilayer network. The degree centrality of a on L is defined as

$$degree(a, L) = |\{\{(a, l), (a', l')\} \in E \; s.t. \; l, l' \in L\}|.$$

- **Degree deviation** is defined as the standard deviation of degrees of a over the layers.

$$\sqrt{\frac{\sum_{l \in L} \left(\text{degree}(a, \{l\}) - \frac{\text{degree}(a,L)}{|L|} \right)^2}{|L|}}$$

Neighborhood Based Measures

- **Neighbors:** Let $a \in A$ be an actor an $L \subseteq \boldsymbol{L}$ a set of layers and $M = (A, \boldsymbol{L}, V, E)$ a multilayer network. The neighbors of a on layers L are defined as

$$neighbors(a, L) = \{a' \in A | \{(a, l), (a', l')\} \in E \; and \; l, l' \in L\}$$

- **Neighborhood Centrality:** The neighborhood of a on layers L is defined as

$$neighborhood(a, L) = |neighbors(a, L)|$$

- **Connective Redundancy:** The connective redundancy of a on layers L is defined as

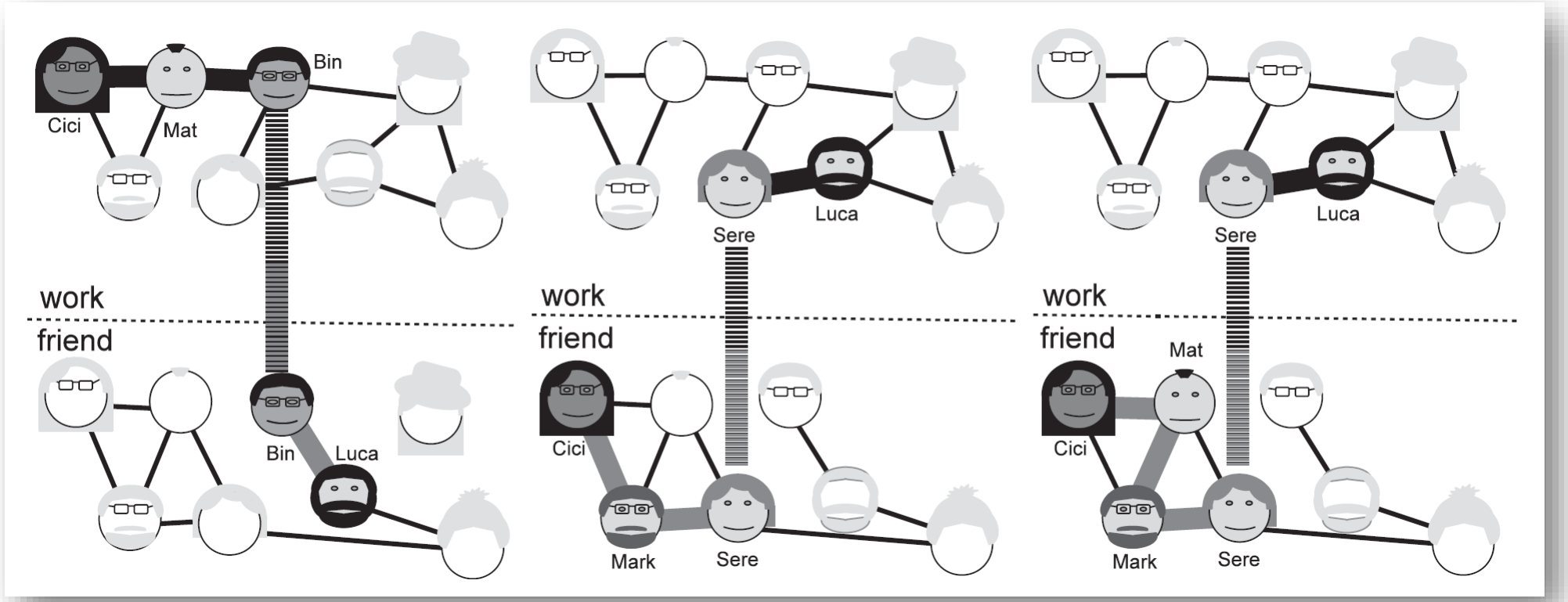
$$connective \; redundancy(a, L) = 1 - \frac{neighborhood(a, L)}{degree(a, L)}$$

- **Exclusive Neighborhood:** The exclusive neighborhood of a on layers L is defined as

$$xneighborhood(a, L) = |neighbors(a, L) \setminus neighbors(a, \boldsymbol{L} \setminus L)|$$

Distance Based Measures

- A more general concept of *Multilayer Distance* that makes a difference between edges on different layers.



Seminar Assignment

- Select one multi-layer network from the source below and compute all the measures given in the presentation for each network actor/node.

- http://multilayer.it.uu.se/datasets.html
- https://manliodedomenico.com/data.php