On non-unique solutions of contact problems with the Coulomb friction

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Abstract

Our contribution deals with contact problems of linear elasticity with the Coulomb friction in two space dimensions. The finite element approximation leads to an algebraic problem whose projective primal-dual formulation is given as an equation described by a piecewise differentiable function. The solution can be computed iteratively by the semi-smooth Newton method [1]. However, there are known examples, in which the iterates do not converge. One of reasons consists in the fact that the Coulomb friction problem may have more than one solution.

We will analyze the solution set of this problem. There are two strategies how to proceed. The first one is based on a theoretical discussion of the piecewise differentiable function describing the discrete problem [3]. It requires to confine ourselves to simple situations, e.g. to one contact node. The strategy presented in our contribution enables us to analyze large-scale problems arising from complex finite element discretizations. The idea is based on a generalization of the continuation technique as introduced in [2]. The continuation algorithm follows the branches of solutions for an appropriately parameterized problem with the Coulomb friction. As this problem is non-smooth, the branches are non-smooth curves (in the space of a high dimension). The continuation algorithm can not only follow branches but it have also to detect non-smooth points, in which the character of solutions is considerably changed. The branches turn back in non-smooth points for problems with non-unique solutions. The continuation algorithm is combined with the semi-smooth Newton method in our numerical experiments.

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References

