VSB TECHNICAL UNIVERSITY OF OSTRAVA | FACULTY OF CIVIL ENGINEERING | DEPARTMENT OF STRUCTURAL MECHANICS

Topic 10:
Structural Reliability - Summary

- Probabilistic approach to assessing the reliability and safety of building structures
- Failure probability calculation
- Design service life of the structure

Probabilistic Approach

The confidence in the methods of level II and III expressed in terms of **probabilistic reliability indicators** (β reliability index, failure probability P_f).

Reliability criterion:

$$P_f \leq P_d \qquad \beta_d < \beta$$

P_f ... probability of failure
P_d ... design value of failure probability

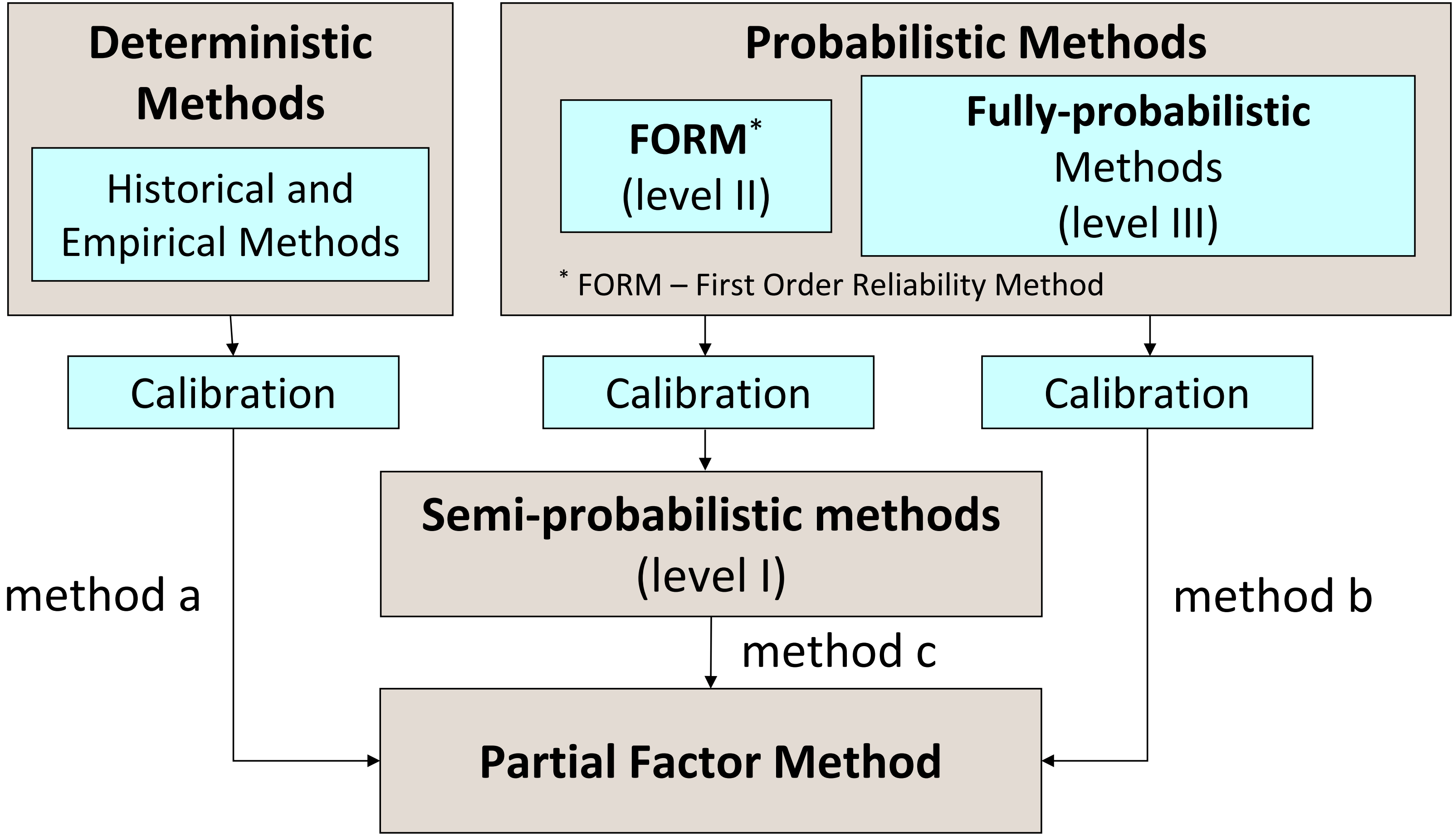
Reliability function:

$$RF = R - E$$

R ... structural resistance
E ... load effect

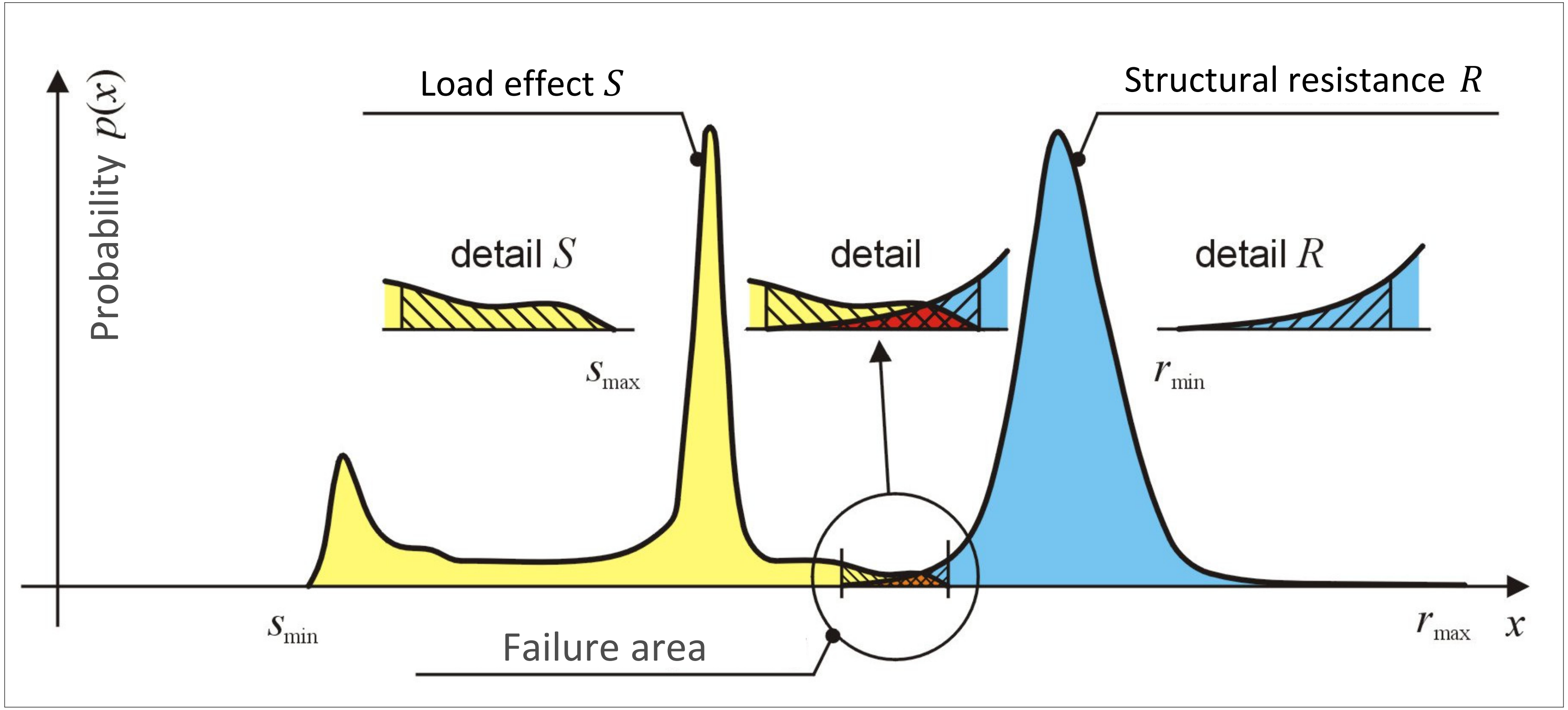
$$P_f = P(RF < 0) = P(R < E)$$

Overview of Reliability Methods



Probability of Failure - Calculation

Failure occurs when the condition is fulfilled: $RF < 0$ $RF = R - S$



Load effect

When determining the effects of loads in probabilistic calculations, the following must be taken into account:

- load source,
- method of action on the structure,
- load intensity,
- load direction,
- load duration,
- environmental influence – e.g. change in temperature or humidity.

The effect of the load S (also denoted by the letter E) must be considered as a random variable, especially due to the **random variability of the load in time and space**.

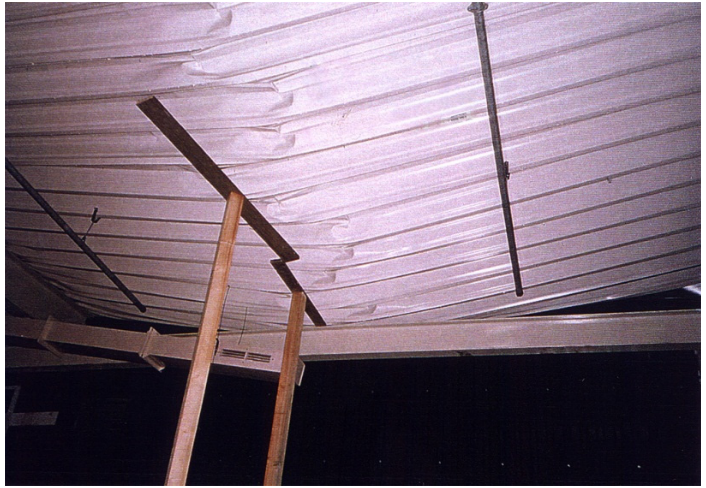
Load effect

The random variables associated with the load are most often expressed by means of **histograms of mean** or **extreme values**.

The so-called **load duration curves** are often used, where the variability of the load is monitored for a certain period of time and the obtained values are finally sorted in ascending order.

The quantity expressing the effect of the load is related to the **limit state** according to which the given probabilistic reliability assessment is performed:

- In the case of the **ultimate limit state**, the effect of the load may represent the actual magnitude of the internal force, or stress.
- At the **serviceability limit state**, the load effect is given by the actual deformation of the structure.



Structural resistance

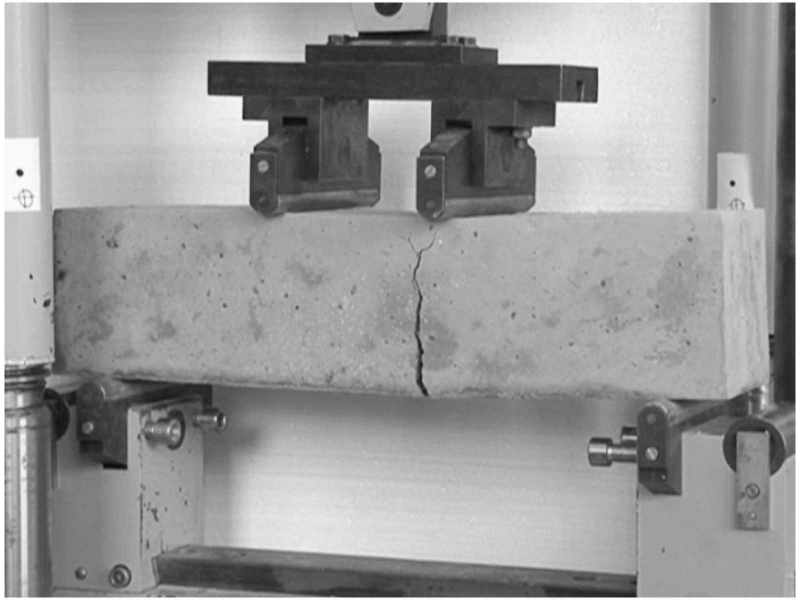
The definition of the **resistance of the structure** depends mainly on:

- **computational model**,
- **material properties of the structure** (material strength characteristics, stiffness characteristics of the material),
- **geometric characteristics of the structure** (shape, size of load-bearing elements, cross-sectional characteristics, manufacturing and assembly inaccuracies).

Structural resistance

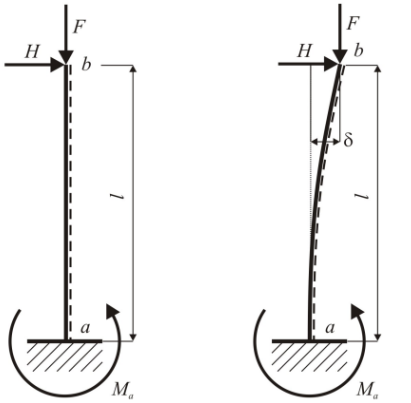
The quantity expressing the **resistance of the structure** is related to the limit state according to which the given probabilistic reliability assessment is performed:

- in the case of the **ultimate limit state**, the resistance of the structure may represent the **load capacity in analyzed stress**, which can be determined at the level of the internal force or stress,
- at the **serviceability limit state**, the resistance of the structure is defined by the limit deformation of the structure, or permissible oscillation frequency.



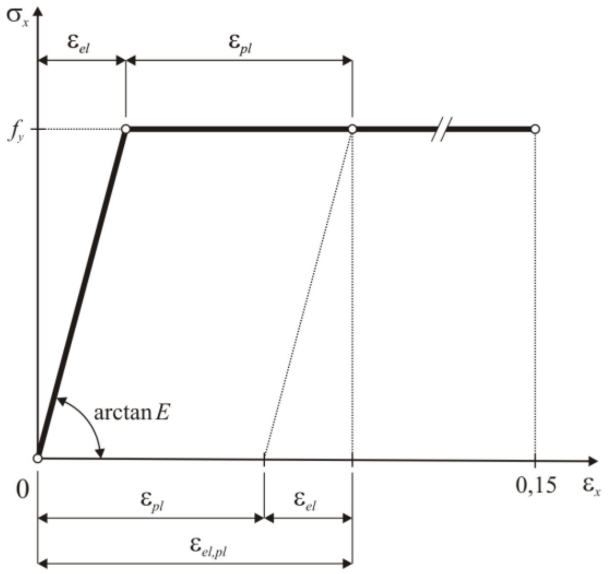
Computational model

The creation of the **computational model** is also related to the limit state within which the probabilistic calculation is performed.



The following plays its role:

- **Calculation methodology** used (1st order theory, 2nd order theory),
- **Mathematical description of construction material behavior** (elastic behavior of the material - the limit state of reaching the yield stress, plastic properties - the limit is plastic load capacity, permissible amount of permanent deformation or elongation of the material).



Computational model

The **calculation model** may also be affected by the fact whether the subject of the probabilistic calculation is a reliability assessment:

- **Part of the supporting structure** (supporting element, the most stressed cross section),
- **The whole supporting system**.

In probabilistic assessment, the computational model leads to the definition of a **reliability function** called RF, which is also called **failure function** G or **safety margin** Z.

Reliability function

The structure must be designed so that the **resistance of the structure** R is greater than the **load effect** E.

Probabilistic reliability assessment is based on the **reliability criterion**, which can be expressed, for example, in the form:

$$RF = R - E \geq 0 \qquad RF = \frac{E}{R} \leq 1 \qquad RF = \frac{R}{E} - 1 \geq 0 \qquad RF = \frac{R}{E} \geq 1 \qquad RF = 1 - \frac{E}{R} \geq 0$$

Failure to meet any **reliability condition** is an unfavorable state in terms of reliability, when the load effect E exceeds the resistance of the structure R.

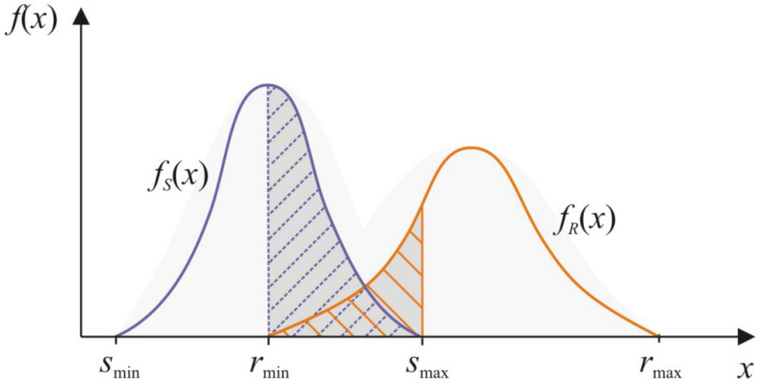
Probability of failure

The variability of the resistance of the structure R and the load effect S can be expressed by:

- **histograms**,
- **probability density function** (PDF) $f_{R(x)}$, $f_{S(x)}$,
- **cumulative distribution function** (CDF) $\Phi_{R(x)}$, $\Phi_{S(x)}$.

Mutual position of load effect density curve $f_{S(x)}$ and density curve of structural resistance $f_{R(x)}$

The relative position of the curves $f_{R(x)}$ and $f_{S(x)}$ characterizes and specifies the area in which a failure can occur, and at the same time allows to determine the **probability of failure** P_f, eg as:



$$P_f = P(RF < 0) = P(R - S < 0) = P(R < S)$$

Probability of failure

The failure occurs if, for example, the condition is met:

$$P_f = P(RF < 0) = P(R - S < 0) = P(R < S) = \iint_{D_f} f_{R,S}(r,s)\,\mathrm{d}r\,\mathrm{d}s$$

where D_f is **failure area** with safety margin $Z(\mathbf{X}) < 0$ and $f(X_1, X_2, \cdots, X_n)$ **probability density function** of **random variables** $\mathbf{X} = X_1, X_2, \cdots, X_n$.

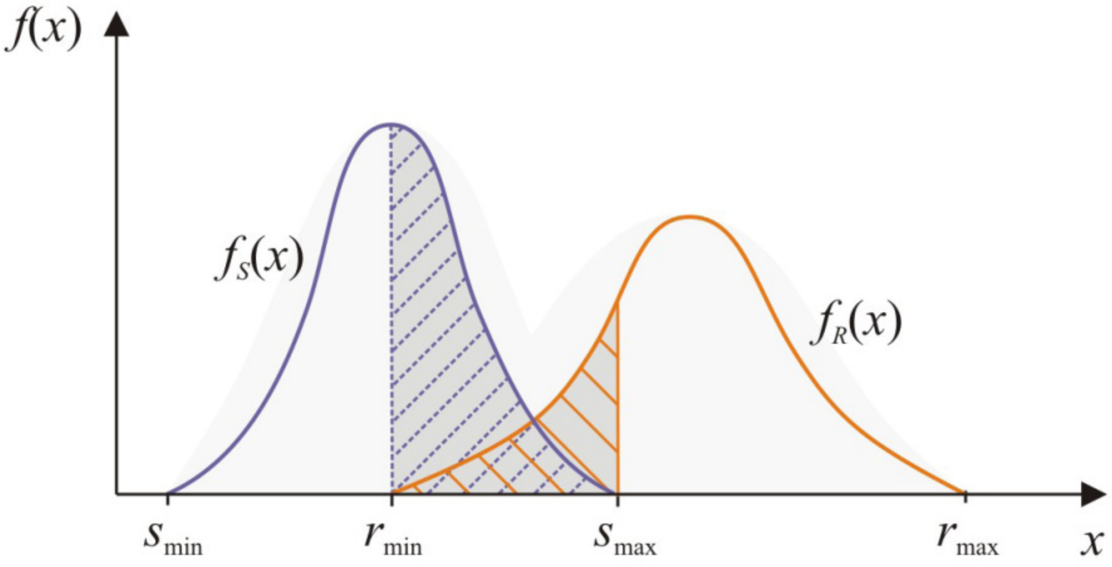
In the case of **statistical independence** of the structural resistance R and the **load effect** S, it is possible to adjust:

$$P_f = P(RF < 0) = P(R - S < 0) = P(R < S) = \int_{-\infty}^{\infty} \int_{-\infty}^{r \le s} f_R(r) \cdot f_S(s)\,\mathrm{d}r\,\mathrm{d}s$$

Probability of failure

If there are intersections of the probability density $f_{R(x)}$ with the x-axis with the value $x = r_{\min}$ and the probability density $f_{S(x)}$ with the x-axis with the value $x = s_{\max}$, then for any value of s applies:

- **the failure does not occur** if $r_{\min} > s_{\max}$. The probability of failure is then $P_f = 0$,
- **the failure can but may not occur** if $r_{\min} \leq s \leq s_{\max}$. The probability of failure can then take the magnitude $0 \leq P_f \leq 1$ for all possible values of s,
- **occurs whenever** $r_{\max} < s_{\min}$. The probability of failure is then $P_f = 1$.



Probability of failure

The probability that the resistance of the structure R is less than the given value x can also be determined on the basis of the **distribution function** $\Phi_R(x)$:

$$P(R \leq x) = \Phi_R(x)$$

The probability that the load effect S will be in the interval $\langle x - \mathrm{d}x; x + \mathrm{d}x\rangle$ is equal to:

$$P\left(x - \frac{\mathrm{d}x}{x} \leq S \leq x + \frac{\mathrm{d}x}{x}\right) = f_S(x)$$

The probability of the simultaneous validity of both expressions is given by the product $\mathrm{d}P_f = f_S(x) \cdot \Phi_R(x)\,\mathrm{d}x$. For x from the interval $(-\infty; \infty)$ the probability of failure P_f is given by the integral:

$$P_f = P(R - S < 0) = P(R < S) = \int_{-\infty}^{\infty} \mathrm{d}P_f = \int_{-\infty}^{\infty} f_S(x) \cdot \Phi_R(x)\,\mathrm{d}x$$

Design value of failure probability

The **degree of structural reliability** in the probabilistic calculation is the **limit design value** of the **failure probability** p_d (hereinafter referred to as the design probability) or the **reliability index** β.

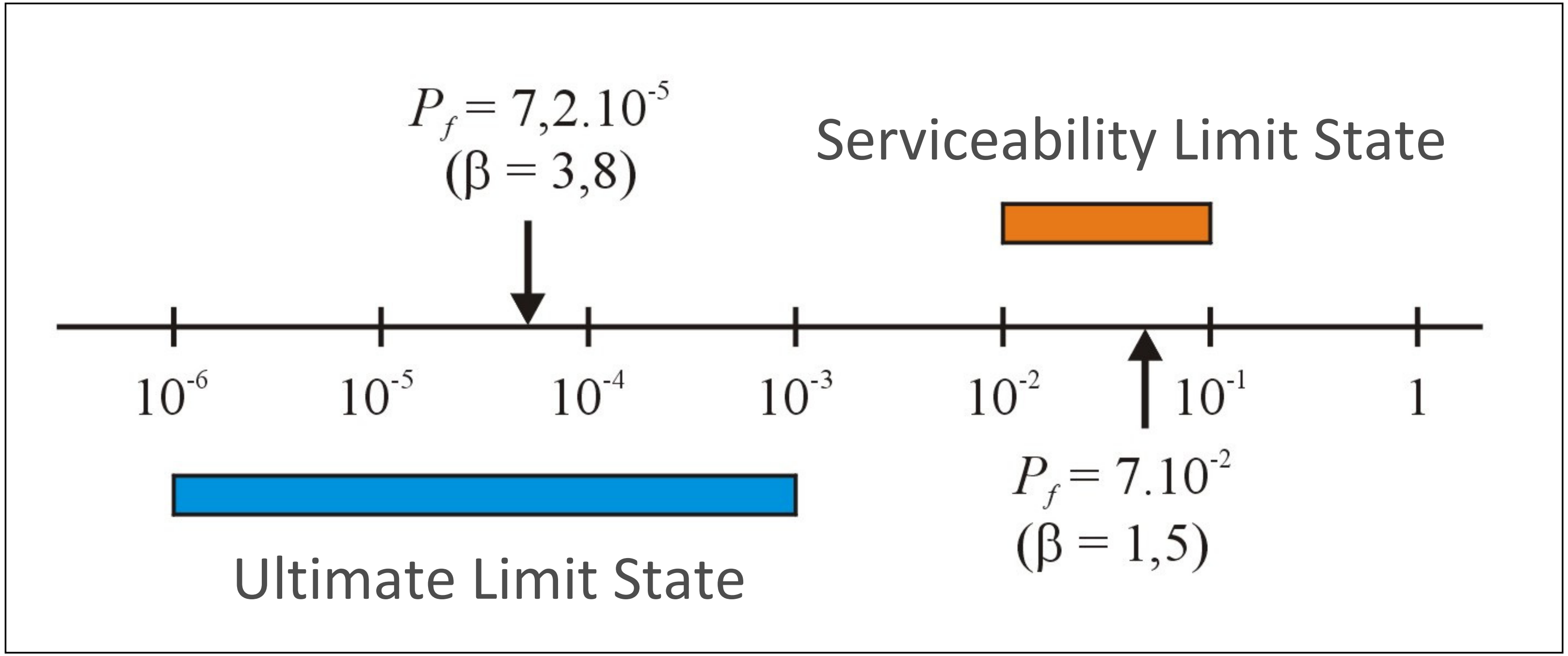
The construction is **reliable** if the **reliability conditions** are fulfill:

$$p_f \leq p_d \qquad \beta_d < \beta$$

The **design value of the failure probability** p_d (or **reliability index** β) is determined on the basis of:

- required **levels of reliability**,
- type of **limit state**,
- expected **service life** of the structure T_d.

Indicator of reliability



The normal range of values for the **probability of failure** P_f
for a design life of 50 years - **Ultimate Limit State** and **Serviceability Limit State**
(and recommended values of the failure probability)

Design value of failure probability

Circumstances should be considered when selecting the **reliability level** of the structure under design, including:

- possible **cause** and / or way of reaching the limit state,
- the possible **consequences of the failure**, expressed by the risk of loss of life, injuries and possible economic losses,
- the **social severity of the failure**,
- the **costs and procedures** necessary to **reduce the risk of failure**.

Guideline values of design probabilities P_d, resp. **reliability index** β_d according to valid European standard documents are given in the following tables.

Consequences classes (EN 1990)

Consequences Class	*Description*	*Examples of buildings and civil engineering works*
CC3	**Serious consequences** for loss of human life, or for economic, social or environmental concerns	Grandstands, public buildings where consequences of failure are high (e.g., a concert hall)
CC2	**Moderate consequence** for loss of human life; economic, social or environmental consequences considerable	Residential and office buildings, public buildings where consequences of failure are medium (e.g., an office building)
CC1	**Low consequence** for loss of human life; economic, social or environmental consequences small or negligible	Agricultural buildings where people do not normally enter (e.g., storage buildings, greenhouses)

Design value of failure probability

Recommended minimum values of the **reliability index** β and **design value of failure probability** P_d (ultimate limit states) according to EN 1990 :

Reliability Class	Minimum Values β		P_d
	1 year reference period	50 years reference period	
RC3 (serious consequences)	5.2	4.3	$8.4 \cdot 10^{-6}$
RC2 (moderate consequence)	4.7	3.8	$7.2 \cdot 10^{-5}$
RC1 (low consequence)	4.2	3.3	$4.8 \cdot 10^{-4}$

Design value of failure probability

Recommended minimum values of the **reliability index** β and **design value of failure probability** P_d (serviceability limit states) according to EN 1990 :

Reliability Class	**Minimum Values** β		P_d
	1 year reference period	50 years reference period	
RC2 (moderate consequence)	2.9	1.5	$6.7 \cdot 10^{-2}$

Relation between β and P_f (EN 1990)

P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-7}	10^{-8}
β	1.28	2.32	3.09	3.72	4.27	4.75	5.20

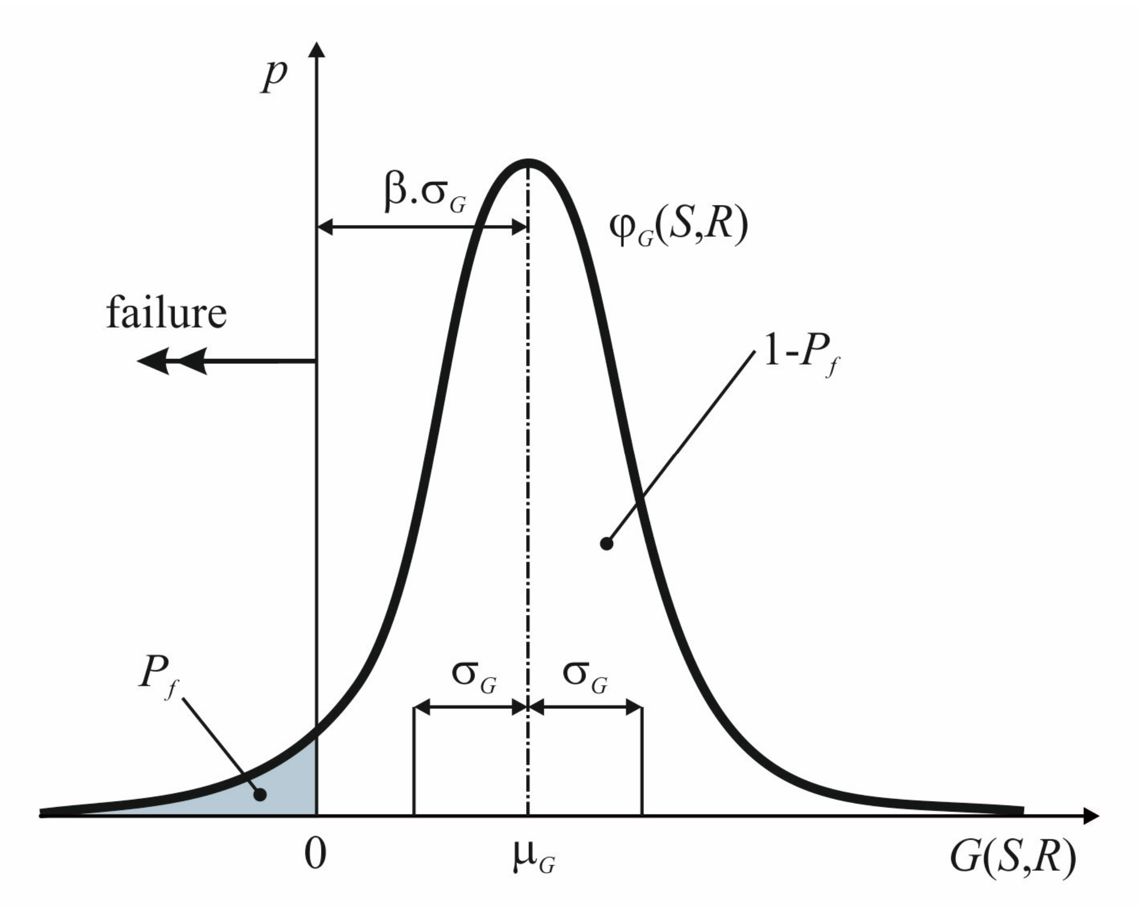
Differentiation reliability of structures

Based on:

- selecting values of **reliability indexes**,
- **adjustment of partial factors** for loads or properties related to resistance,
- level of **control in the design**,
- level of **control during implementation**,
- level of **inspection and compliance** with procedures referred to project documentation.

For the purposes of reliability differentiation EC recommended three classes of consequences **CC1 to CC3** (*consequences classes*).

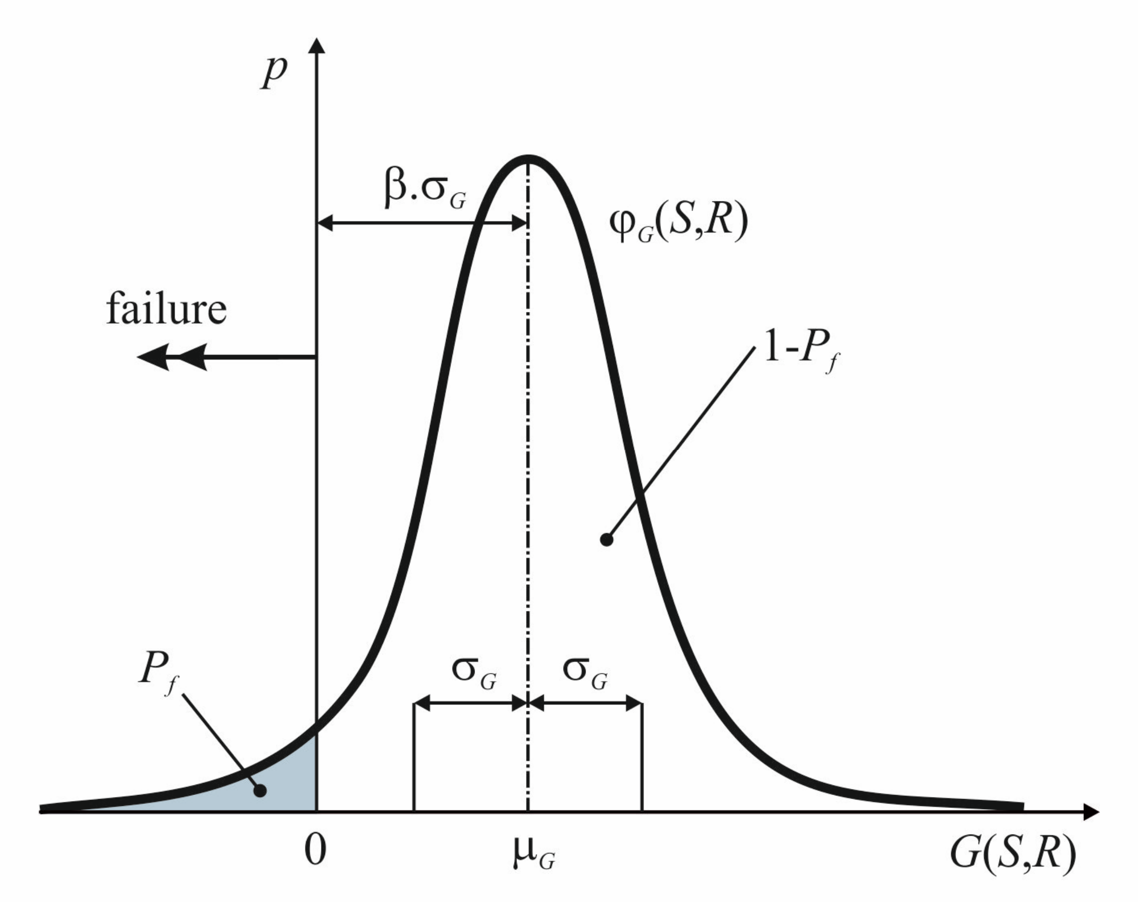
Reliability index



The scheme is based on the definition of the **reliability function** G taking into account the **normal probability distribution** of the **structural resistance** R and the **load effect** S.

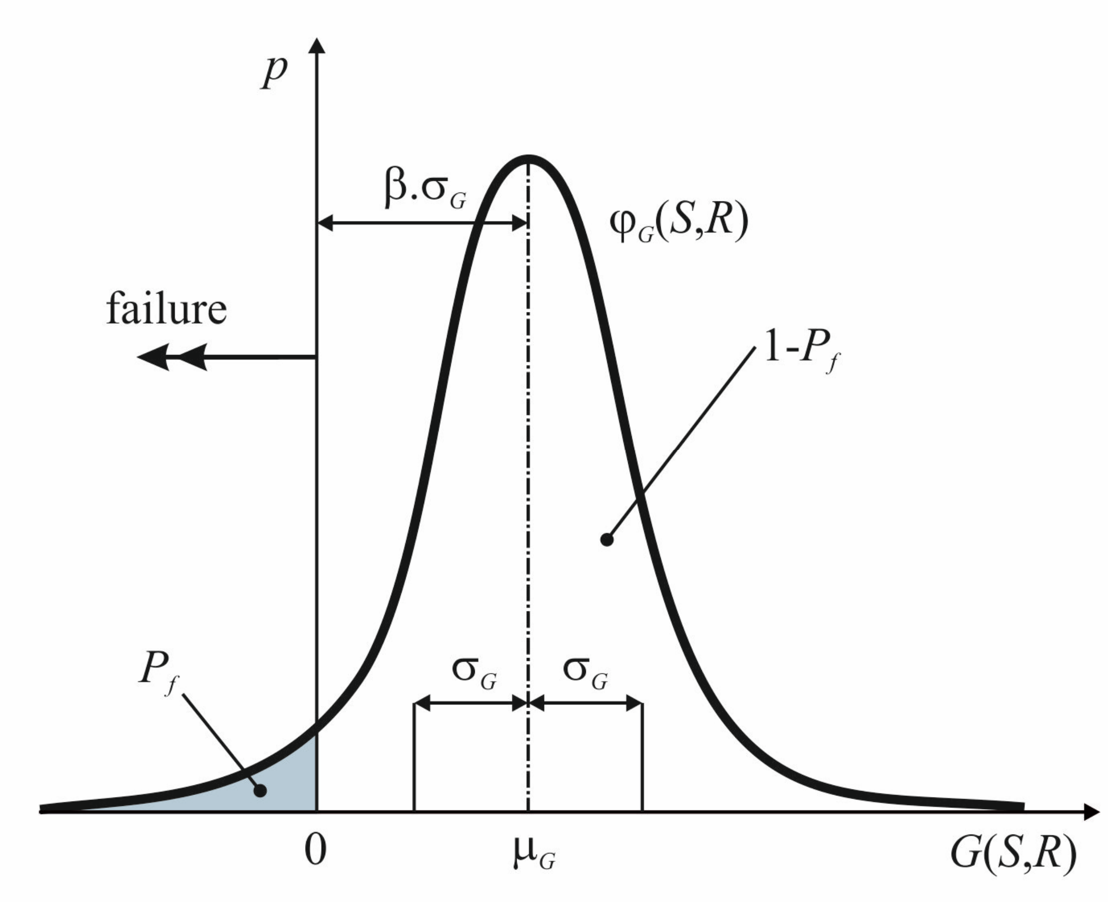
A design failure occurs when $G < 0$.

Reliability index



The **reliability index** β is then determined as the distance of the **mean value** of the reliability function G from the origin, determined in units of the **standard deviation** σ_G.

Reliability index



The following applies to the **reliability index** β:

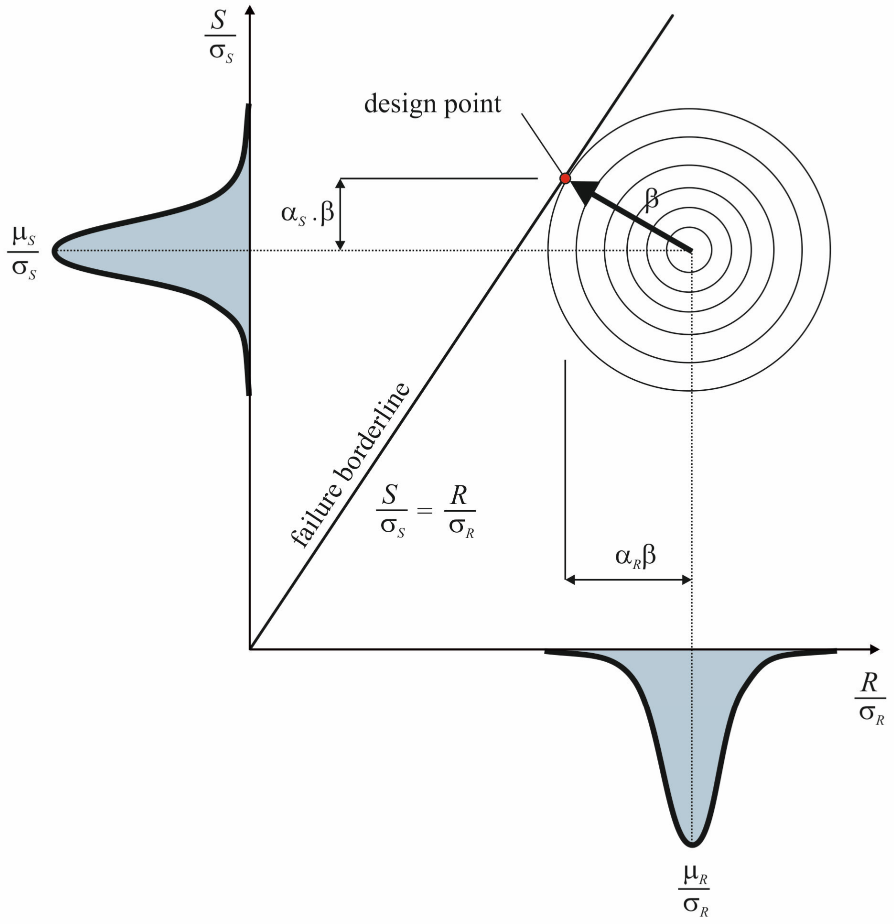
$$\beta = \frac{\mu_G}{\sigma_G}$$

where the **mean value** μ_G and the **standard deviation** of σ_G are given by:

$$\mu_G = \mu_R - \mu_S$$

$$\sigma_G = \sqrt{\sigma_R^2 - \sigma_S^2}$$

Reliability index



Graphical interpretation of the **reliability index** β:

The line defining the failure area is described by:

$$\mu_G - \beta \cdot \sigma_G = 0$$

The quantities $\alpha_{R,S}$ represent the separation coefficients of the random variable S from R and vice versa.

Reliability index β, which is linked to the so-called design point:

$$\beta = \frac{\mu_G}{\sigma_G} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 - \sigma_S^2}} \geq \beta_d$$

Useful transfer relationships

Direct relationship between reliability index β and failure probability p_f:

$$P_f = \Phi(-\beta) \qquad P_d = \Phi(-\beta_d)$$

where Φ denotes the **distribution function of the normalized normal distribution** ($\mu_N = 0, \sigma_N = 1$), resp.

$$\beta = -\Phi^{-1}(p_f) \qquad \beta_d = -\Phi^{-1}(p_d)$$

where Φ^{-1} is the **inverse distribution function of the normalized normal distribution** for the failure probability P_f resp. P_d.

Relation between β and P_f (EN 1990)

P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-7}	10^{-8}
β	1.28	2.32	3.09	3.72	4.27	4.75	5.20

Useful approximation relations

Approximation expressions for the calculation of the **failure probability** P_f and the **reliability index** β for the interval $10^{-7} \leq P_d \leq 10^{-1}$:

$$P_f \approx 10^{-0.1981\cdot(-\beta-0.605)^2-0.297}$$

$$\beta \approx -0.605 + \sqrt{-\frac{\log P_f}{0.198} - 1.5}$$

Example: The design value of the reliability index β_d for the reliability class RC2 and the design life 50 is 3.8. The same value can be obtained by substituting the design probability $P_d = 7.2 \cdot 10^{-5}$ of the same reliability class RC2 into the approximation or conversion relation:

$$\beta_d \approx -0.605 + \sqrt{-\frac{\log(7.2 \cdot 10^{-5})}{0.198} - 1.5} \cong 3.802104 \approx 3.8$$

$$\beta_d = -\Phi^{-1}(7.2 \cdot 10^{-5}) \cong 3.801195 \approx 3.8$$

NORMSINV table function can be used

Useful approximation relations

Example:

On the other hand, the value of the design probability P_d can be determined by substituting the reliability index $\beta_d = 3.8$ (reliability class RC2 with a design life of 50 years) into the approximation or conversion relation:

$$P_f \approx 10^{-0.1981\cdot(-3.8-0.605)^2-0.297} \cong 7.228741 \cdot 10^{-5} \approx 7.2 \cdot 10^{-5}$$

NORMSDIST table function can be used:

$$P_f = \Phi(-3.8) = 7.234804 \cdot 10^{-5}$$

Design service life of the structure

When determining the value of the **design probability** P_d , the design life of the structure can also be taken into account, ie the expected time for which the structure or its part is to be used during routine maintenance for the specified purpose, but without the need for major repairs.

Five informative categories of **design life** and indicative values of construction life in years can be defined, including examples of constructions according to the recommendations of the national annex EN 1990.

Design service life of the structure

Design life category	Informative design service life T_d (in years)	Examples
1	10	Temporary structures
2	10 to 20	Interchangeable components, eg crane girders, bearings
3	25 to 50	Agricultural and similar buildings
4	80	Buildings and other common structures
5	100	Monumental buildings, bridges and other engineering structures

Informative design lifetimes according to EC 1990.

Design service life of the structure

For structures with a design life T_{dx} other than the reference design life T_d, the design probability P_{dx} can be determined:

$$P_{dx} = 1 - (1 - P_d)^{\frac{T_{dx}}{T_d}},$$

where p_d is the design probability for the design lifetime T_d and P_{dx} is the design probability at time $T_{dx} \leq T_d$.

Note: the probability of failure is a **function of time** $P_f(t)$.