

Topic 9: Applications of DOProC method

- Probabilistic calculation of fatigue damage prediction in cyclically loaded steel structures
- Reliability assessment of arch supports in underground and mining workings

Usage of DOProC method

- Probabilistic assessment of load combinations,
- Probabilistic reliability assessment of cross-sections and systems of statically (in)definite load-bearing constructions,
- Probabilistic approach to assessment of mass concrete and fibrous concrete mixtures,
- Reliability assessment of arch supports in underground and mining workings,
- Reliability assessment of load-bearing constructions under impact loads,
- Probabilistic calculation of fatigue damage prediction in cyclically loaded steel structures.



Fatigue crack propagation

Fatigue crack propagation using linear fracture mechanics

Paris-Erdogan law: $\frac{da}{dN} = C \cdot \Delta K^m$ where C, m are material constants (determined experiment

Having modified:

$$\int_{a_1}^{a_2} \frac{\mathrm{d}a}{\left(\sqrt{\pi \cdot a} \cdot f_{(a)}\right)^m} = \int_{N_1}^{N_2} C \cdot \Delta \sigma^m \,\mathrm{d}N$$

C, *m* are material constants (determined experimentally), *a* is fatigue crack length, *N* is number of fatigue loading cycles,

 ΔK is range of the stress intensity factor.

$$\Delta K = \Delta \sigma \cdot \sqrt{\pi \cdot a} \cdot f_{(a)}$$

where number of fatigue cycles from N_1 to N_2 is needed to increase the **fatigue crack length** from the a_1 to a_2 ,

 $\Delta\sigma$ is constant stress range,

 $f_{(a)}$ is the **calibration function** - represents the course of propagation of the crack (e.g. from the edge or from the surface, determined experimentally).

Probabilistic calculation of fatigue crack propagation

Resistance of the structure:

where

- a_0 is **initial** fatigue crack length,
- is **detectable** length of the fatigue crack, a_d
- a_{ac} is **acceptable** length of the fatigue crack.

Cumulated effect

Cumulated effect of loads:
where N is total number of fatigue loading
cycles of constant stress range
$$\Delta \sigma$$
 needed to increase the crack from a_0 to a_d or a_{ac} ,

 $R_{(a_d)} = \int_{a_0}^{a_d} \frac{\mathrm{d}a}{\left(\sqrt{\pi \cdot a} \cdot f_{(a)}\right)^m}$

 $R_{(a_{ac})} = \int_{a_{ac}}^{a_{ac}} \frac{\mathrm{d}a}{\sqrt{1-1}}$

 N_0 is total number of fatigue loading cycles of constant stress range in time of fatigue crack initialization.

Safety margin, probability of failure:

where X is vector of random physical properties.

$$G_{fail}_{(\mathrm{X})} = R_{(a_{ac})} - E_{(N)}$$

$$P_f = P(G_{fail}(X) < 0) = P(R_{(a_{ac})} - E_{(N)} < 0)$$

Probabilistic growth of the fatigue crack in time



Probability distribution of increased fatigue crack in time

Probability of defined random events

Probability of **crack undetection** in time *t*: $P(U_{(t)}) = P(a_{(t)} < a_d)$ where a_d is minimal **detectable** length of the crack.

Probability of crack detection in time t, crack size $a_{(t)}$ is less than acceptable length of the crack a_{ac} : $P(D_{(t)}) = P(a_d \le a_{(t)} < a_{ac})$

Probability of crack detection in time t, crack size $a_{(t)}$ is equal or greater than acceptable length a_{ac} : $P(F_{(t)}) = P(a_{(t)} \ge a_{ac})$

 $P(F_{(t)}) \ge P_d \rightarrow \text{inspection}$

All of these three events creates **full space of random events**, which can come in time *t*, can be applied: $P(U_{(t)}) + P(D_{(t)}) + P(F_{(t)}) = 1$

Bayes' theorem

Bayes' theorem describes the probability of an event, based on prior knowledge of conditions that might be related to the selected event.

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} \qquad P(B) > 0$$

Probability of event *F* in time *T* with respect to the **results of structural inspection** in time $t_I < T$:

$$P\left(F_{(T)} \mid U_{(t_{I})}\right) = \frac{P(F_{(T)}) - P(F_{(t_{I})}) - P(D_{(t_{I})}) \cdot P\left(F_{(T)} \mid D_{(t_{I})}\right)}{P(U_{(t_{I})})}$$
$$P\left(F_{(T)} \mid D_{(t_{I})}\right) = \frac{P(F_{(T)}) - P(F_{(t_{I})}) - P(U_{(t_{I})}) \cdot P\left(F_{(T)} \mid U_{(t_{I})}\right)}{P(D_{(t_{I})})}$$

Design of structural inspections



Design of structural inspections based on probability of failure P_f , conditional probability and required degree of reliability P_d .

Reliability assessment of steel flange in tension



Look to the reviewed road bridge, photo: Assoc. Prof. J. Odrobiňák

> Detail of solved steel bridge's flange, photo: Assoc. Prof. J. Odrobiňák





Fatigue damage danger's concentration

Crack's propagations **from the edge** or **from the surface** are possible to monitor according to initial crack position.



Weakness of the same flange increased from the edge is quicker then from the surface.

Crack's propagations from the edge



Steel railway bridge near Hodonín built in 1929, photo: prof. V. Tomica, 1998.

Fatigue crack propagation from the edge in the wall of longitudinal beam



Crack's propagations from the edge



Fatigue crack in the weld of the connected crossbeam

Fisher at al, A Fatigue Primer for Structural Engineers, 1998.

Crack's propagations from the edge



View of the fatigue crack, propagating from the edge, arising at the left edge of the weld (60x magnification)

Corrosion Testing Laboratories, Inc., 2007

Crack's propagations from the surface



Section of the load-bearing element with an example of fatigue crack growth from the surface

Fisher J.W. at al. (1998) A Fatigue Primer for Structural Engineers

Probabilistic calculation of fatigue damage prediction in cyclically loaded steel structures

Crack's propagations from the surface



Semi-elliptical fatigue crack from the surface of a propeller

Sanford R.J. (2003) Principles of Fracture Mechanics

Probabilistic calculation of fatigue damage prediction in cyclically loaded steel structures

Overview of variable input quantities

Some of input values - no way to obtain using measurement, can be approximate only.

Quantity	Type of parametric	Parameters				
Quantity	distribution	Mean Value	Standard Deviation			
Oscillation of stress peaks $\Delta\sigma$ [MPa]	Normal	30	3			
Total number of oscillation of stress peaks per year N [-]	Normal	106	10 ⁵			
Initial size of the crack a_0 [mm]	Lognormal	0.2	0.05			
Smallest measurable size of the crack a_d [mm]	Normal	10	0.6			
Yield stress of material f_y [MPa]	Lognormal	280	28			
Nominal stress in flange σ [MPa]	Normal	200	20			

Quantity	Mean Value
Constant of material <i>m</i>	3
Constant of material C [MPa ^{<i>m</i>} m ^{(<i>m</i>/2)+1}]	2.2.10-13
Flange width b_f [mm]	400
Flange thickness t_f [mm]	25

Real values

Approximate values

Probabilistic calculation of fatigue damage prediction in cyclically loaded steel structures

List of **constant**

input variables

Program FCProbCalc

Probabilistic calculation of fatigue crack propagility	gation in flang	ge in tensio	n of the cyclic loa	aded stru	uctures (Ve	rsion 1. 2. 1.	0) 🗖	
Function Set up Help								
Input data Results Inspections								
Fatigue crack progression from the	e edge	•		Par	ameter epsi	on for bounde	ed parametric	: histogram
Number of years n starting / step / end values :	0 / 1 /	150						1E-8
Design value of the limit probability pd	2.277E-2						Number	of intervals
Width of the flange in tension bf [mm]	400	Thickness (of the flange in tension	tf [mm]	25		1	100
Constant of material C	2.2E-13	Constant of	material m		3			
	Parametric	/ Raw data	Parametric distribu	ition	Mi	Sigma	N int	
Oscillation of stress peaks DeltaS [MPa]	Parametric	•	Normal	•	30	3	100	
Total number of oscillation of stress peaks per year	Parametric	•	Normal	-	1E6	1E5	100	
Yield stress of material Fy [MPa]	Parametric	•	LogNormal_2P	•	280	28	100	
Nominal stress in flange in tension Sigma [MPa]	Parametric	•	Normal	•	200	20	100	
Initial size of the crack a0 [mm]	Parametric	•	LogNormal_2P	•	0.2	0.05	98	
Detectable size of the crack ad [mm]	Parametric	•	Normal	-	10	0.6	100	
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FCProbCalc program desktop - entry of input

quantities

Cumulated load effect

Determined for each year of the bridge operation using time step equals **1 year**.

FCProb	Calc : Graph of h	istogram, reliabili	ty					1		×
	<u>i</u>									
							INT	f (z)	pf (z)	
	Histogram	: C*DeltaS^m*	Nn (C=2.2E-1	3, m=3, n=	49)		0	1.71291E-2	1.12657E-17	
		(26.4.20	12 , 11:47:52)				1	1.84317E-2	6.05388E-10	
0,0085		1					2	1.97342E-2	3.67905E-09	
0,008 -						5.555	3	2.10368E-2	4.88110E-13	
0,0075							4	2.23394E-2	1.84525E-09	
0,007		·					5	2.36420E-2	1.30240E-08	
0,0065		·····					6	2.49445E-2	4.18003E-11	
0.006							7	2.62471E-2	5.23440E-09	
0.0055							8	2.75497E-2	5.03988E-08	
0.005 -							9	2.88522E-2	1.01471E-09	
0.0045							10	3.01548E-2	5.62264E-09	
0,0045 7							11	3.14574E-2	6.11808E-08	
0,004 -							12	3.27600E-2	3.91024E-09	
0,0035 -		· · · · · · · · · · · · · · · · · · ·					13	3.40625E-2	2.98368E-08	
0,003 -		h					14	3.53651E-2	3.00885E-07	
0,0025		.					15	3.66677E-2	4.45624E-08	
0,002		1 4				2222	16	3.79702E-2	2.99332E-08	
0,0015		La					17	3.92728E-2	3.23716E-07	-
0,001			·····				Number	r of intervals	1021	
0,0005							Min		1.71291E-2	
0 3							Max		1.34575	
1.712	91E-2 1.90371E-1	3.81849E-1 5.720	25E-1 7.62200E-1	9.52376E-1	1.11389 1.248	06	Probabi	lity summation	1.00000	
		international constants in the second second	Loury Actives International Additional Second		1743989999 - 24097843		Frequer	ncy summation	8.16577E+27	



Total number oscillation of stress peak per **110 years**

Total number oscillation of stress peak per **49 years**

Structural resistance

Yet possible to select 5 types of integration methods

FCProbCalc : Graph of histogram, reliability		[×
	INT	f (z)	pf (z)	
Histogram : Resistance Rad	0	7.16697E-3	7.57558E-3	
(26.4.2012, 11:52:47)	1	1.43339E-2	4.17128E-07	
	2	2.15009E-2	3.11990E-07	
0,055	3	2.86679E-2	4.36572E-07	
0.05	4	3.58349E-2	5.40980E-07	
n,00	5	4.30018E-2	7.00445E-07	
,045	6	5.01688E-2	9.01654E-07	
	7	5.73358E-2	1.26523E-06	
U,U4	8	6.45028E-2	1.49573E-06	
035	9	7.16697E-2	1.88407E-06	
	10	7.88367E-2	2.35054E-06	
0,03	11	8.60037E-2	2.87768E-06	
025	12	9.31707E-2	6.32507E-08	
	13	1.00338E-1	3.58137E-06	
0,02	14	1.07505E-1	4.23428E-06	
015	15	1.14672E-1	6.84571E-07	
,013	16	1.21839E-1	5.36172E-06	
0,01	17	1.29006E-1	5.23801E-06	•
005	Numbe	r of intervals	256	
	Min		7.16697E-3	
₀╨ ┍╶┍╶┥╶╱╝╝╝╝╝╝╝╝╝╝╝╝╝╝╝╝╝	Max		1.82758	
7.16697E-3 2.43677E-1 5.08855E-1 7.66866E-1 1.00338 1.18255 1.36889 1.55523 1.74157	Probabi	lity summation	1.00000	
	Freque	ncy summation	4.56943E+27	



Propagation of the crack **from the surface**

Propagation of the crack from the edge

Probability of defined random events



The crack **from the edge** 0 to 80 years of operation

The crack **from the surface** 0 to 150 years of operation

Probability of the event U, D and F, depending on years of operation of the bridge



Estimation of inspection's times

Fatigue crack from the edge

Input data Results Inspections	
Choice of integration method : Adaptive integration (tol0 = 0.0001)	Eirst inspection time Charts
Fatigue crack progression from : the edge	Fatigue crack progression from the edge
Resistance Rad	0,35 F
Resistance Raac	0,3
Total number of oscillation of stress peaks Nn per n years	0,25
Cumulated effect of Loads Sn = C * (DeltaS^m) * Nn	0,2
Reliability function U (RE = Rad - Sn)	0,15
Reliability function E (RE = Raac - Sn)	0,1
	0,05
Year	02 5 8 12 17 22 27 32 37 42 47 52 57 62 67 72 77
P(U) 6.17882E-01 80 ▼ P(D) 1.02294E-01	Time of the first inspection :
p(F) 2.79824E-01	between years : 49 - 50

FCProbCalc program desktop

FCProbCalc code - application

Fatigue crack from the surface

The resulting probabilities and times of construction inspections

			n)									
nput d	ata Result	s Inspections										
	Yea	ar of inspection	F	robabilities of U of the fir	,D,F events ir st inspection t	n calculation ime		Table of th	of probabilities ne selected 2	of D,F,Fiever th inspection	nts in calculat time (p(U)=0	ion)
	Inspection	between the years	N	p(U)	p(D)	p(F)		N	p(D)	p(F)	F2	
	1	110 - 111	0	9.99631E-01	2.60401E-05	3.42732E-04		110	1.00000E+00	0.00000E+00	0.00000E+00	
	2	123 - 124	1	9.96581E-01	8.83999E-05	3.33062E-03		111	9.38915E-01	6.10849E-02	9.39932E-04	
	3	131 - 132	2	9.93190E-01	4.50787E-05	6.76536E-03		112	8.87852E-01	1.12148E-01	2.06066E-03	
	4	137 - 138	3	9.92627E-01	1.60310E-05	7.35663E-03		113	8.27690E-01	1.72310E-01	3.14432E-03	
	5	142 - 143	4	9.92536E-01	3.20665E-06	7.46070E-03		114	7.71347E-01	2.28653E-01	4.33300E-03	
	6	147 - 148	5	9.92521E-01	9.87538E-07	7.47809E-03		115	7.28553E-01	2.71447E-01	5.72985E-03	
	7	not calculated	6	9.92518E-01	-4.20406E-07	7.48236E-03		116	6.84529E-01	3.15471E-01	7.13395E-03	
	8	not calculated	7	9.92517E-01	3.24942E-08	7.48303E-03		117	6.48193E-01	3.51807E-01	8.72668E-03	
	9	not calculated	8	9.92517E-01	2.64749E-09	7.48325E-03		118	6.18161E-01	3.81839E-01	1.05204E-02	
	10	not calculated	9	9.92517E-01	2.90549E-09	7.48334E-03		119	5.86515E-01	4.13485E-01	1.23499E-02	
			10	9.92517E-01	2.82780E-09	7.48337E-03		120	5.55518E-01	4.44482E-01	1.41981E-02	
			11	9.92517E-01	-1.44328E-09	7.48339E-03		121	5.27895E-01	4.72105E-01	1.62183E-02	
			12	9.92517E-01	5.08245E-09	7.48339E-03		122	4.99782E-01	5.00218E-01	1.82961E-02	
			13	9.92517E-01	1.13449E-09	7.48340E-03	-	123	4.80482E-01	5.19518E-01	2.06319E-02	Ŧ
piect										E		



The dependence of the probability of failure P_f and years of operation of the bridge (adaptive numerical integration method chosen with a parameter $tol_0 = 1 \cdot 10^{-4}$)

Estimation of inspection's times

Fatigue crack from the edge

atigue crack	Probabilistic calculation of fatigue crack propagation in f	lange in tension of the cyclic loaded structures (Version 1. 2. 1. 0)
	Function Set up Help	
rom the edge		
	Input data Results Inspections	
	Choice of integration method : Adaptive integration (tol0 = 0.0001) Eirst inspection time Charts
	Fatigue crack progression from : the edge	Fatigue crack progression from the edge
	Resistance Rad	0,35
	Resistance Raac	0,3
	Total number of oscillation of stress peaks Nn per n years	0,25
	Cumulated effect of Loads Sn = C * (Delta S^m) * Nn	0,2
	Reliability function U (RF = Rad - Sn)	0,15
	Reliability function F (RF = Raac - Sn)	0,1
	Year	0,05 0 02 5 8 12 17 22 27 32 37 42 47 52 57 62 67 72 77
	80 - p(D) 1.02294E-01	Time of the first inspection :
	p(F) 2./9824E-01	between years : 49 - 50
FCProbCalc program	Project	BUN
desktop	11:42:53	

Estimation of inspection's times

Fatigue crack **from the surface**

The resulting probabilities and times of construction inspections

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ut data Result	Inspections										
Ye	ar of inspection	P	robabilities of U of the fir	,D,F events ir st inspection t	n calculation ime		Table of th	of probabilities ne selected 2	of D,F,Fiever thinspection	nts in calculat time (p(U)=0	ior)
Inspection	between the years	N	p(U)	p(D)	p(F)		N	p(D)	p(F)	F2	
1	110 - 111	0	9.99631E-01	2.60401E-05	3.42732E-04		110			0.00000E+00	
2	123 - 124	1	9.96581E-01	8.83999E-05	3.33062E-03		111	9.38915E-01	6.10849E-02	9.39932E-04	
3	131 - 132	2	9.93190E-01	4.50787E-05	6.76536E-03		112	8.87852E-01	1.12148E-01	2.06066E-03	
4	137 - 138	3	9.92627E-01	1.60310E-05	7.35663E-03		113	8.27690E-01	1.72310E-01	3.14432E-03	
5	142 - 143	4	9.92536E-01	3.20665E-06	7.46070E-03		114	7.71347E-01	2.28653E-01	4.33300E-03	
6	147 - 148	5	9.92521E-01	9.87538E-07	7.47809E-03		115	7.28553E-01	2.71447E-01	5.72985E-03	
7	not calculated	6	9.92518E-01	-4.20406E-07	7.48236E-03		116	6.84529E-01	3.15471E-01	7.13395E-03	
8	not calculated	7	9.92517E-01	3.24942E-08	7.48303E-03		117	6.48193E-01	3.51807E-01	8.72668E-03	
9	not calculated	8	9.92517E-01	2.64749E-09	7.48325E-03		118	6.18161E-01	3.81839E-01	1.05204E-02	
10	not calculated	9	9.92517E-01	2.90549E-09	7.48334E-03		119	5.86515E-01	4.13485E-01	1.23499E-02	
		10	9.92517E-01	2.82780E-09	7.48337E-03		120	5.55518E-01	4.44482E-01	1.41981E-02	
		11	9.92517E-01	-1.44328E-09	7.48339E-03		121	5.27895E-01	4.72105E-01	1.62183E-02	
		12	9.92517E-01	5.08245E-09	7.48339E-03		122	4.99782E-01	5.00218E-01	1.82961E-02	
		13	9.92517E-01	1.13449E-09	7.48340E-03	-	123	4.80482E-01	5.19518E-01	2.06319E-02	
			-						F		



The dependence of the probability of failure P_f and years of operation of the bridge (Adaptive numerical integration method chosen with a parameter $tol_0 = 1 \cdot 10^{-4}$)

- 1. Rectangular method (default number of diferences 1000)
- 2. Simpson method (default number of diferences 1000)
- 3. Romberg method (default parameter n = 10)
- 4. Adaptive method (default value of tolerated inaccuracy $tol_0 = 10^{-4}$)
- 5. Gaussian quadrature





Comparison of the calculated 1st time inspection

Fatigue crack from the edge



Calculated times of the 1st inspection of the bridge construction with a particular attention on used numerical integration method

Time of the calculation

Fatigue crack from the edge



Calculation time for each type of numerical integration and the specified number of intervals of input random variables

Comparison of the calculated 1st time inspection

Fatigue crack from the edge

Calculated times of the 1st inspection of the bridge construction for Simpson method of numerical integration and the specified number of intervals of input random variables and number of diferences



Comparison of the calculated 1st time inspection

Fatigue crack from the edge

Calculated times of the 1st inspection of the bridge construction for Romberg method of numerical integration and the specified number of intervals of input random variables and specified parameter *n*



Calibration functions for short edge cracks under selected loads w is width of rectangular cross-section, f_v is yield stress.



Calibration functions for short edge cracks under selected loads w is width of rectangular cross-section, f_y is yield stress.



Calibration functions for short edge cracks under selected loads

3-Point Bending



$$f\left(\frac{a}{h}\right)_{3PBT\frac{l}{h}=2} = +1.0259 - 1.4659 \cdot \left(\frac{a}{h}\right) + 4.9318 \cdot \left(\frac{a}{h}\right)^2 - 2.4637 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{3PBT\frac{l}{h}=4} = +1.0691 - 1.3496 \cdot \left(\frac{a}{h}\right) + 5.1865 \cdot \left(\frac{a}{h}\right)^2 - 3.3509 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{3PBT\frac{l}{h}=8} = +1.0963 - 1.3052 \cdot \left(\frac{a}{h}\right) + 5.2829 \cdot \left(\frac{a}{h}\right)^2 - 3.5972 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{3PBT\frac{l}{h}=16} = +1.1079 - 1.2328 \cdot \left(\frac{a}{h}\right) + 5.0551 \cdot \left(\frac{a}{h}\right)^2 - 3.2837 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{3PBT\frac{l}{h}=80} = +1.118 - 1.1964 \cdot \left(\frac{a}{h}\right) + 5.0176 \cdot \left(\frac{a}{h}\right)^2 - 3.3127 \cdot \left(\frac{a}{h}\right)^3$$

w is width of rectangular cross-section, f_y is yield stress.

Calibration functions for short edge cracks under selected loads

4-Point Bending



$$f\left(\frac{a}{h}\right)_{4PBT\frac{l}{h}=2} = +1.2505 - 1.7928 \cdot \left(\frac{a}{h}\right) + 6.3295 \cdot \left(\frac{a}{h}\right)^2 - 4.4492 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{4PBT\frac{l}{h}=4} = +1.1535 - 1.2847 \cdot \left(\frac{a}{h}\right) + 5.1957 \cdot \left(\frac{a}{h}\right)^2 - 3.5502 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{4PBT\frac{l}{h}=8} = +1.1202 - 1.1634 \cdot \left(\frac{a}{h}\right) + 4.8443 \cdot \left(\frac{a}{h}\right)^2 - 3.0085 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{4PBT\frac{l}{h}=16} = +1.1222 - 1.2277 \cdot \left(\frac{a}{h}\right) + 5.2654 \cdot \left(\frac{a}{h}\right)^2 - 3.7958 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{4PBT\frac{l}{h}=80} = +1.1179 - 1.1235 \cdot \left(\frac{a}{h}\right) + 4.5993 \cdot \left(\frac{a}{h}\right)^2 - 2.5619 \cdot \left(\frac{a}{h}\right)^3$$

$$W \text{ is width of rectangular cross-section,}$$

 f_y is **yield stress**.

Overview of variable input quantities

	Type of parametric	Parameters				
Quantity	distribution	Mean Value	Standard Deviation			
Total number of fatigue loading cycles per 1 year N	Normal	106	10 ⁵			
Initial size of the crack a_0	Lognormal	0.2 mm	0.05 mm			
Smallest detectable size of the crack a_d	Normal	10 mm	0.6 mm			
Yield stress of material f_y	Lognormal	280 MPa	28 MPa			
Loading force in three-point bending test F_{3PB}	Normal	6 kN	0.6 kN			

Quantity	Mean Value
Constant of material m	3
Constant of material C	$2.2 \cdot 10^{-13} \text{ MPa}^m \text{m}^{(m/2)+1}$
Height of the rectangular cross-section h	0.1 m
Width of the rectangular cross-section <i>b</i>	0.01 m
Span of the element <i>l</i>	0.4 m
Designed probability of failure P_d	$0.02277 \ (\beta_d = 2)$

List of random input variables

3-Point Bending

List of deterministic input variables

Calibration functions for short edge cracks under selected loads

3-Point Bending

Inspection No.	Time of inspection [years]
1	35
2	46
3	48
4	50
5	51

Calculated times for the first five inspection of the structural element





Resulting histogram of the calculation for t = 35 years of structural operation: safety margin G_{fail} .

Resulting **probabilities of random events** U, D and F for the first 60 years of operation under various load.

Parallel computing

DOProC method is able to parallelize the calculation (tested on supercomputer, used only 12 cores yet).



IT4Innovations National Supercomputing Center, VSB-Technical University of Ostrava, rank **69th** worldwide in Top500 list (2021)

Parallel computing in MatLab

Results of the probabilistic reliability assessment of the element under **3-point bending** in Matlab **128 intervals in each histogram**



Probability of failure: $P_f = 0.01641$



Resulting histogram of the calculation for t = 35 years of structural operation: safety margin G_{fail} .

Parallel computing in MatLab

Number of intervals	64		128			256	1024		
Core count	Time [min]	Probability of failure	Time [min]	Probability of failure	Time [min]	Probability of failure	Time [min]	Probability of failure	
1	0.39	0.013495	2.71	0.01641	21.91	0.018427	-	-	
3	0.13	0.013495	0.90	0.01641	7.10	0.018427	-	-	
6	0.08	0.013495	0.50	0.01641	3.77	0.018427	-	-	
9	0.07	0.013495	0.36	0.01641	2.62	0.018427	-	-	
12	0.06	0.013495	0.32	0.01641	2.03	0.018427	88.03	0.019917	

Comparison of **calculation time** depending on the number of cores and the number of classes in input histograms including the resulting probability of failure

Parallel computing in MatLab



Parallel algorithm scaling: decrease of **computing time** with increasing **number of processor units**, input histograms described by **128 classes** (left) and **256 classes** (right)

Reliability assessment of existing structures





Reliability aassessment of old crane tracks in a metallurgical company after **85 years of operation**



Reliability assessment of arch supports in underground and mining workings











- Design of anchor reinforcement required for the conditions determined in particular:
- length of anchors (bolts),
- their number and location around the mine or underground work,
- anchors parameters (type, material, diameter, etc.) for determining the structural resistance,
- load anchors.





Using the empirical-analytic methods and a set of input random variables was developed SW **Anchor** for probabilistic designing and assessment of the anchor reinforcement using DOProC method.

Output:

- length of bolts l
- number *n*
- resistance Q_{sv} of anchor reinforcement



Histogram of the reliability function RF, probability of failure $P_f = 1.19 \cdot 10^{-2}$ for 4 anchors Ø18 mm / 1 m of mining work

10				>					
Zadání 1						Zadání 2			
Název	Vysledky Kotveni [Ver	sion 1. U.17. U 7 2	3.5.2009 J			q : Odpor podpěrné	výztuže (kombi	novaná výztuž) [MPa]	G
E	8 : Šířka důlního díla [m]		Graf	B* [m]	4.99805	1			
1 Si	rka_Dulniho_dila.dis			28* [m]	9,99609			Délka svorníku	חר
Hornir	nové noměru nad důlní	n dílem :		CD (**)				Dona oronina	
monim	tore policity had danin					gamma : Objernová	hmotnost horni	n [t/m3]	G
Vrstva	Druh hominy	Pevnost v jedno	osém tlaku [MPa]		Mocnost [m] Graf	1 Objernova_Hmotnost	dis.		
	Prachovec	Pevnost_PRACH(DVEC.dis		5.39609				
2	Uhlí_popel_do_10%	Pevnost_UHLI.dis			.1			Zatizeni svornikū	
3	Jílovec	Pevnost_JILOVE0	C. dis		2	d1 vnější průměr s	vorníků [mm]	18	
4	Uhlí_popel_10-20%	Pevnost_UHLI.dis Pevnost_PISKOVEC_JEMNOZRNY.dis			1			1 0	
5	Pískovec_jemnozmý				.2	az vnitrni prumer s	vomiku [mm]		
6	Pískovec_hrubozrný	Pevnost_PISKOVEC_HRUB0ZRNY.dis			.3	n počet kotev		4	
7	Pískovec_střednězmitý	Pevnost_PISKOV	EC_STREDNEZRNY.di	\$	1	ds vzdálenost kote	ev [mm]	1000	
8									
10						SigmaH : Pevnost s	vomíku [MPa	1]	G
10			7.000005.4			1 HK24-Fy295-PLP-rok	. 2000.dis		_
β	Beta souč.vrstevnatosti (z tabulkyj	7.600002-1	<u>R</u> eduk	ovaná pevnost 🔺		0.	• únoenost svorníků	ſ
	H efektivní hloubka pod	povrchern [m]	800					Gilbaribat avoirintu	
	K : konuprapnění kodi	niont	lent	1			<u>P</u> e	osudek spolehlivosti	
1	KonvergKoef dis	GIGH	Circi	-					
						 Výběr a uložení his 	togramu		
	Kn : součinitel		Graf				*		
1	SoučinitelKn.dis					do tabulky B	- Sířka o	iùlmiho dila	•
				1		vúhěr tunu Param	etrické roz	dělení	-
-	HMH : Lieomechancký	Klasifikaćni koefici	ent Graf	Т	orba BMB	1950 (pa			5
	HMH Vytvoren (1 vorb	анмн ј						🚽 Ulož	

Desktop of the program

Suma četnosti

Hodnota f(z)

0.999

Velikost kvantilu F

Results: Length of bolts *l*



1.59447E+08

2.08317E+09

1.70442E+10

1.14004E+11

5.60714E+11

2.38803E+12

Histogram of the **bolt** length *l* [m]

Reliability assessment of arch supports in underground and mining workings

0.0000

0.0000

2.45785

6.52688E+18

pf(z) :

f(z) :

Kvantil

5 71463E-

5 86349E.

6.01235E-

6.16120E-1

6.31006E-

6.45892E-

6.60778E-

5 86349E-1

6.01235E-1

6.16120E-1

6.31006E-1

6.45892E-1

6.60778E-

6.75664E-1

1 1620E-12

2 4429E-1

3.1917E-10

2.6114E-09

1.7467E-0

8.5908E-0

3.6588E-0

Results: Bolt loading Q



Histogram of **bolt loading** Q [kN/m]

Results: Loading capacity of bolts Q_{sv}

$$Q_{sv} = \frac{n \cdot q_{sv}}{d_s} =$$
$$= \frac{n \cdot \pi \cdot (d_1 - d_2)^2 \cdot \sigma_{sv}}{4 \cdot d_s}$$

🔗 🛛 KOTVENÍ : 🛛 graf histogramu : SigmaR - pevnost svorníku [MPa] 🕞 🚺 Char 🎊 👖 Marks Value HK24-Fv295-PLP-rok 2000.dis [MPa] 8.2.2010 . 11:49:18 0.3 0.3 0,34 0.3 0 0.28 0.2 0.24 0.2 0 0.1 0.16 0.1 0.12 0.1 0,08 0,06 0.04 0,02 324.83333 338,50000 352.16667 365.83333 393.16667 406.83333 420.50000 434,16667 379.50000 Do Pravděpodobnost Četnos 318.00000 331.66667 324.83333 1.21951E-1 20 331 66667 345.33333 4 14634F-1 68 434,16667 345.33333 359 00000 1 64634E-1 27 Suma pravděpodobnosti 1.00000 372 66667 1 46341E-1 24 359 00000 Suma četnost 164 372 6666 386 33333 8 53659E-2 14 Hodnota f(z) 386 33333 400 00000 2 43902E-2 4 pf(z) : 0.0000 4 400.00000 413 66667 2 43902E-2 Kvantil 0.00000 413 66667 427.33333 1.21951E-2 2 Velikost kvantilu P

A histogram of loading capacity of bolts Q_{sv} [kN]

0.00000

f(z) :

427.33333

441.00000

Celkem

6.09756E-3

1.00000

1

164

Reliability assessment of anchor reinforcement

$$RF = Q_{sv} - Q$$

A histogram of the reliability function RF with the resultant failure probability P_f



Počet intervalů	251		INT Od		Do	Pravděpod	obnost	Četnost	
Min	-144.09185 277.15114 1.00000 5,75676E+18		0	-144.93433	-143.24936	2.9277E-1	9 2		
Мах			1	-143.24936	-141.56439	7.6876E-1	9 4	8	
Suma pravděpodobnosti			2	-141.56439	-139.87942	1.4633E-1	8 8		
Suma četnosti			3 -139.87942		-138.19445	9.2480E-1	8 53	53	
Hodnota f(z) :			Pravděpodobnost Kvantil Úroveň spolehlivosti		Zvýšená	Obvyklá	Snížená		
0	Kvantil	1.661//E-3 1.18853E-2			8.0000E-07	7.0000E-06	5.0000E-05	1.1885E-0	2
Velikost kvantilu P :					-6.4056E+01	-5.3946E+01	-4.3836E+01	0.00000	
	f(z) :	0.000000			NEVYHOVUJÍCÍ !				