Topic 9:

Applications of DOProC method

- Probabilistic calculation of fatigue damage prediction in cyclically loaded steel structures
- Reliability assessment of arch supports in underground and mining workings

Usage of DOProC method

- Probabilistic assessment of load combinations,
- Probabilistic reliability assessment of cross-sections and systems of statically (in)definite load-bearing constructions,
- Probabilistic approach to assessment of mass concrete and fibrous concrete mixtures,
- **Reliability assessment of arch supports in underground and mining workings**,
- Reliability assessment of load-bearing constructions under impact loads,
- **Probabilistic calculation of fatigue damage prediction in cyclically loaded steel structures**.

Fatigue crack propagation

Fatigue crack propagation using **linear fracture mechanics**

Paris-Erdogan law: $\frac{\mathrm{d}a}{\mathrm{d}N} = C \cdot \Delta K^m$ where

C, m	are **material constants** (determined experimentally),
a	is **fatigue crack length**,
N	is **number of fatigue loading cycles**,
ΔK	is range of the **stress intensity factor**.

$$\Delta K = \Delta\sigma \cdot \sqrt{\pi \cdot a} \cdot f_{(a)}$$

Having modified:

$$\int_{a_1}^{a_2} \frac{\mathrm{d}a}{\left(\sqrt{\pi \cdot a} \cdot f_{(a)}\right)^m} = \int_{N_1}^{N_2} C \cdot \Delta\sigma^m \, \mathrm{d}N$$

where number of fatigue cycles from N_1 to N_2 is needed to increase the **fatigue crack length** from the a_1 to a_2,

$\Delta\sigma$ is constant **stress range**,

$f_{(a)}$ is the **calibration function** - represents the course of propagation of the crack (e.g. from the edge or from the surface, determined experimentally).

Probabilistic calculation of fatigue crack propagation

Resistance of the structure:

where

a_0 is **initial** fatigue crack length,

a_d is **detectable** length of the fatigue crack,

a_{ac} is **acceptable** length of the fatigue crack.

$$R_{(a_d)} = \int_{a_0}^{a_d} \frac{\mathrm{d}a}{\left(\sqrt{\pi \cdot a} \cdot f_{(a)}\right)^m}$$

$$R_{(a_{ac})} = \int_{a_0}^{a_{ac}} \frac{\mathrm{d}a}{\left(\sqrt{\pi \cdot a} \cdot f_{(a)}\right)^m}$$

Cumulated effect of loads:

$$S_{(N)} = \int_{N_0}^{N} C \cdot \Delta\sigma^m \, \mathrm{d}N = C \cdot \Delta\sigma^m \cdot (N - N_0)$$

where N is **total number of fatigue loading cycles** of constant stress range $\Delta\sigma$ needed to increase the crack from a_0 to a_d or a_{ac},

N_0 is total number of fatigue loading cycles of constant stress range in time of fatigue crack initialization.

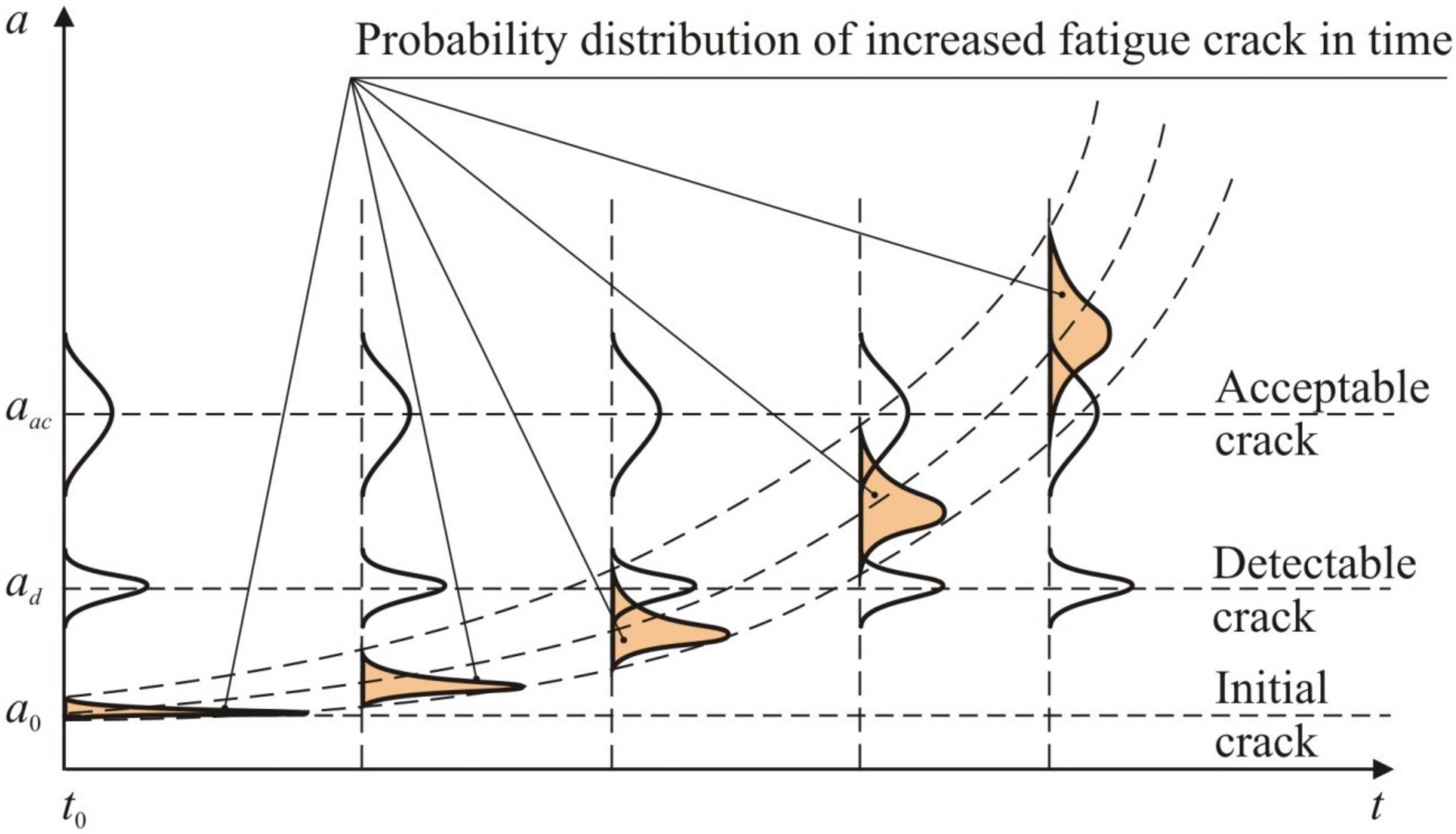
Safety margin, probability of failure:

$$G_{fail\,(\mathrm{X})} = R_{(a_{ac})} - E_{(N)}$$

$$P_f = P\left(G_{fail\,(\mathrm{X})} < 0\right) = P\left(R_{(a_{ac})} - E_{(N)} < 0\right)$$

where X is vector of random physical properties.

Probabilistic growth of the fatigue crack in time



Probability distribution of increased fatigue crack in time

Probability of defined random events

Probability of **crack undetection** in time t: $P(U_{(t)}) = P(a_{(t)} < a_d)$

where a_d is minimal **detectable** length of the crack.

Probability of **crack detection** in time t, crack size $a_{(t)}$ is **less than acceptable length** of the crack a_{ac}: $P(D_{(t)}) = P(a_d \leq a_{(t)} < a_{ac})$

Probability of **crack detection** in time t, crack size $a_{(t)}$ is **equal or greater than acceptable length** a_{ac}:

$$P(F_{(t)}) = P(a_{(t)} \geq a_{ac})$$

$$P(F_{(t)}) \geq P_d \rightarrow \text{inspection}$$

All of these three events creates **full space of random events**, which can come in time t, can be applied:

$$P(U_{(t)}) + P(D_{(t)}) + P(F_{(t)}) = 1$$

Bayes' theorem

Bayes' theorem describes the probability of an event, based on prior knowledge of conditions that might be related to the selected event.

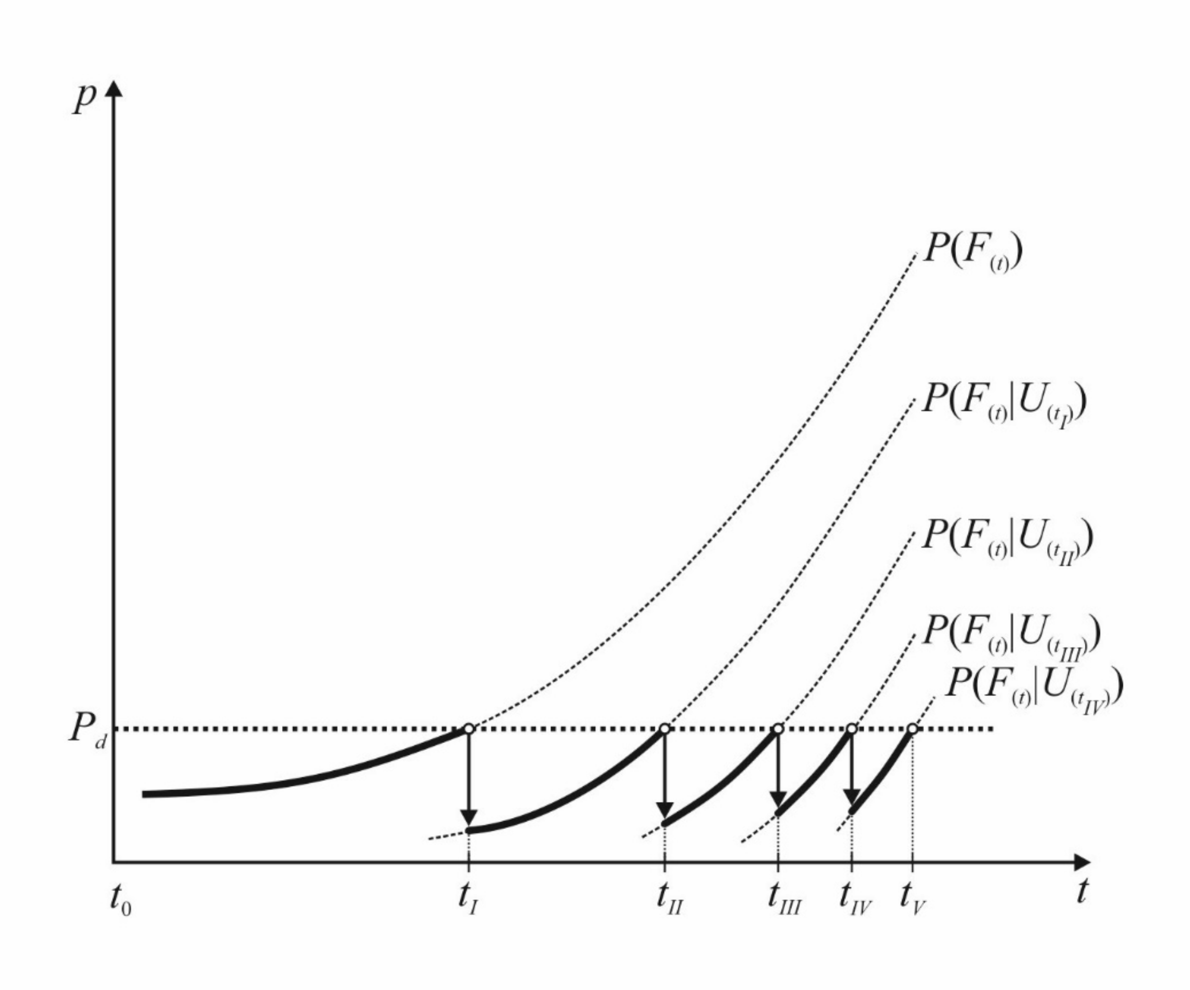
$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} \qquad P(B) > 0$$

Probability of event F in time T with respect to the **results of structural inspection** in time $t_I < T$:

$$P\left(F_{(T)} \middle| U_{(t_I)}\right) = \frac{P\left(F_{(T)}\right) - P\left(F_{(t_I)}\right) - P\left(D_{(t_I)}\right) \cdot P\left(F_{(T)} \middle| D_{(t_I)}\right)}{P\left(U_{(t_I)}\right)}$$

$$P\left(F_{(T)} \middle| D_{(t_I)}\right) = \frac{P\left(F_{(T)}\right) - P\left(F_{(t_I)}\right) - P\left(U_{(t_I)}\right) \cdot P\left(F_{(T)} \middle| U_{(t_I)}\right)}{P\left(D_{(t_I)}\right)}$$

Design of structural inspections

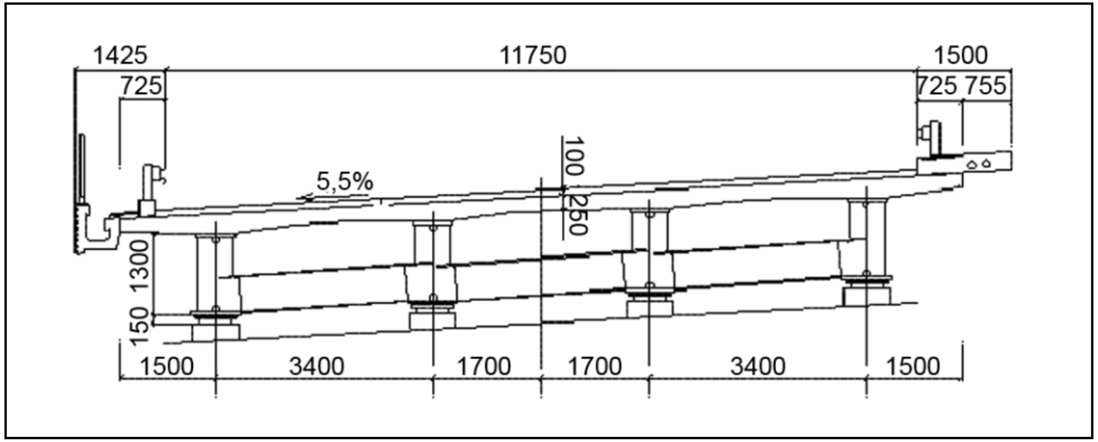


Design of **structural inspections** based on probability of failure P_f, conditional probability and required degree of reliability P_d.

Reliability assessment of steel flange in tension

Look to the reviewed road bridge, photo: Assoc. Prof. J. Odrobiňák

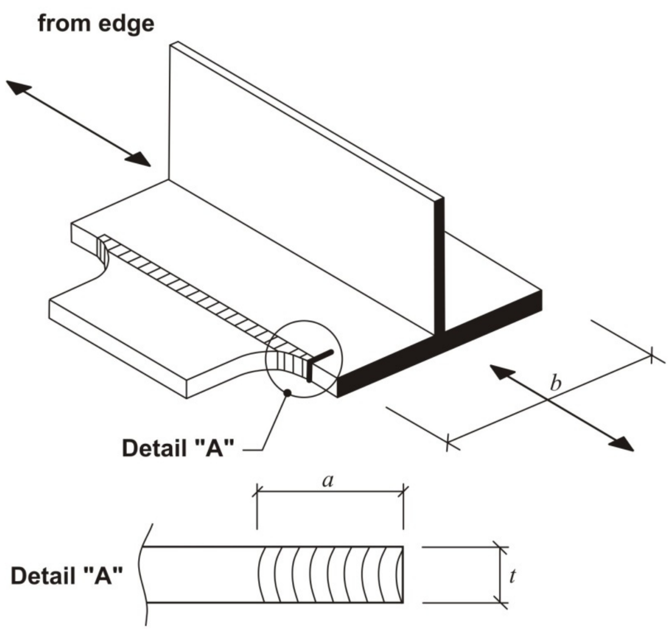
Detail of solved steel bridge's flange, photo: Assoc. Prof. J. Odrobiňák

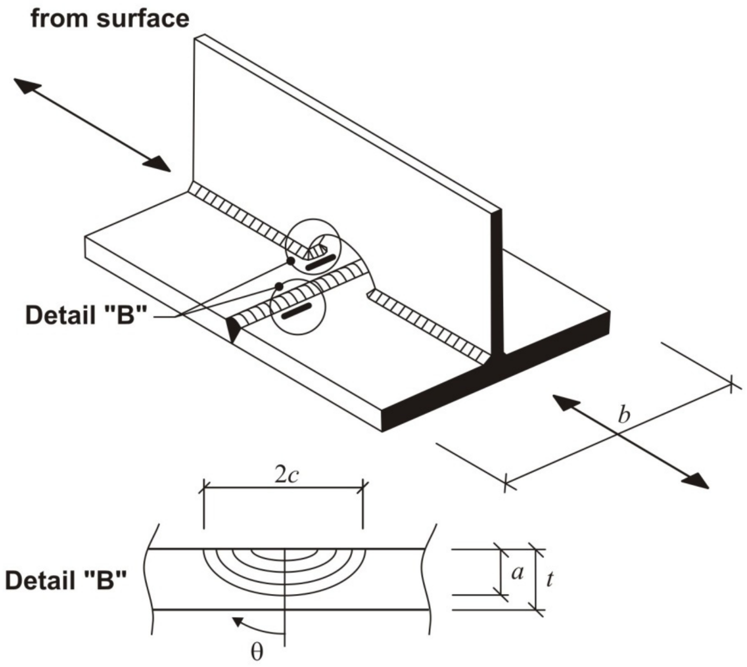


Bridge's crosswise cut

Fatigue damage danger's concentration

Crack's propagations **from the edge** or **from the surface** are possible to monitor according to initial crack position.



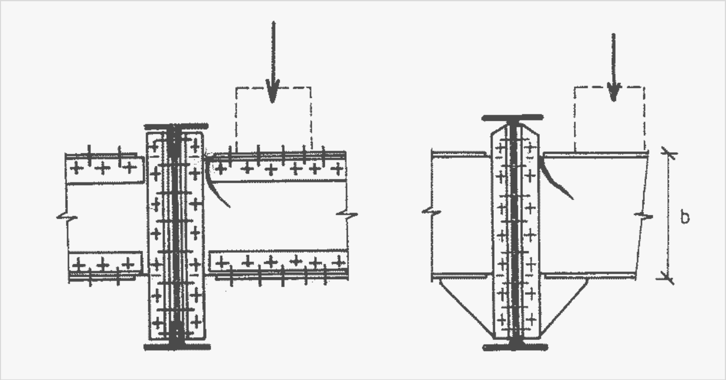


Weakness of the same flange increased **from the edge is quicker** then from the surface.

Crack's propagations from the edge

Steel railway bridge near Hodonín built in 1929,
photo: prof. V. Tomica, 1998.

Fatigue crack propagation from the edge in the wall of longitudinal beam



Crack's propagations from the edge

Fatigue crack in the weld of the connected crossbeam

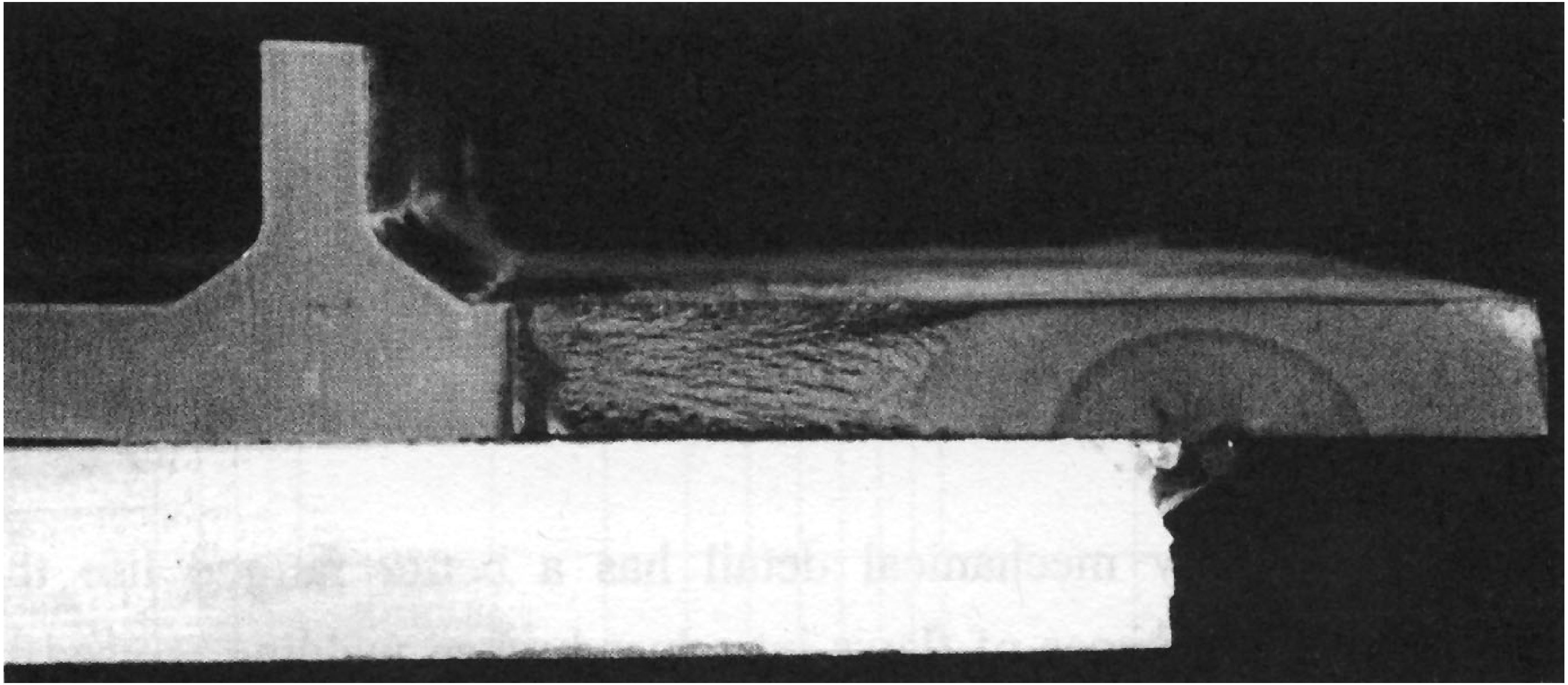
Fisher at al,
A Fatigue Primer for Structural Engineers, 1998.

Crack’s propagations from the edge

View of the fatigue crack,
propagating from the edge,
arising at the left edge of the weld
(60x magnification)

Corrosion Testing Laboratories, Inc., 2007

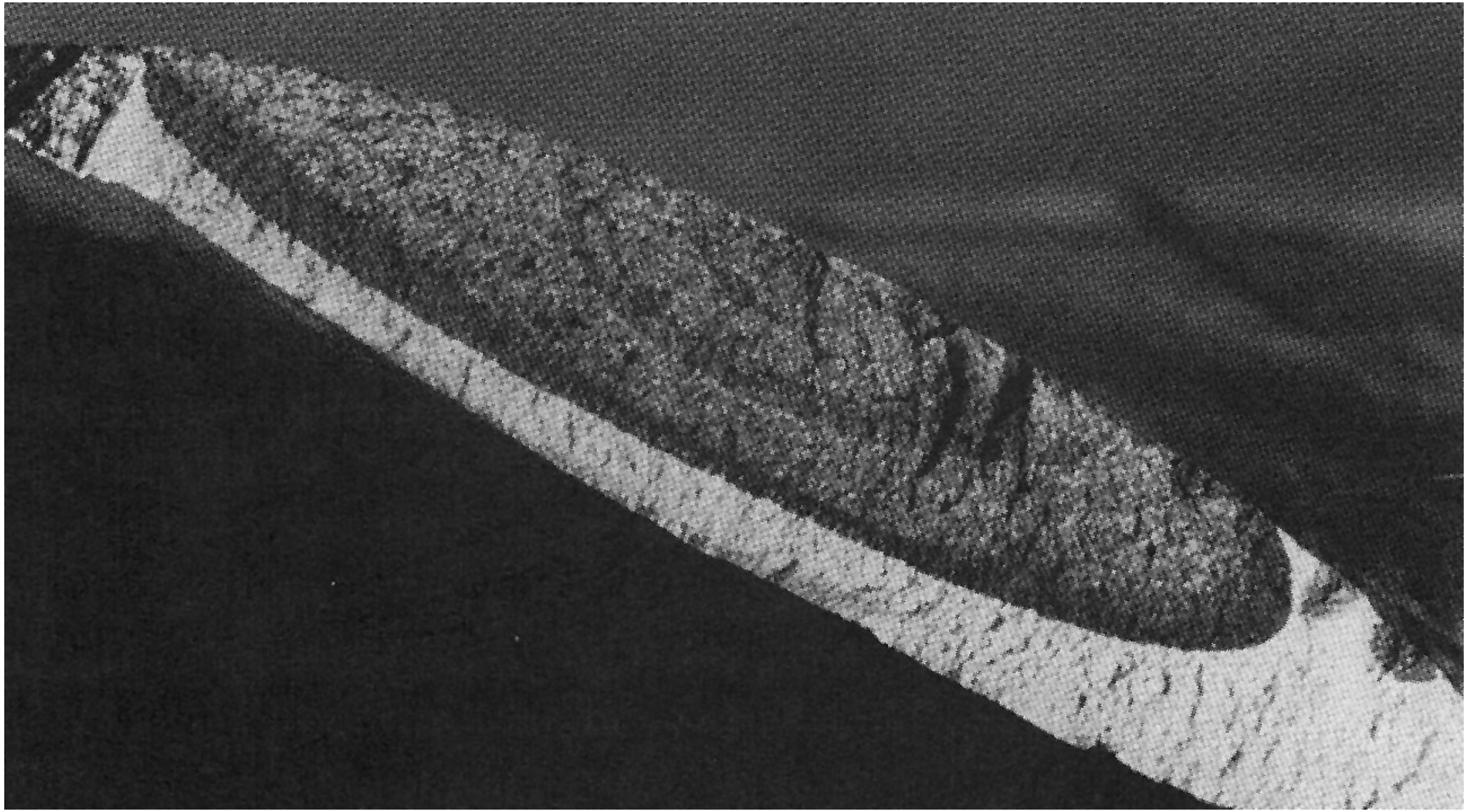
Crack's propagations from the surface



Section of the load-bearing element with an example of fatigue crack growth from the surface

Fisher J.W. *at al.* (1998) A Fatigue Primer for Structural Engineers

Crack's propagations from the surface



Semi-elliptical fatigue crack from the surface of a propeller

Sanford R.J. (2003) Principles of Fracture Mechanics

Overview of variable input quantities

Some of input values - no way to obtain using measurement, can be approximate only.

List of **random** input variables

Quantity	Type of parametric distribution	Parameters	
		Mean Value	Standard Deviation
Oscillation of stress peaks $\Delta\sigma$ [MPa]	Normal	30	3
Total number of oscillation of stress peaks per year N [-]	Normal	10^6	10^5
Initial size of the crack a_0 [mm]	Lognormal	0.2	0.05
Smallest measurable size of the crack a_d [mm]	Normal	10	0.6
Yield stress of material f_y [MPa]	Lognormal	280	28
Nominal stress in flange σ [MPa]	Normal	200	20

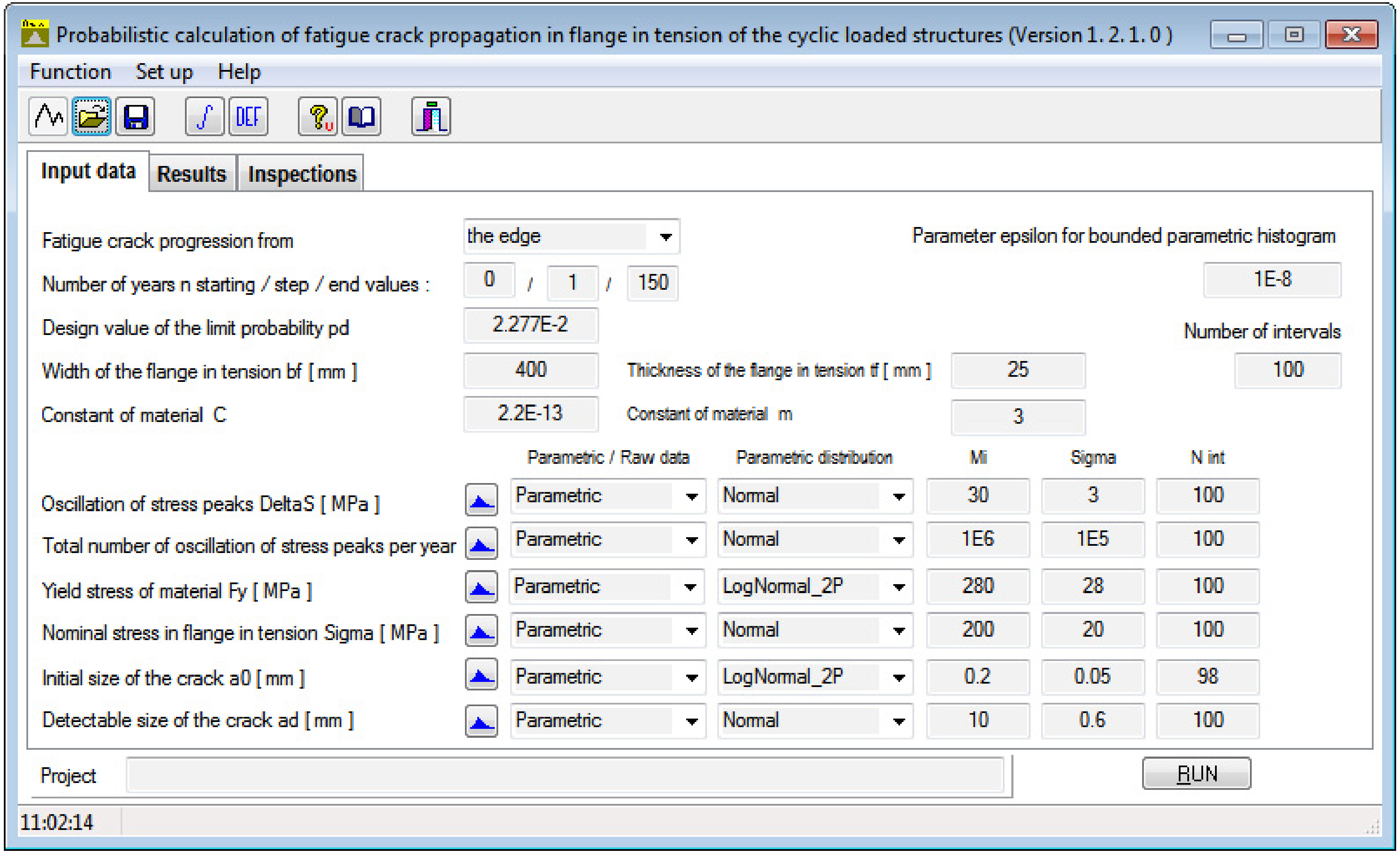
Quantity	Mean Value
Constant of material m	3
Constant of material C [$MPa^m m^{(m/2)+1}$]	$2.2.10^{-13}$
Flange width b_f [mm]	400
Flange thickness t_f [mm]	25

List of **constant** input variables

Real values

Approximate values

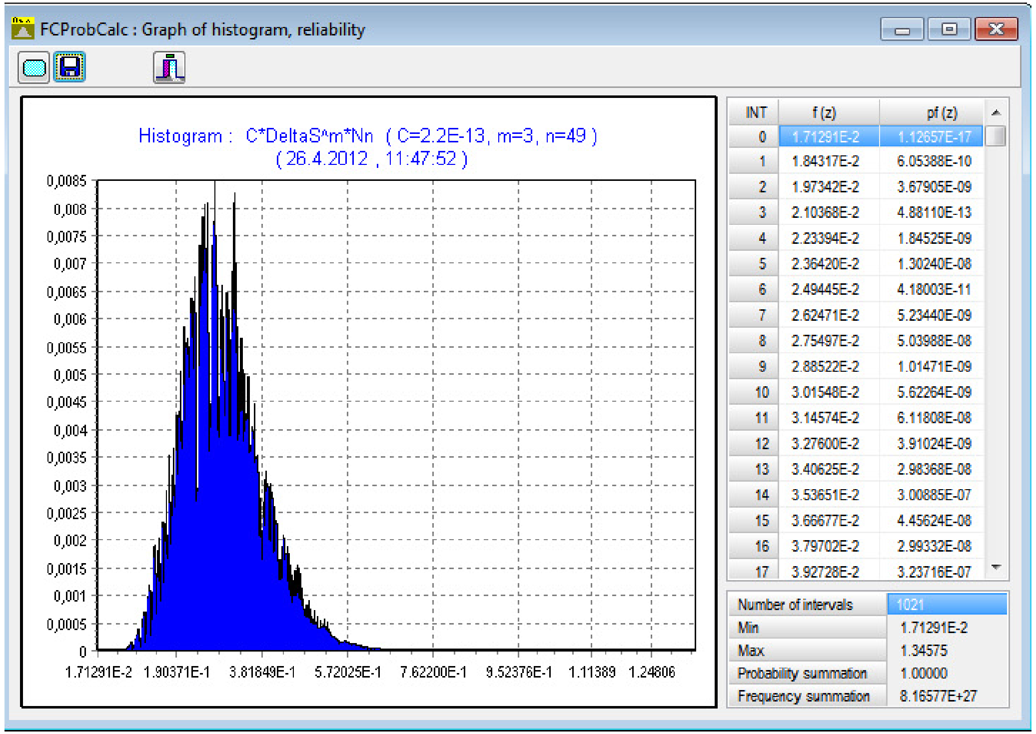
Program FCProbCalc

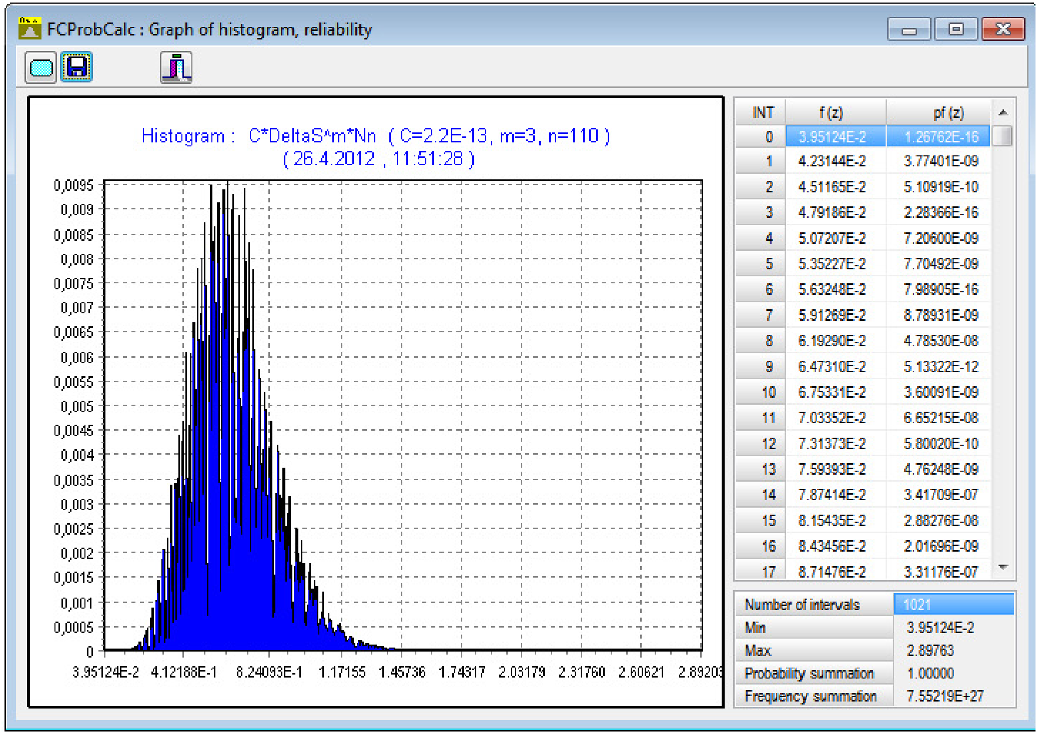


FCProbCalc program desktop - entry of input quantities

Cumulated load effect

Determined for each year
of the bridge operation using time step equals **1 year**.



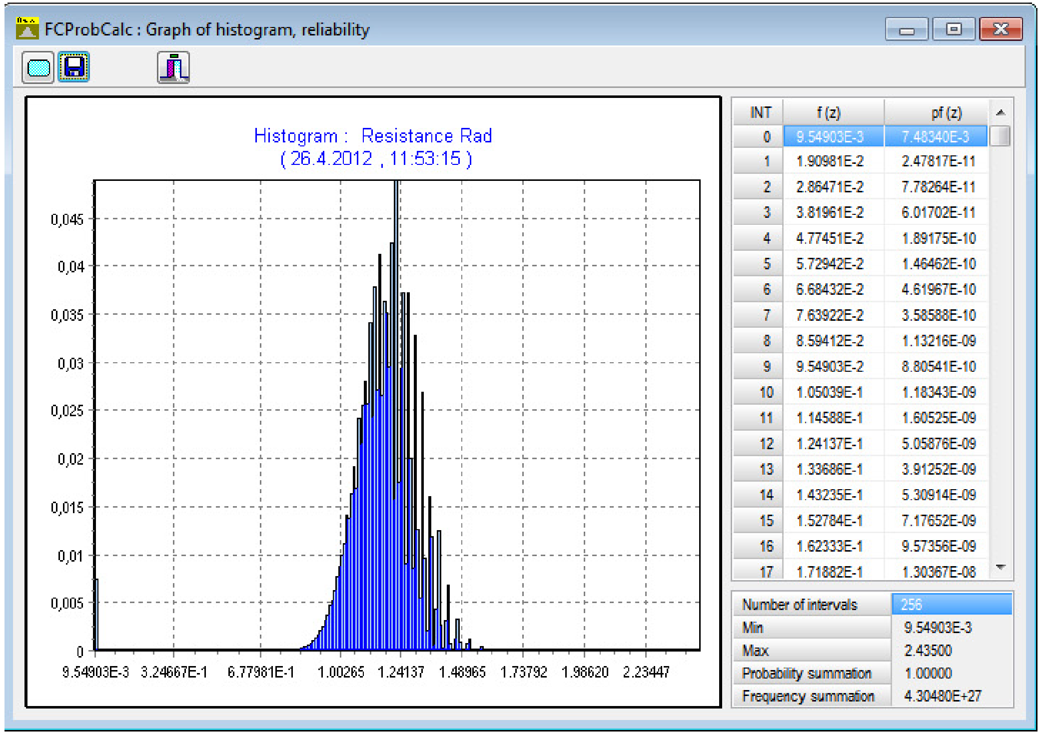


Total number oscillation of stress peak per **110 years**

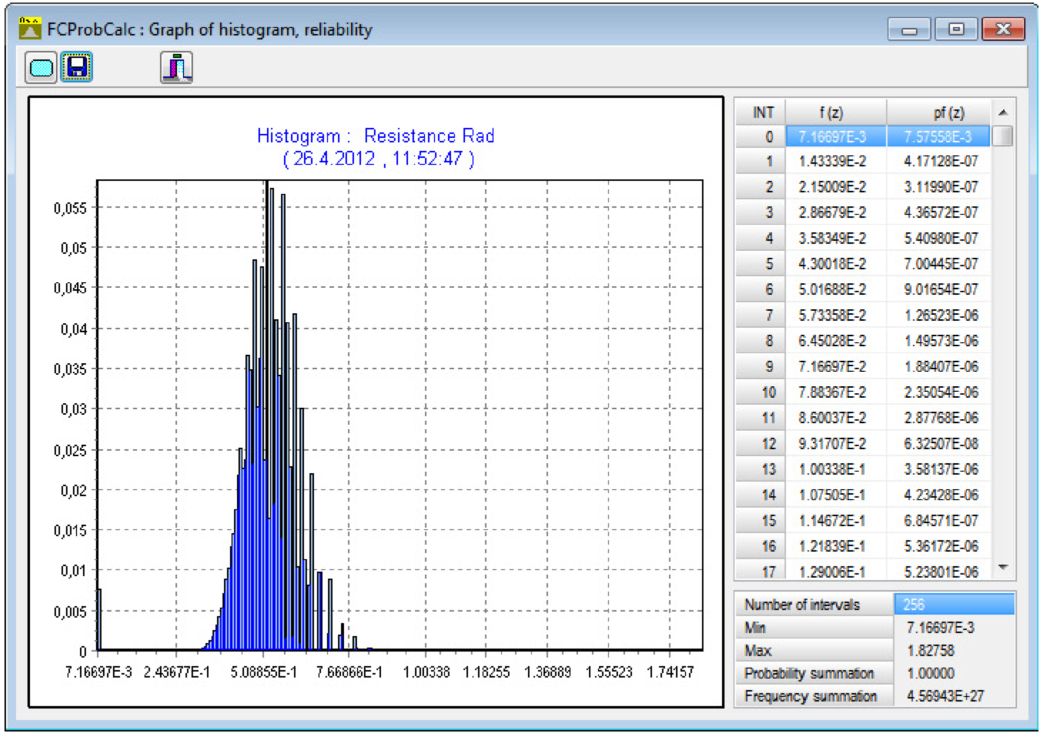
Total number oscillation of stress peak per **49 years**

Structural resistance

Yet possible to select
5 types of integration methods

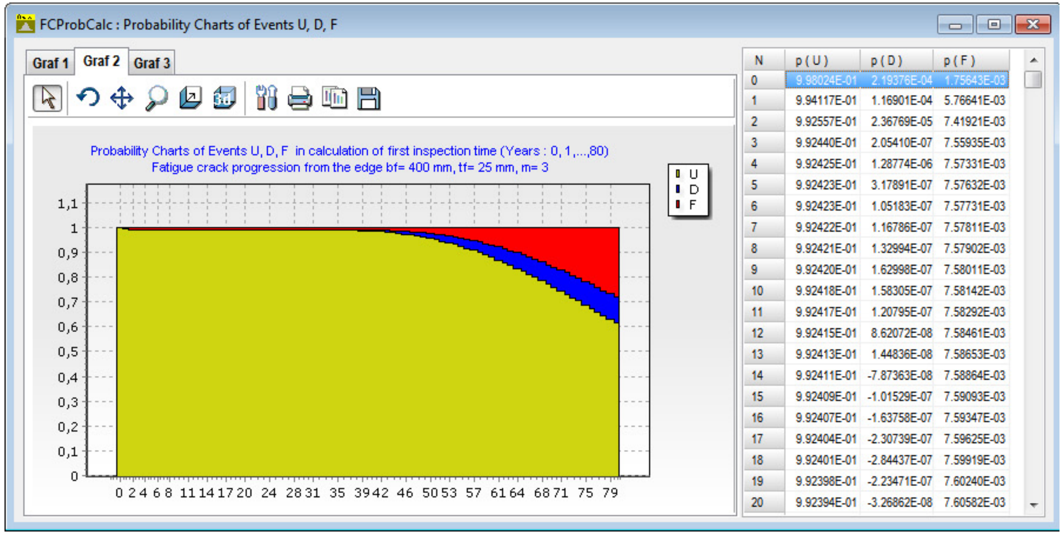


Propagation of the crack **from the surface**



Propagation of the crack
from the edge

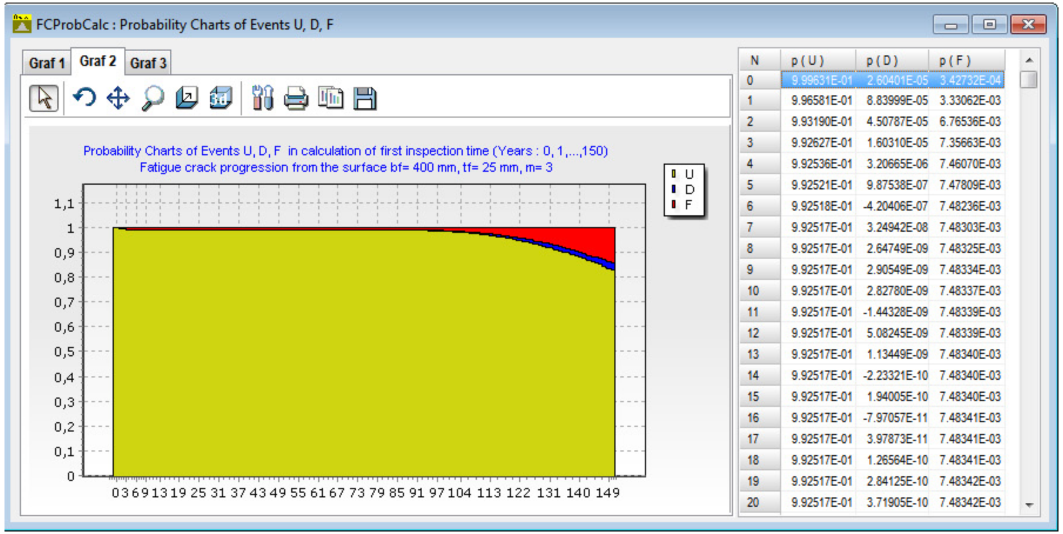
Probability of defined random events



Probability of the event U, D and F, depending on years of operation of the bridge

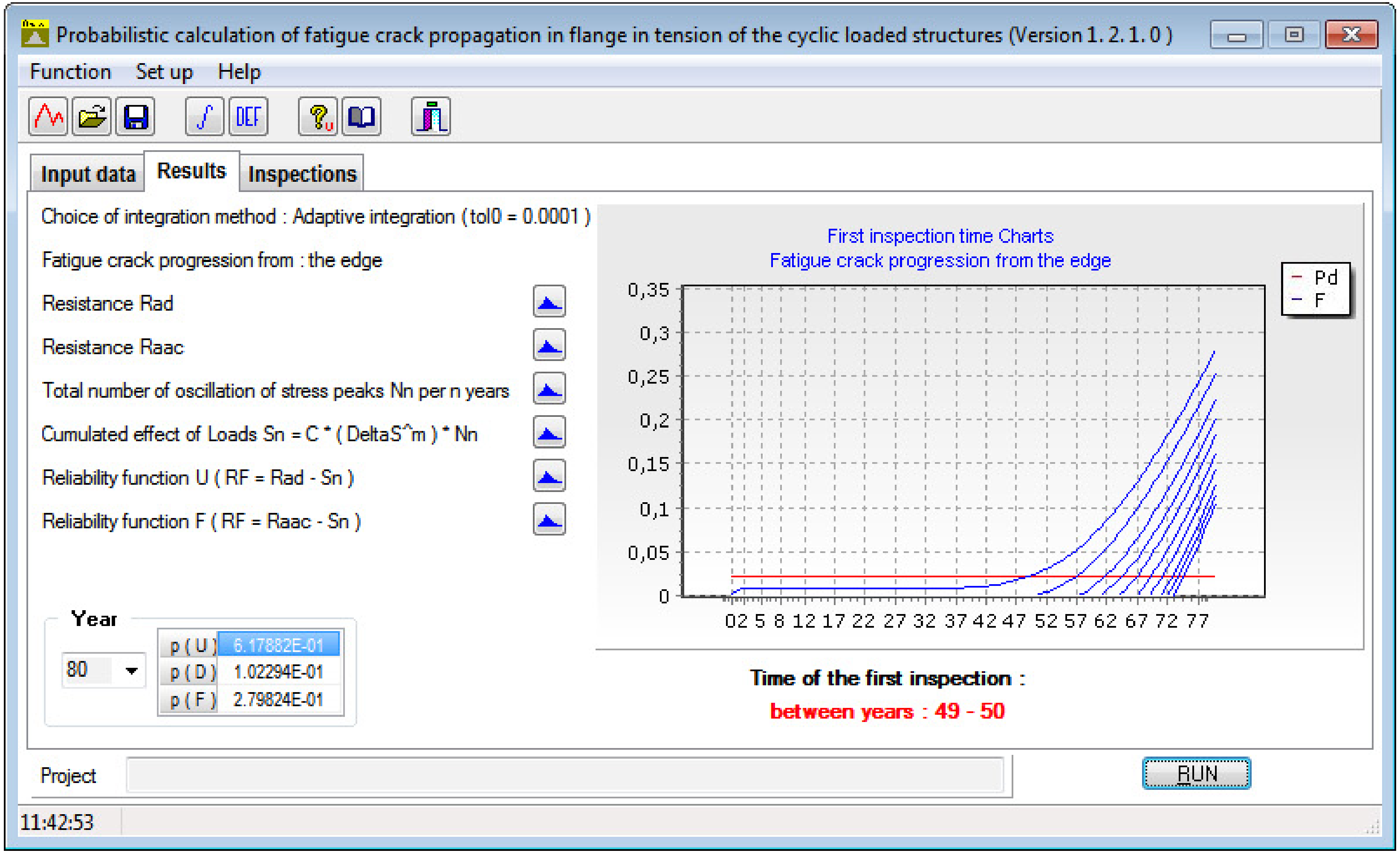
The crack **from the edge** 0 to 80 years of operation

The crack **from the surface** 0 to 150 years of operation



Estimation of inspection's times

Fatigue crack
from the edge

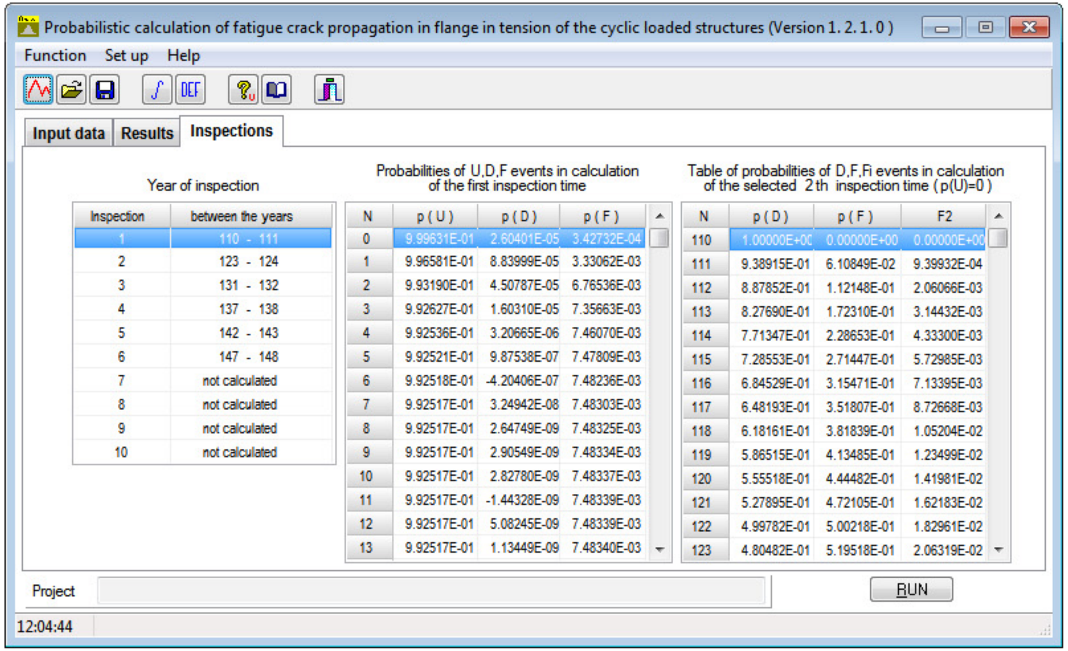


FCProbCalc program desktop

FCProbCalc code - application

Fatigue crack **from the surface**

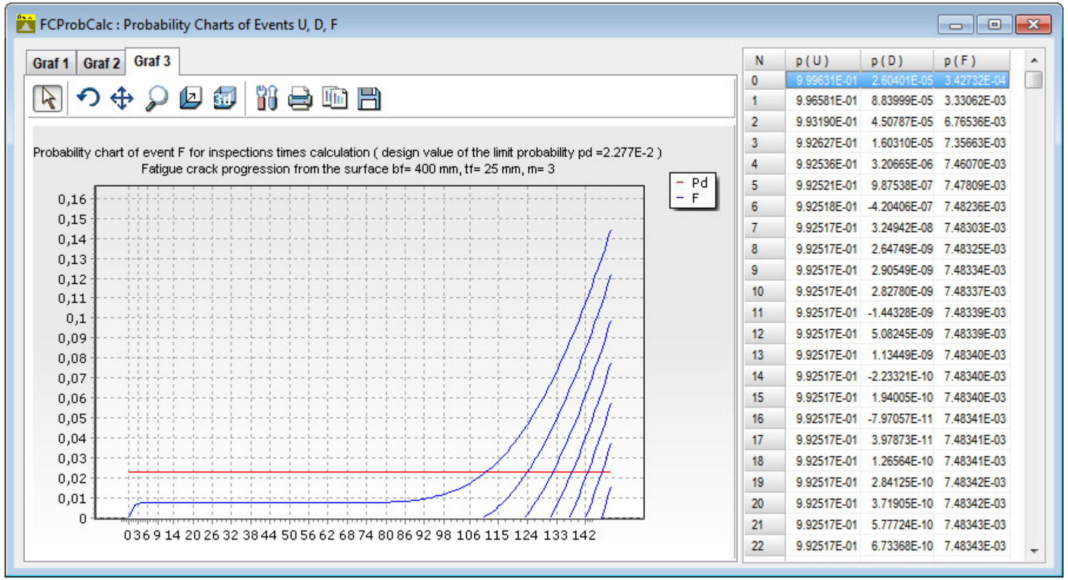
The resulting probabilities and times of construction inspections



Inspection	between the years
1	110 - 111
2	123 - 124
3	131 - 132
4	137 - 138
5	142 - 143
6	147 - 148
7	not calculated
8	not calculated
9	not calculated
10	not calculated

N	p (U)	p (D)	p (F)
0	9.99631E-01	2.60401E-05	3.42732E-04
1	9.96581E-01	8.83999E-05	3.33062E-03
2	9.93190E-01	4.50787E-05	6.76536E-03
3	9.92627E-01	1.60310E-05	7.35663E-03
4	9.92536E-01	3.20665E-06	7.46070E-03
5	9.92521E-01	9.87538E-07	7.47809E-03
6	9.92518E-01	-4.20406E-07	7.48236E-03
7	9.92517E-01	3.24942E-08	7.48303E-03
8	9.92517E-01	2.64749E-09	7.48325E-03
9	9.92517E-01	2.90549E-09	7.48334E-03
10	9.92517E-01	2.82780E-09	7.48337E-03
11	9.92517E-01	-1.44328E-09	7.48339E-03
12	9.92517E-01	5.08245E-09	7.48339E-03
13	9.92517E-01	1.13449E-09	7.48340E-03

N	p (D)	p (F)	F2
110	1.00000E+00	0.00000E+00	0.00000E+00
111	9.38915E-01	6.10849E-02	9.39932E-04
112	8.87852E-01	1.12148E-01	2.06066E-03
113	8.27690E-01	1.72310E-01	3.14432E-03
114	7.71347E-01	2.28653E-01	4.33300E-03
115	7.28553E-01	2.71447E-01	5.72985E-03
116	6.84529E-01	3.15471E-01	7.13395E-03
117	6.48193E-01	3.51807E-01	8.72668E-03
118	6.18161E-01	3.81839E-01	1.05204E-02
119	5.86515E-01	4.13485E-01	1.23499E-02
120	5.55518E-01	4.44482E-01	1.41981E-02
121	5.27895E-01	4.72105E-01	1.62183E-02
122	4.99782E-01	5.00218E-01	1.82961E-02
123	4.80482E-01	5.19518E-01	2.06319E-02



N	p (U)	p (D)	p (F)
0	9.99631E-01	2.60401E-05	3.42732E-04
1	9.96581E-01	8.83999E-05	3.33062E-03
2	9.93190E-01	4.50787E-05	6.76536E-03
3	9.92627E-01	1.60310E-05	7.35663E-03
4	9.92536E-01	3.20665E-06	7.46070E-03
5	9.92521E-01	9.87538E-07	7.47809E-03
6	9.92518E-01	-4.20406E-07	7.48236E-03
7	9.92517E-01	3.24942E-08	7.48303E-03
8	9.92517E-01	2.64749E-09	7.48325E-03
9	9.92517E-01	2.90549E-09	7.48334E-03
10	9.92517E-01	2.82780E-09	7.48337E-03
11	9.92517E-01	-1.44328E-09	7.48339E-03
12	9.92517E-01	5.08245E-09	7.48339E-03
13	9.92517E-01	1.13449E-09	7.48340E-03
14	9.92517E-01	-2.23321E-10	7.48340E-03
15	9.92517E-01	1.94005E-10	7.48340E-03
16	9.92517E-01	-7.97057E-11	7.48341E-03
17	9.92517E-01	3.97873E-11	7.48341E-03
18	9.92517E-01	1.26564E-10	7.48341E-03
19	9.92517E-01	2.84125E-10	7.48342E-03
20	9.92517E-01	3.71905E-10	7.48342E-03
21	9.92517E-01	5.77724E-10	7.48343E-03
22	9.92517E-01	6.73368E-10	7.48343E-03

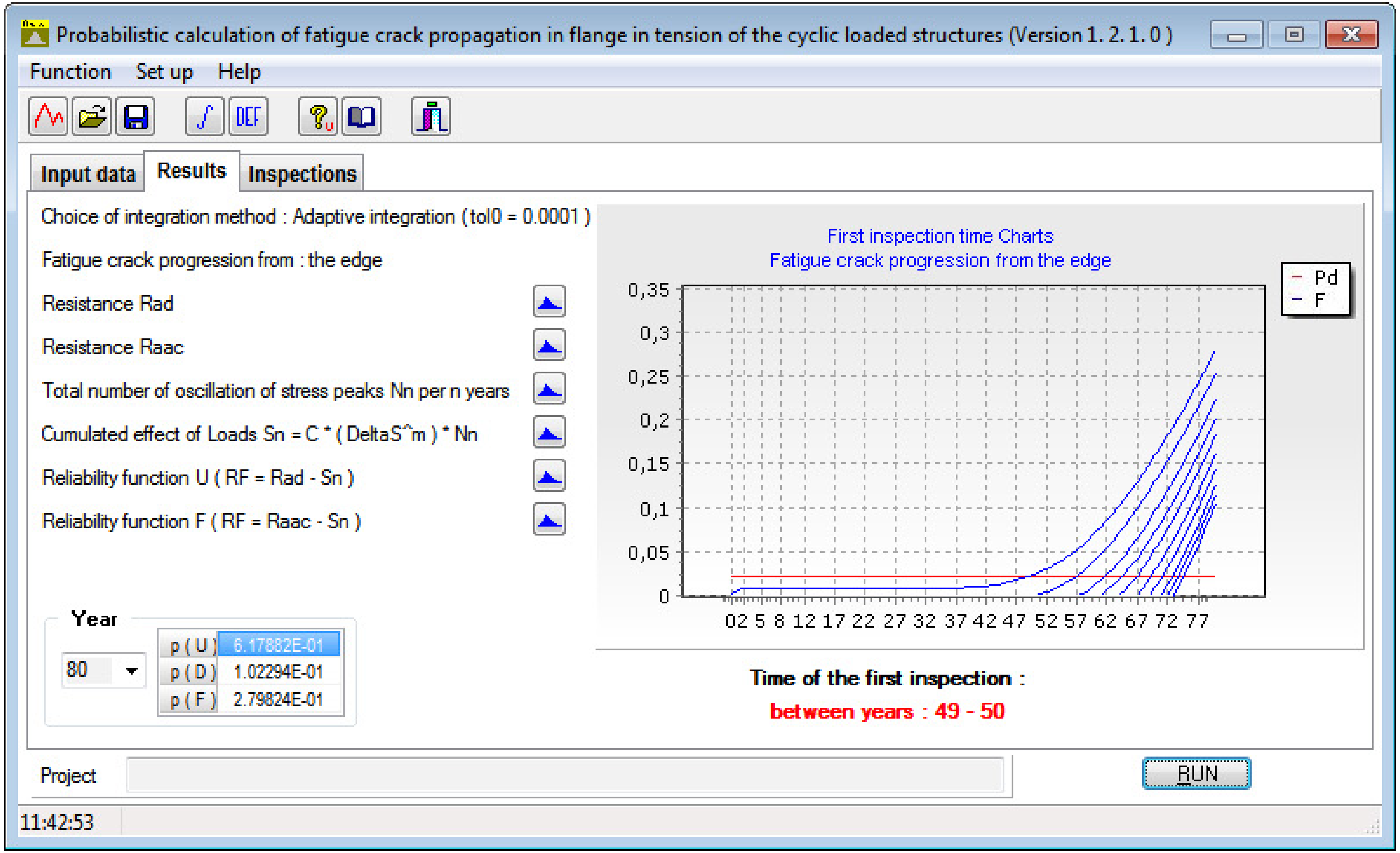
The dependence of the probability of failure P_f and years of operation of the bridge

(adaptive numerical integration method chosen with a parameter $tol_0 = 1 \cdot 10^{-4}$)

Estimation of inspection's times

Fatigue crack

from the edge

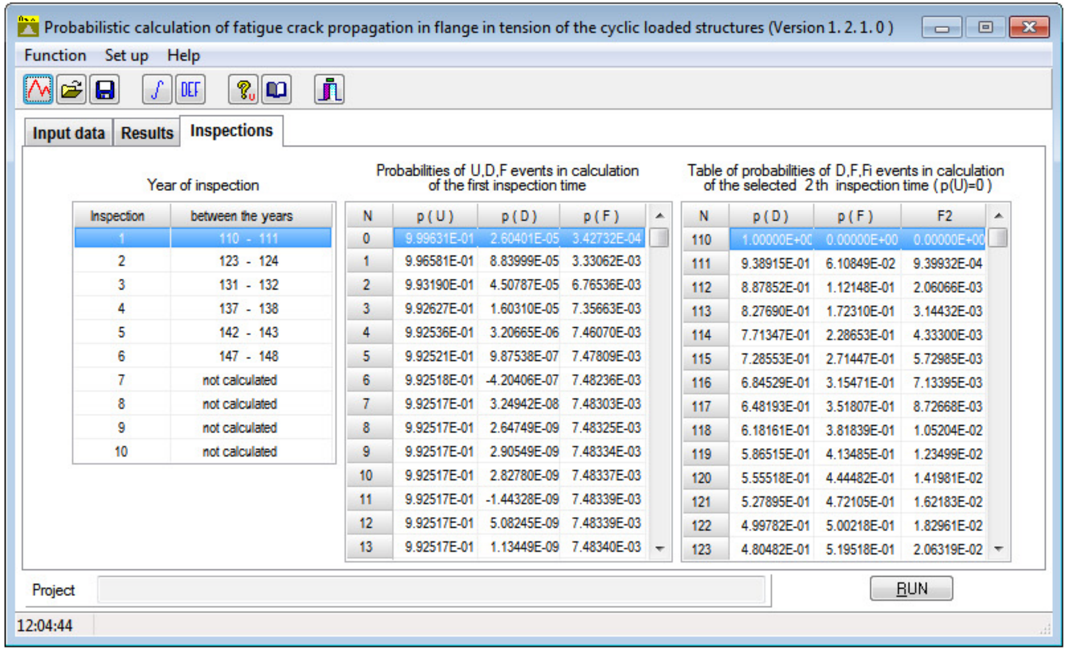


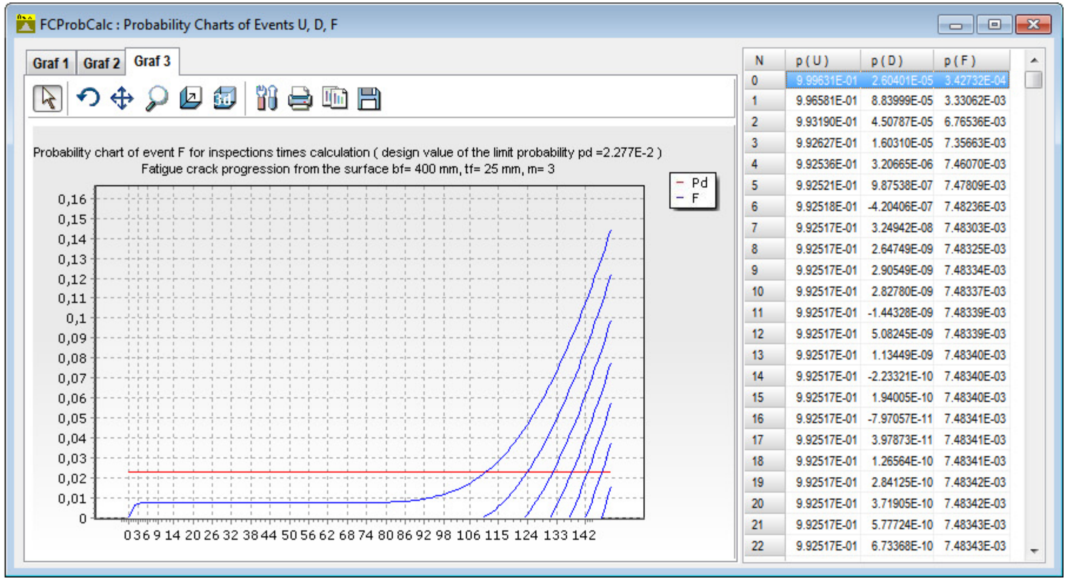
FCProbCalc program desktop

Estimation of inspection's times

Fatigue crack **from the surface**

The resulting probabilities and times of construction inspections



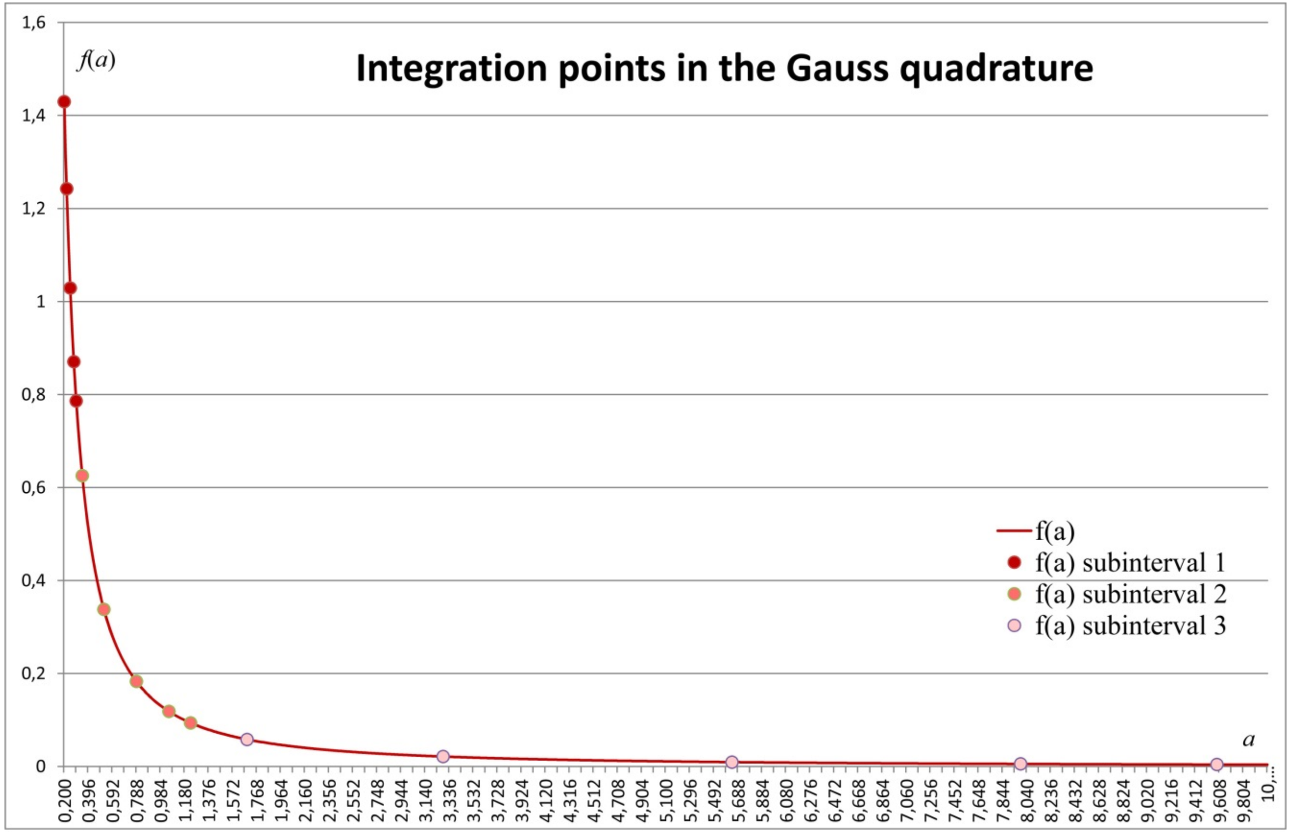


The dependence of the probability of failure P_f and years of operation of the bridge

(Adaptive numerical integration method chosen with a parameter $tol_0 = 1 \cdot 10^{-4}$)

Numerical integration

1. **Rectangular method** (default number of diferences 1000)
2. **Simpson method** (default number of diferences 1000)
3. **Romberg method** (default parameter $n = 10$)
4. **Adaptive method** (default value of tolerated inaccuracy $tol_0 = 10^{-4}$)
5. **Gaussian quadrature**

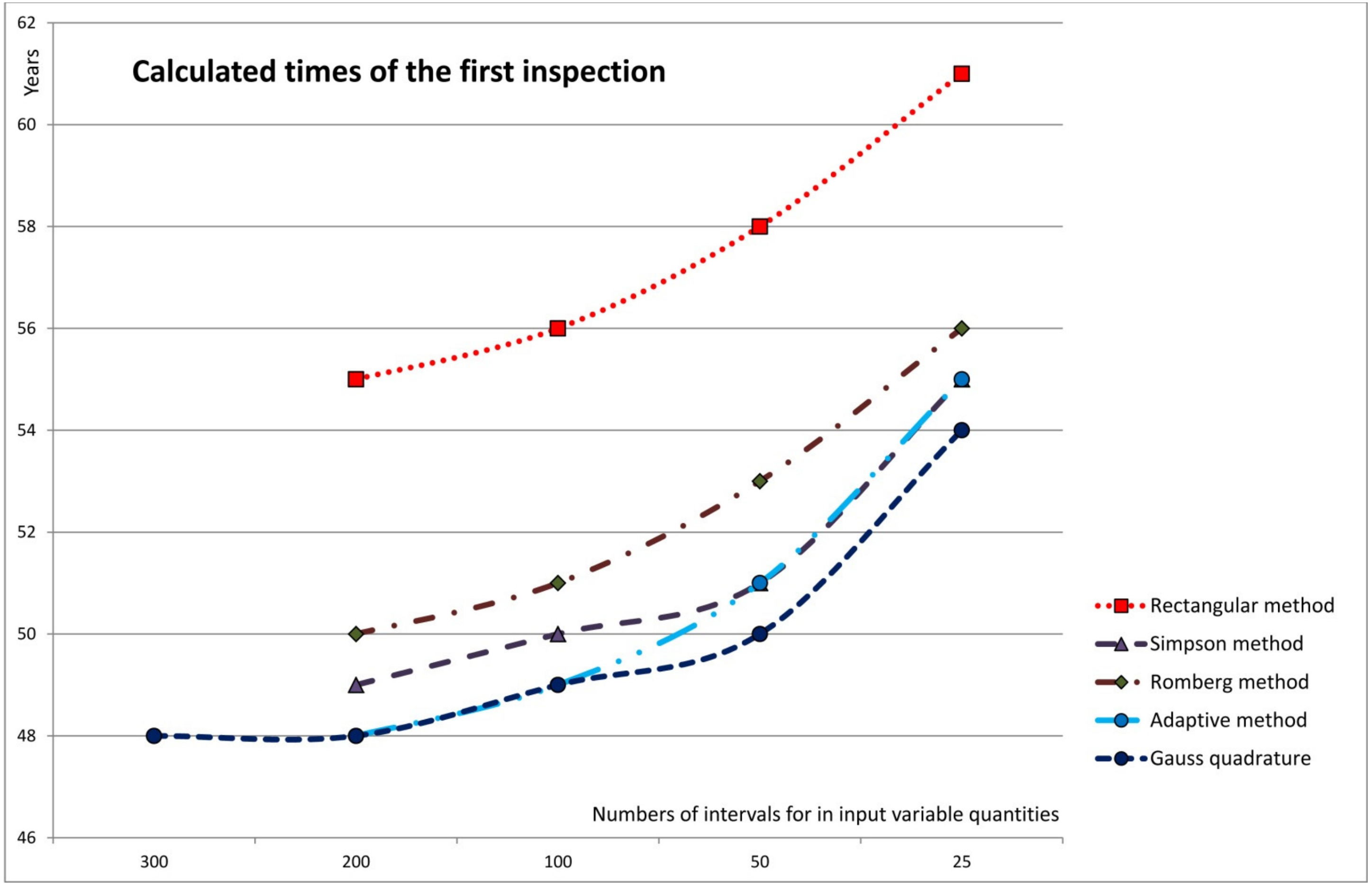


Gaussian quadrature with selected integration points

Numerical integration

Comparison of the calculated 1st time inspection

Fatigue crack **from the edge**

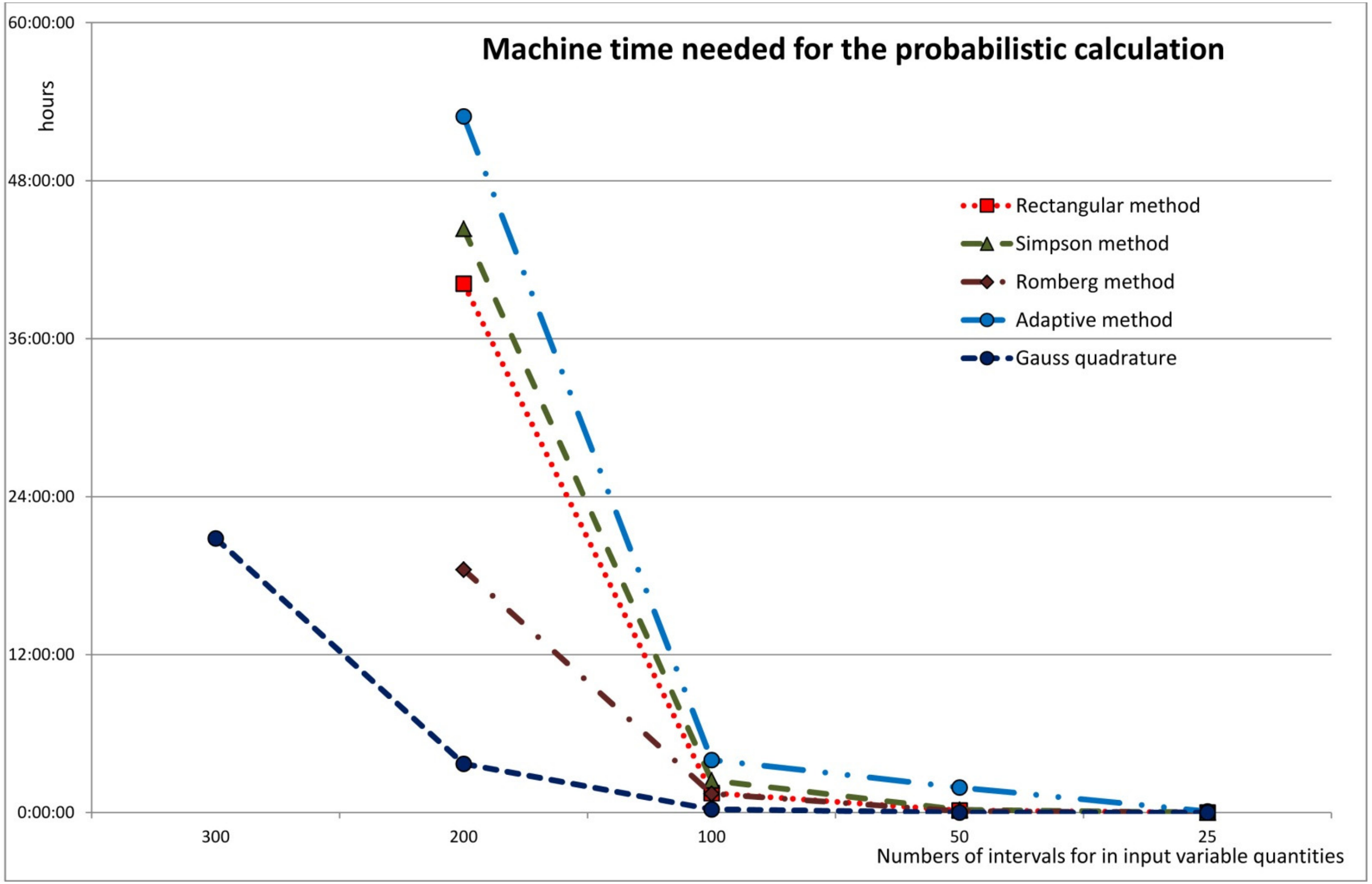


Calculated times of the 1st inspection of the bridge construction with a particular attention on **used numerical integration method**

Numerical integration

Time of the calculation

Fatigue crack **from the edge**

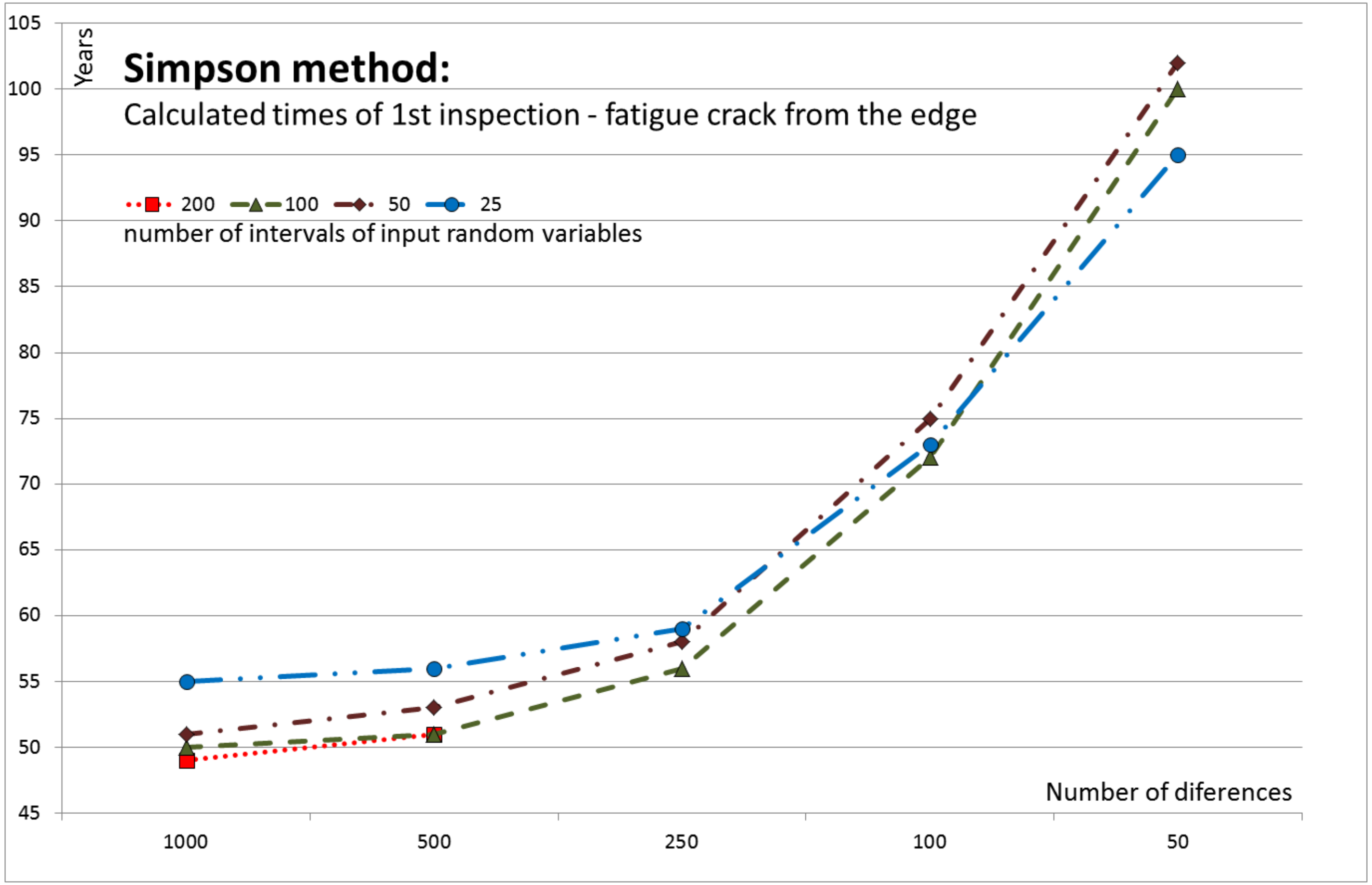


Calculation time for each **type of numerical integration** and the specified **number of intervals** of input random variables

Numerical integration

Comparison of the calculated 1st time inspection

Fatigue crack **from the edge**



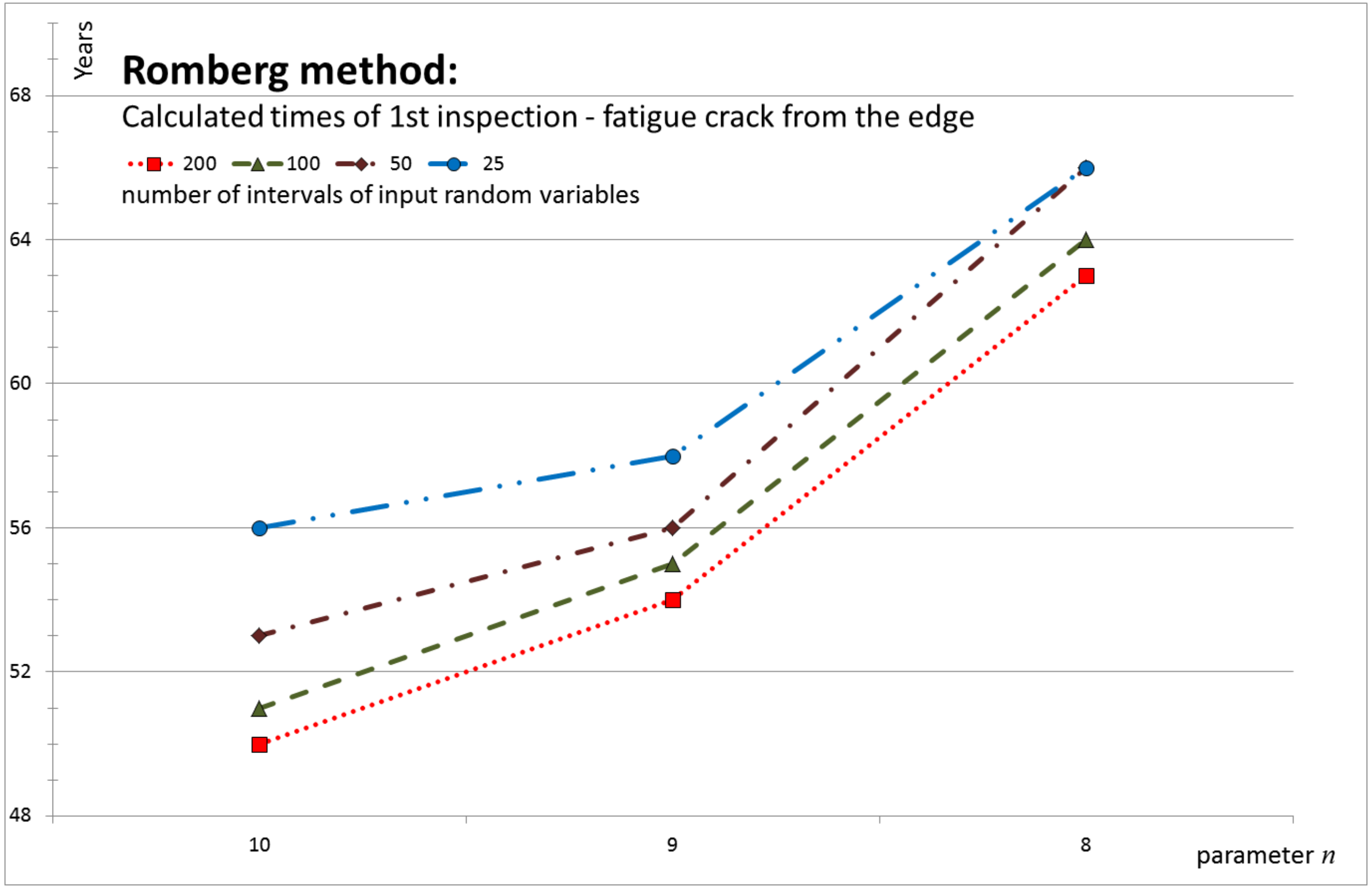
Calculated times of the 1st inspection of the bridge construction for **Simpson method** of numerical integration and the specified **number of intervals** of input random variables and **number of diferences**

Numerical integration

Comparison of the calculated 1st time inspection

Fatigue crack **from the edge**

Calculated times of the 1st inspection of the bridge construction for **Romberg method** of numerical integration and the specified **number of intervals** of input random variables and specified **parameter n**

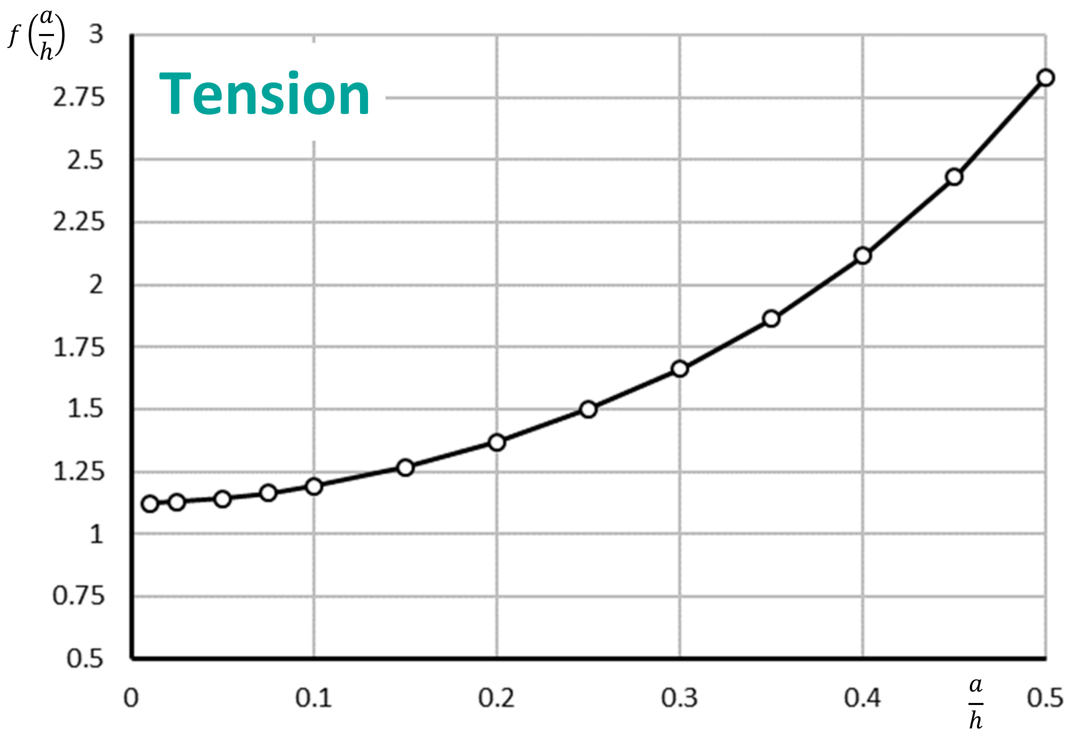


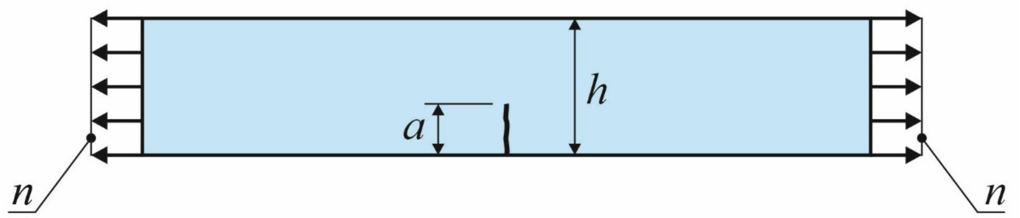
Calibration functions for short edge cracks

Calibration functions for short edge cracks under selected loads

w is **width** of rectangular cross-section,

f_y is **yield stress**.





$$\Delta\sigma = n = \frac{N}{w \cdot h}$$

$$N = n \cdot w \cdot h$$

$$a_{ac} = h - \frac{N}{w \cdot f_y}$$

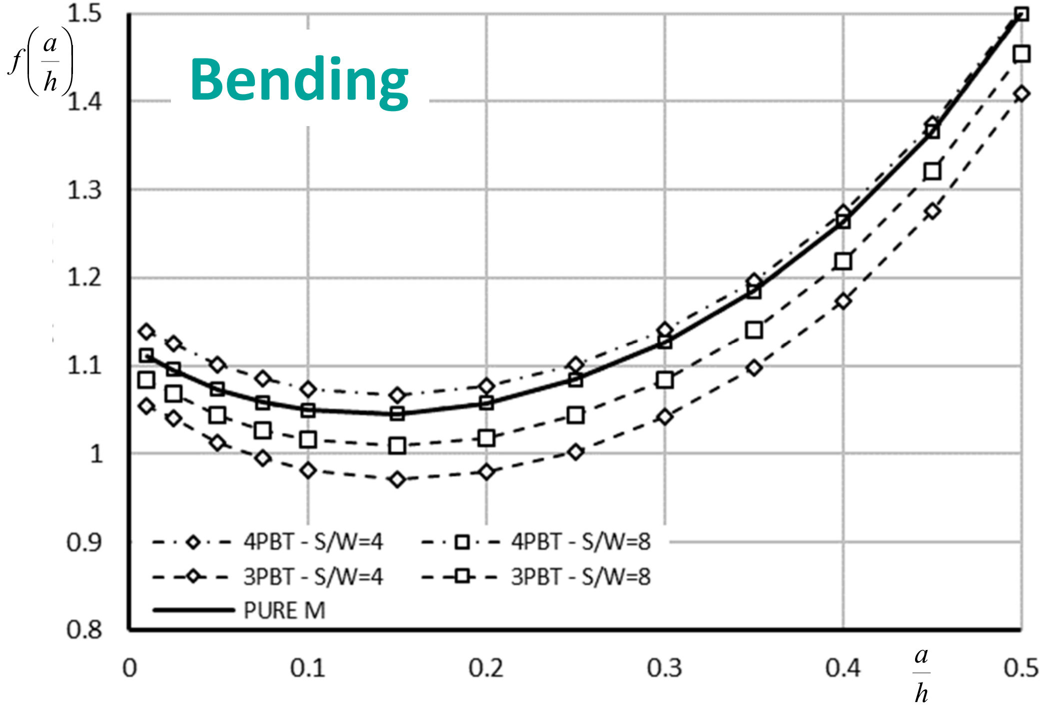
$$f\left(\frac{a}{h}\right)_{tension} = +1.1082 + 0.6956 \cdot \left(\frac{a}{h}\right) + 1.2486 \cdot \left(\frac{a}{h}\right)^2 + 8.415 \cdot \left(\frac{a}{h}\right)^3$$

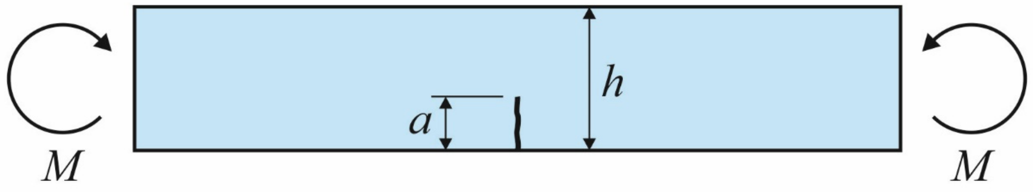
Calibration functions for short edge cracks

Calibration functions for short edge cracks under selected loads

w is **width** of rectangular cross-section,

f_y is **yield stress**.





$$\Delta\sigma = \frac{6 \cdot M}{w \cdot h^2}$$

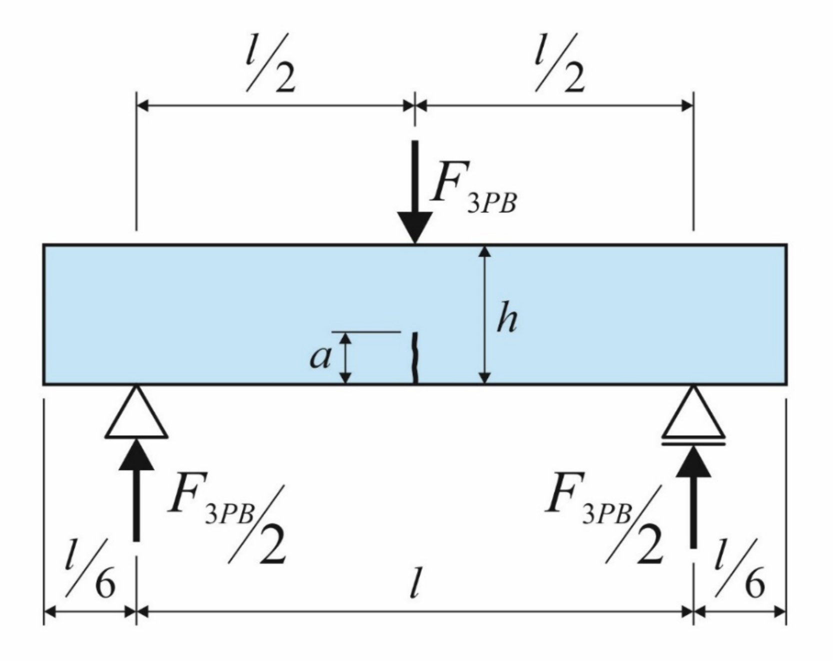
$$a_{ac} = h - \sqrt{\frac{6 \cdot M}{w \cdot f_y}}$$

$$f\left(\frac{a}{h}\right)_{pure} = +1.114 - 0.8975 \cdot \left(\frac{a}{h}\right) + 2.752 \cdot \left(\frac{a}{h}\right)^2 - 1.1323 \cdot \left(\frac{a}{h}\right)^3$$

Calibration functions for short edge cracks

Calibration functions for short edge cracks under selected loads

3-Point Bending



$$f\left(\frac{a}{h}\right)_{3PBT\frac{l}{h}=2} = +1.0259 - 1.4659 \cdot \left(\frac{a}{h}\right) + 4.9318 \cdot \left(\frac{a}{h}\right)^2 - 2.4637 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{3PBT\frac{l}{h}=4} = +1.0691 - 1.3496 \cdot \left(\frac{a}{h}\right) + 5.1865 \cdot \left(\frac{a}{h}\right)^2 - 3.3509 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{3PBT\frac{l}{h}=8} = +1.0963 - 1.3052 \cdot \left(\frac{a}{h}\right) + 5.2829 \cdot \left(\frac{a}{h}\right)^2 - 3.5972 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{3PBT\frac{l}{h}=16} = +1.1079 - 1.2328 \cdot \left(\frac{a}{h}\right) + 5.0551 \cdot \left(\frac{a}{h}\right)^2 - 3.2837 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{3PBT\frac{l}{h}=80} = +1.118 - 1.1964 \cdot \left(\frac{a}{h}\right) + 5.0176 \cdot \left(\frac{a}{h}\right)^2 - 3.3127 \cdot \left(\frac{a}{h}\right)^3$$

$$\Delta\sigma = \frac{3 \cdot F_{3PB} \cdot l}{2 \cdot w \cdot h^2}$$

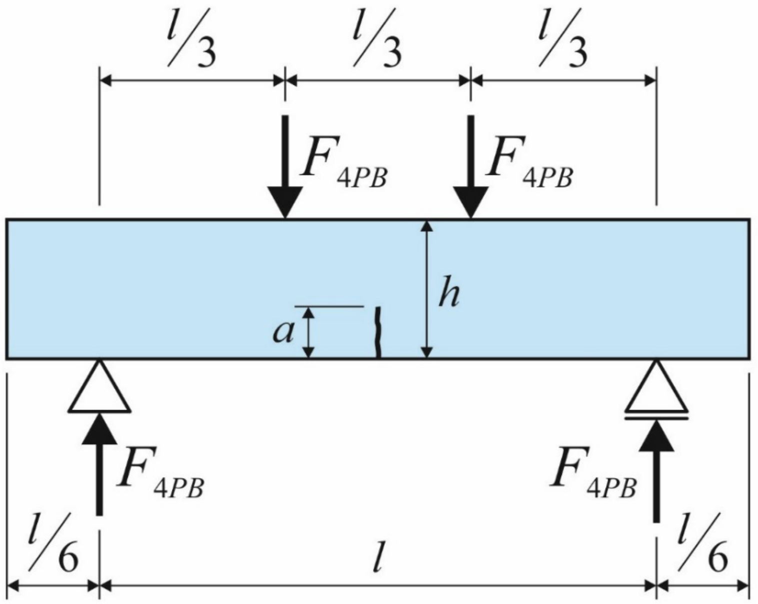
$$a_{ac} = h - \sqrt{\frac{3 \cdot F_{3PB} \cdot l}{2 \cdot w \cdot f_y}}$$

w is **width** of rectangular cross-section,
f_y is **yield stress**.

Calibration functions for short edge cracks

Calibration functions for short edge cracks under selected loads

4-Point Bending



$$f\left(\frac{a}{h}\right)_{4PBT\frac{l}{h}=2} = +1.2505 - 1.7928 \cdot \left(\frac{a}{h}\right) + 6.3295 \cdot \left(\frac{a}{h}\right)^2 - 4.4492 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{4PBT\frac{l}{h}=4} = +1.1535 - 1.2847 \cdot \left(\frac{a}{h}\right) + 5.1957 \cdot \left(\frac{a}{h}\right)^2 - 3.5502 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{4PBT\frac{l}{h}=8} = +1.1202 - 1.1634 \cdot \left(\frac{a}{h}\right) + 4.8443 \cdot \left(\frac{a}{h}\right)^2 - 3.0085 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{4PBT\frac{l}{h}=16} = +1.1222 - 1.2277 \cdot \left(\frac{a}{h}\right) + 5.2654 \cdot \left(\frac{a}{h}\right)^2 - 3.7958 \cdot \left(\frac{a}{h}\right)^3$$

$$f\left(\frac{a}{h}\right)_{4PBT\frac{l}{h}=80} = +1.1179 - 1.1235 \cdot \left(\frac{a}{h}\right) + 4.5993 \cdot \left(\frac{a}{h}\right)^2 - 2.5619 \cdot \left(\frac{a}{h}\right)^3$$

$$\Delta\sigma = \frac{2 \cdot F_{4PB} \cdot l}{w \cdot h^2}$$

$$a_{ac} = h - \sqrt{\frac{2 \cdot F_{4PB} \cdot l}{w \cdot f_y}}$$

w is **width** of rectangular cross-section,
f_y is **yield stress**.

Overview of variable input quantities

Quantity	Type of parametric distribution	Parameters	
		Mean Value	Standard Deviation
Total number of fatigue loading cycles per 1 year N	Normal	10^6	10^5
Initial size of the crack a_0	Lognormal	0.2 mm	0.05 mm
Smallest detectable size of the crack a_d	Normal	10 mm	0.6 mm
Yield stress of material f_y	Lognormal	280 MPa	28 MPa
Loading force in three-point bending test F_{3PB}	Normal	6 kN	0.6 kN

List of **random input variables**

Quantity	Mean Value
Constant of material m	3
Constant of material C	$2.2 \cdot 10^{-13}\ \mathrm{MPa}^m \mathrm{m}^{(m/2)+1}$
Height of the rectangular cross-section h	0.1 m
Width of the rectangular cross-section b	0.01 m
Span of the element l	0.4 m
Designed probability of failure P_d	0.02277 ($\beta_d = 2$)

3-Point Bending

List of **deterministic input variables**

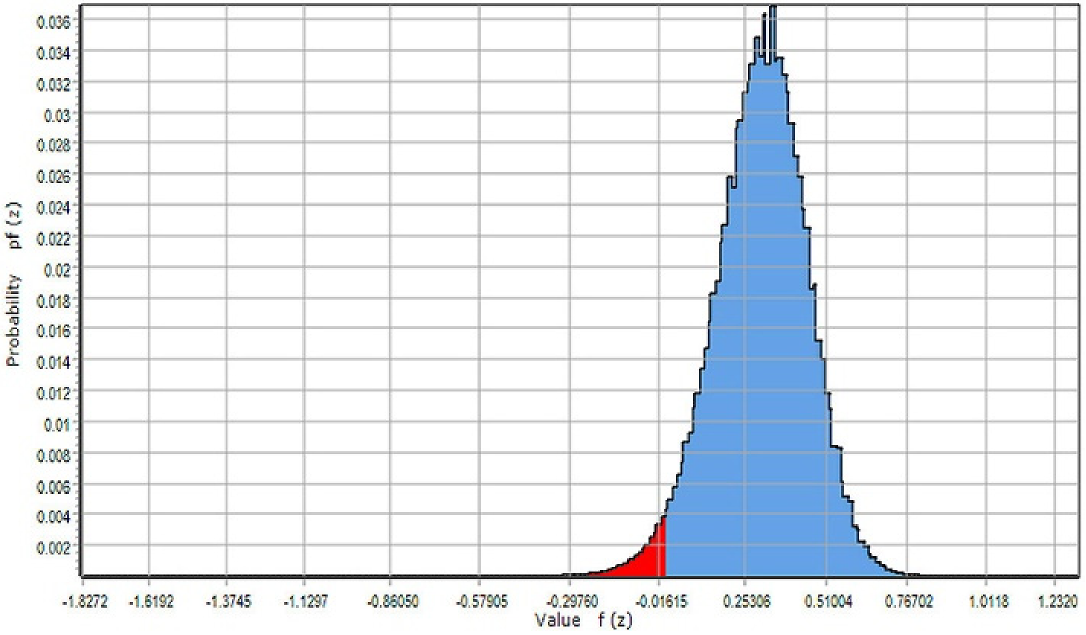
Calibration functions for short edge cracks

Calibration functions for short edge cracks under selected loads

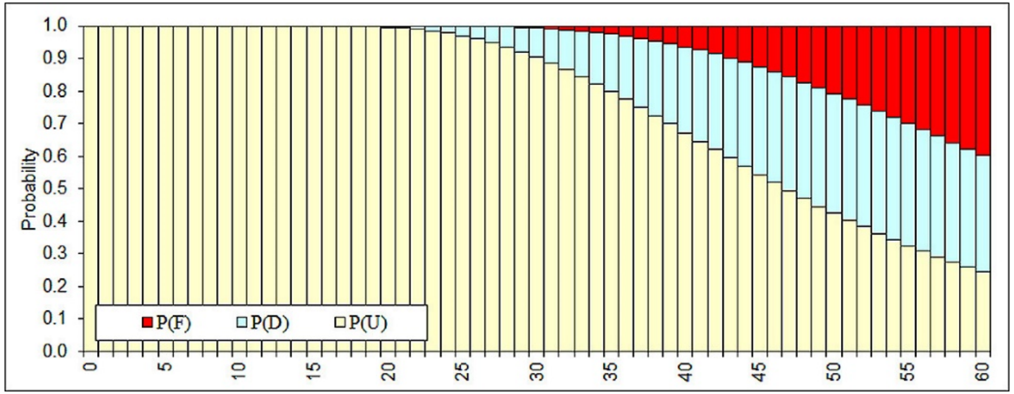
3-Point Bending

Inspection No.	**Time of inspection** [years]
1	**35**
2	**46**
3	**48**
4	**50**
5	**51**

Calculated times for the first five inspection of the structural element



Resulting histogram of the calculation for $t = 35$ years of structural operation: **safety margin** G_{fail} .



Resulting **probabilities of random events** U, D and F for the first 60 years of operation under various load.

Parallel computing

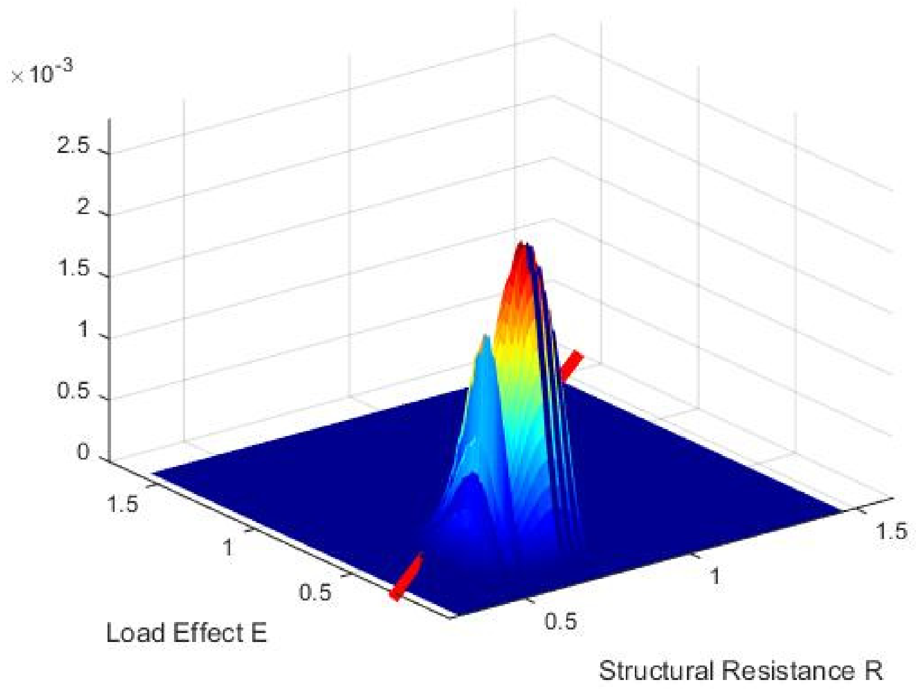
DOProC method is able to parallelize the calculation (tested on supercomputer, used only 12 cores yet).

IT4Innovations National Supercomputing Center,
VSB-Technical University of Ostrava, rank **69th** worldwide in Top500 list (2021)

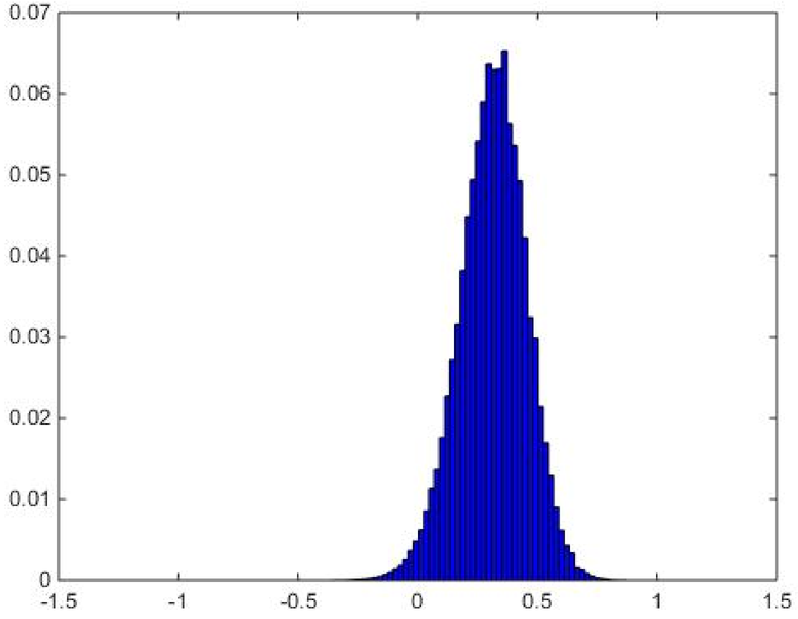
Parallel computing in MatLab

Results of the probabilistic reliability assessment of the element under **3-point bending** in Matlab

128 intervals in each histogram



Probability of failure: $P_f = 0.01641$



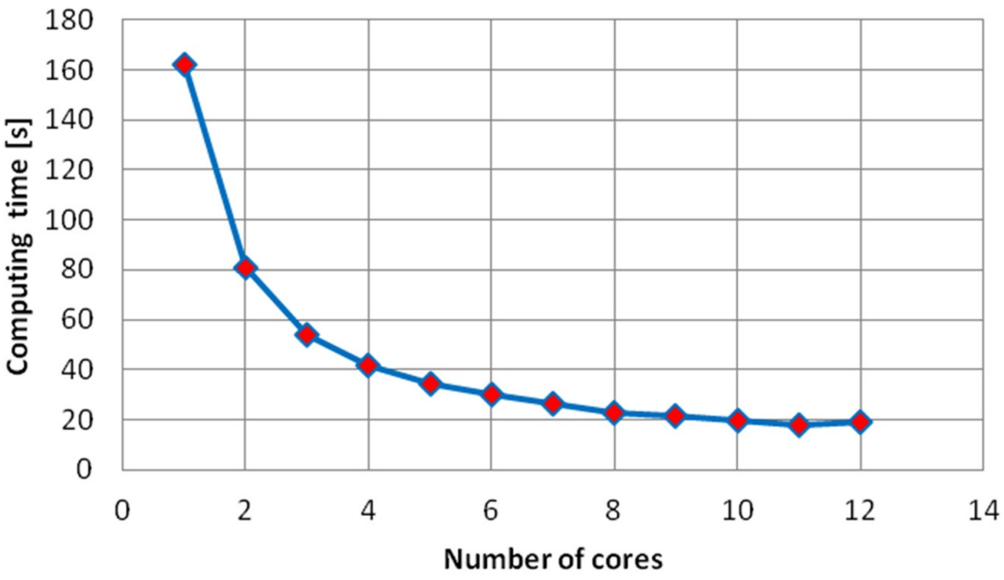
Resulting histogram of the calculation for $t = 35$ years of structural operation: safety margin G_{fail}.

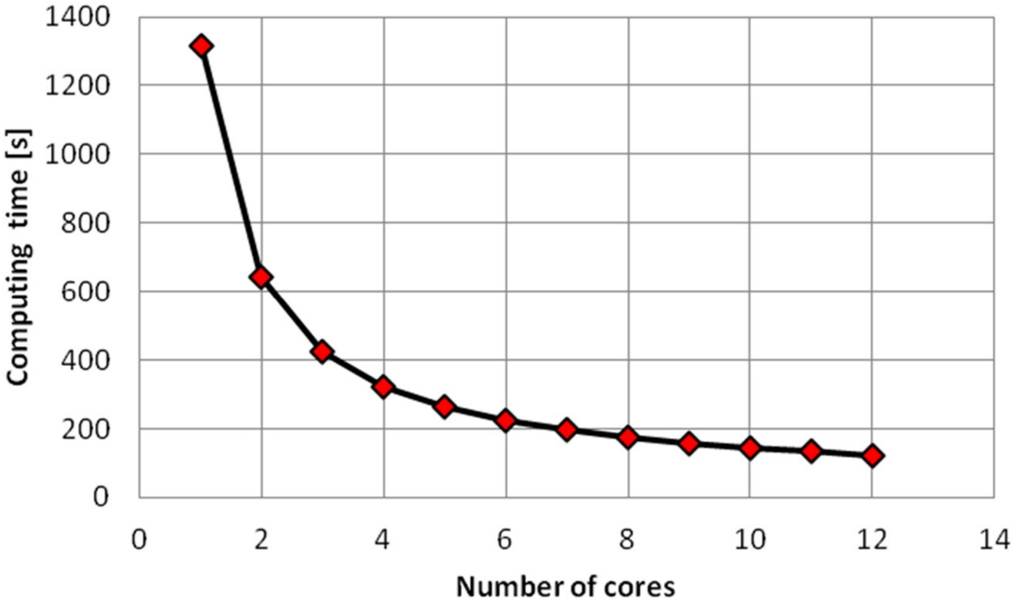
Parallel computing in MatLab

Number of intervals	64		128		256		1024	
Core count	Time [min]	Probability of failure	Time [min]	Probability of failure	Time [min]	Probability of failure	Time [min]	Probability of failure
1	**0.39**	0.013495	**2.71**	0.01641	**21.91**	0.018427	-	-
3	**0.13**	0.013495	**0.90**	0.01641	**7.10**	0.018427	-	-
6	**0.08**	0.013495	**0.50**	0.01641	**3.77**	0.018427	-	-
9	**0.07**	0.013495	**0.36**	0.01641	**2.62**	0.018427	-	-
12	**0.06**	0.013495	**0.32**	0.01641	**2.03**	0.018427	**88.03**	0.019917

Comparison of **calculation time** depending on the number of cores and the number of classes in input histograms including the resulting probability of failure

Parallel computing in MatLab



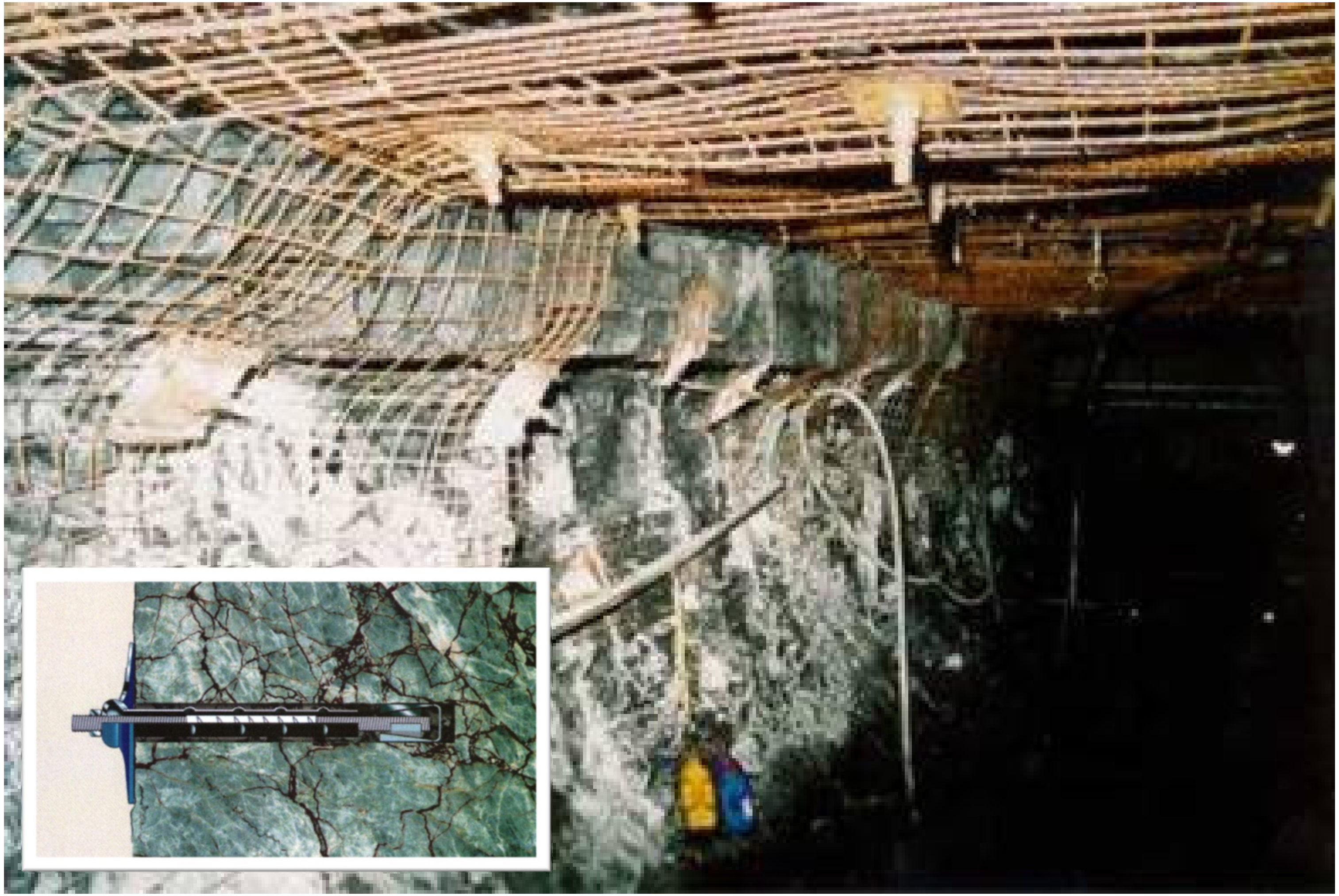


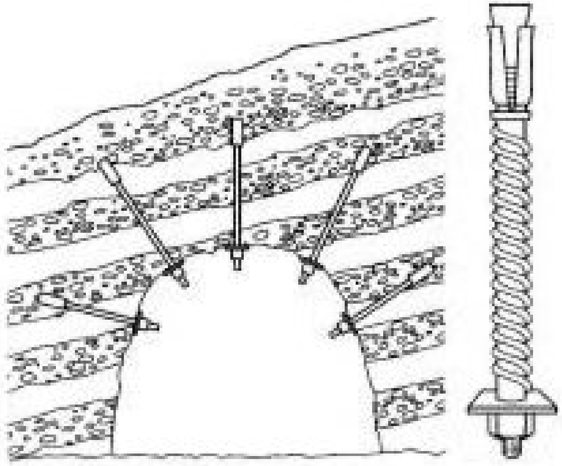
Parallel algorithm scaling: decrease of **computing time** with increasing **number of processor units**, input histograms described by **128 classes** (left) and **256 classes** (right)

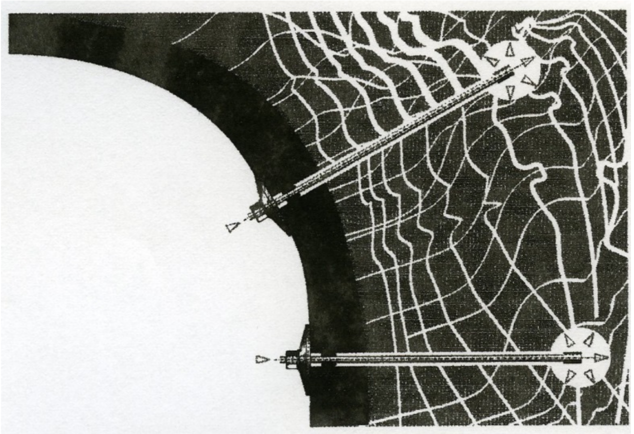
Reliability assessment of existing structures

Reliability aassessment of old crane tracks in a metallurgical company after **85 years of operation**

Anchoring the reinforcement of underground works







Anchoring the reinforcement of underground works

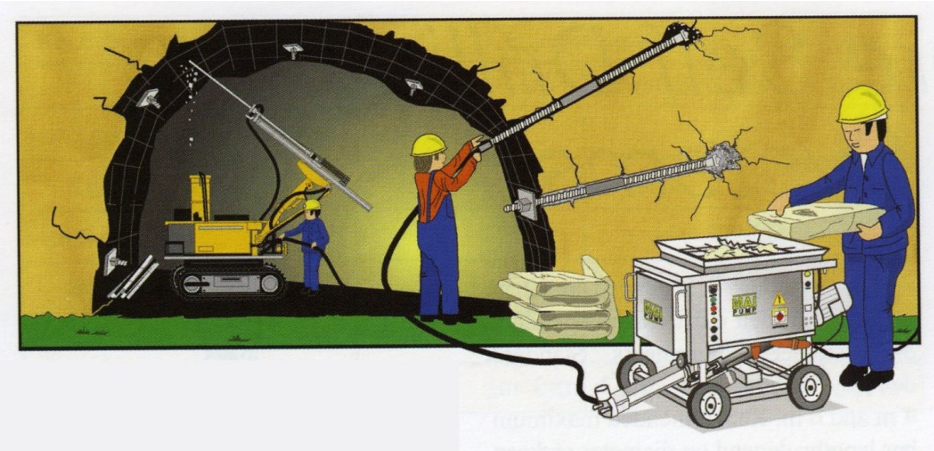
Anchoring the reinforcement of underground works

Anchoring the reinforcement of underground works

Anchoring the reinforcement of underground works

Design of anchor reinforcement required for the conditions determined in particular:

- length of anchors (bolts),
- their number and location around the mine or underground work,
- anchors parameters (type, material, diameter, etc.) for determining the structural resistance,
- load anchors.

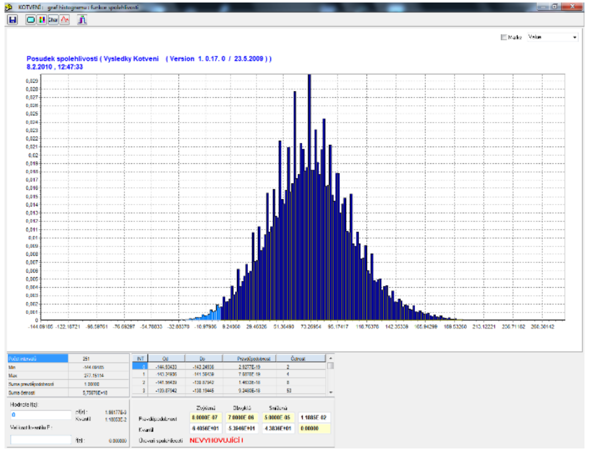


Software utility for probabilistic design

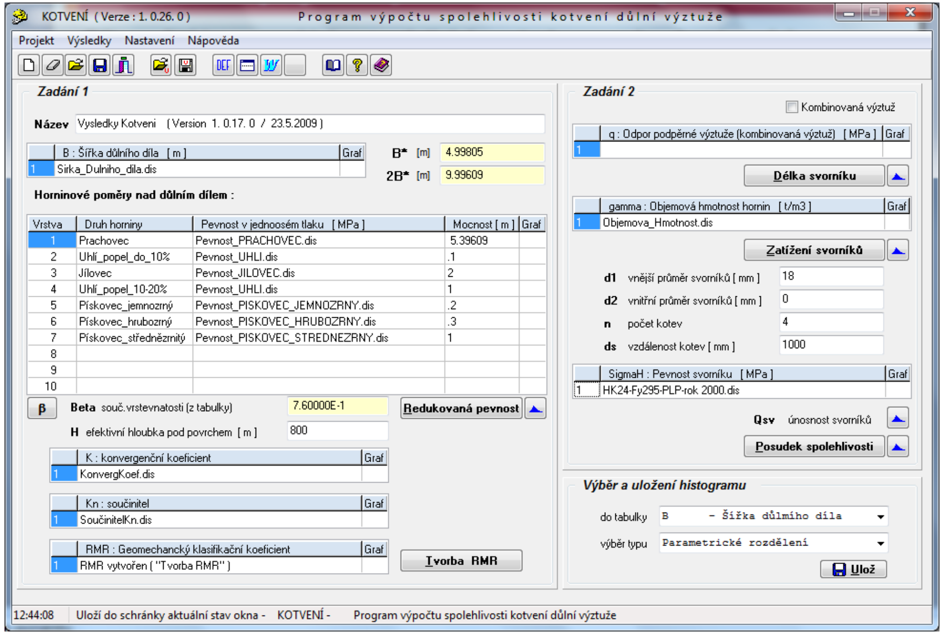
Using the empirical-analytic methods and a set of input random variables was developed SW **Anchor** for probabilistic designing and assessment of the anchor reinforcement using DOProC method.

Output:

- length of bolts l
- number n
- resistance Q_{sv} of anchor reinforcement



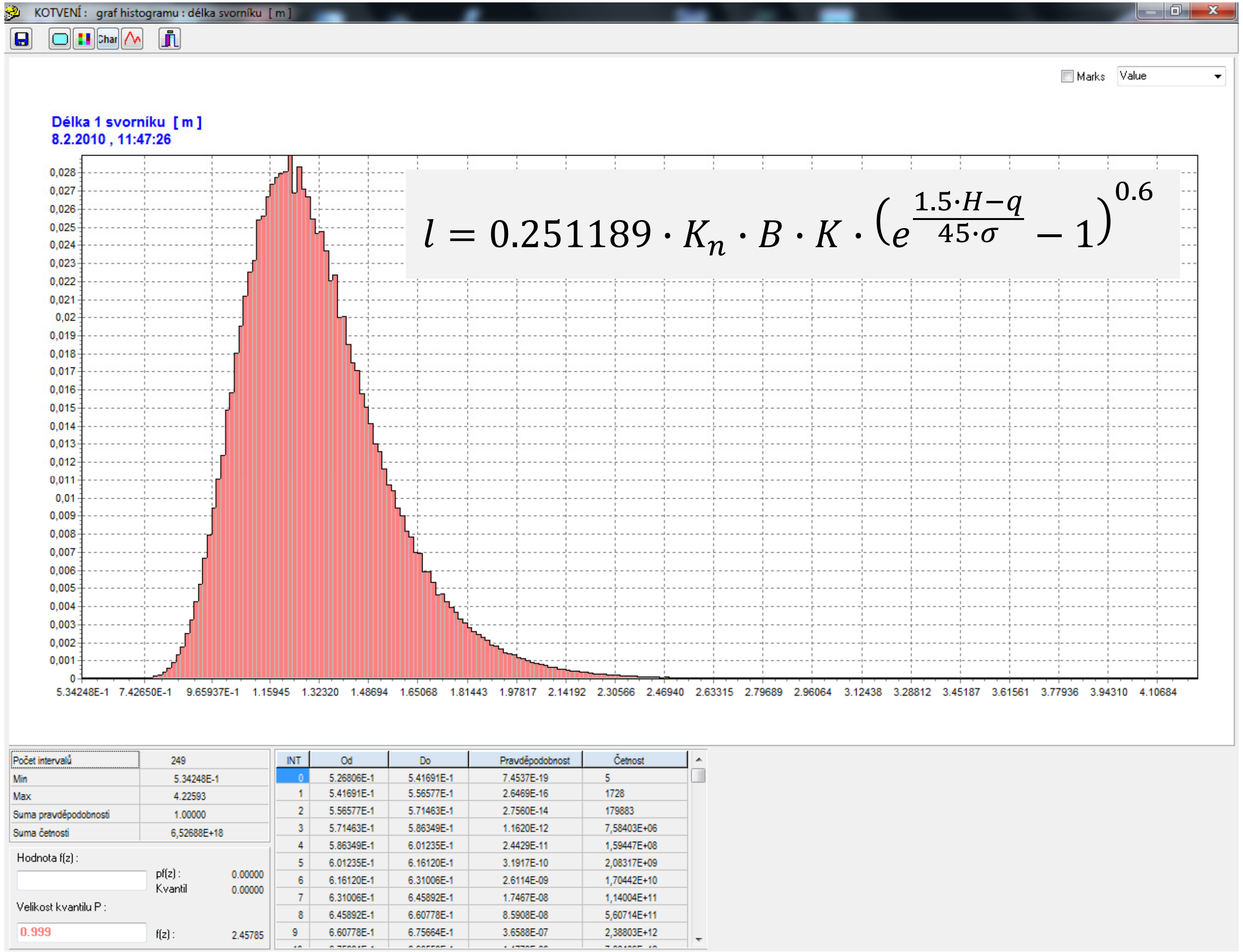
Histogram of the reliability function RF, probability of failure $P_f = 1.19 \cdot 10^{-2}$ for 4 anchors Ø18 mm / 1 m of mining work



Desktop of the program

Software utility for probabilistic design

Results: Length of bolts l

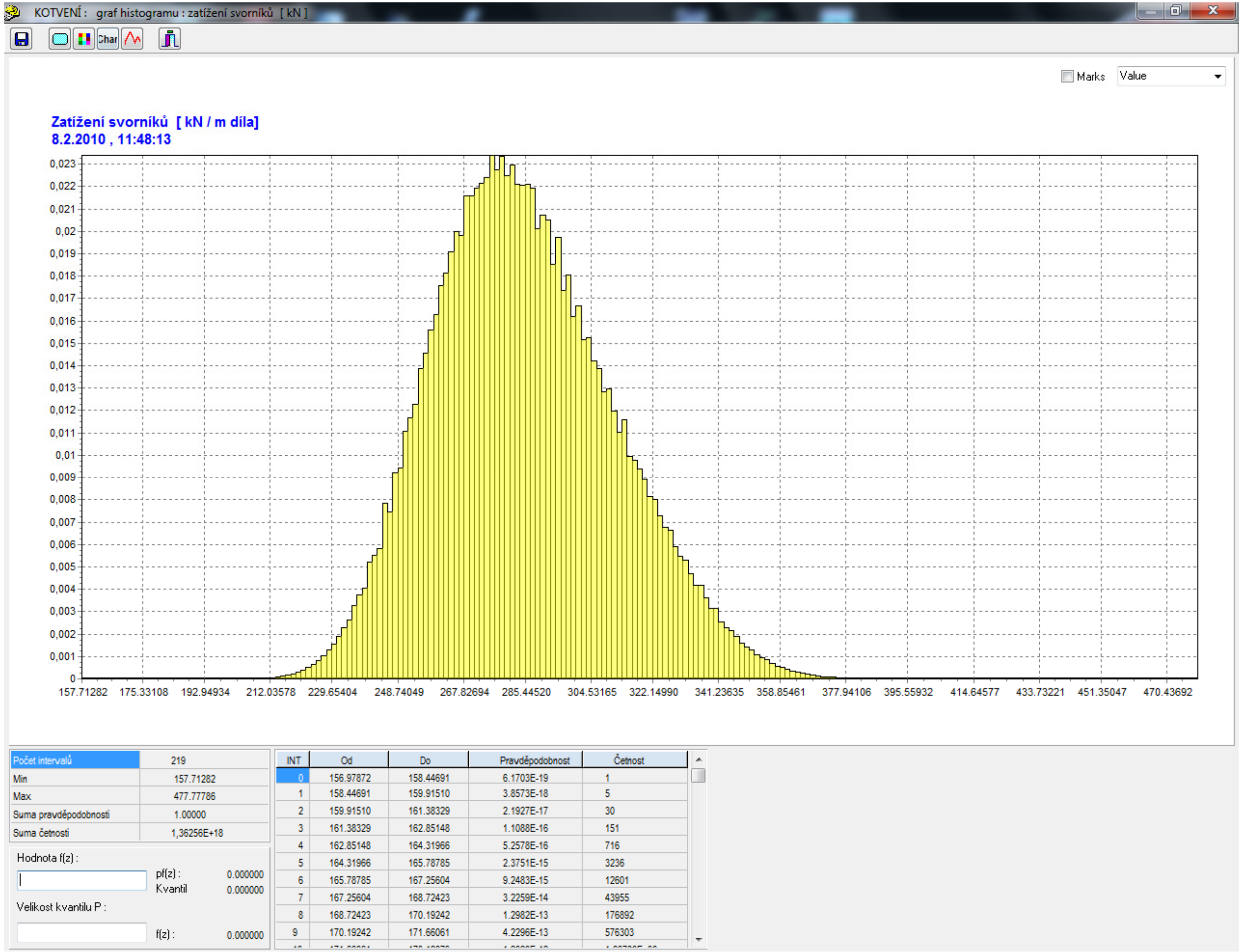


$$l = 0.251189 \cdot K_n \cdot B \cdot K \cdot \left(e^{\frac{1.5 \cdot H - q}{45 \cdot \sigma}} - 1\right)^{0.6}$$

Histogram of the **bolt length** l [m]

Software utility for probabilistic design

Results: Bolt loading Q



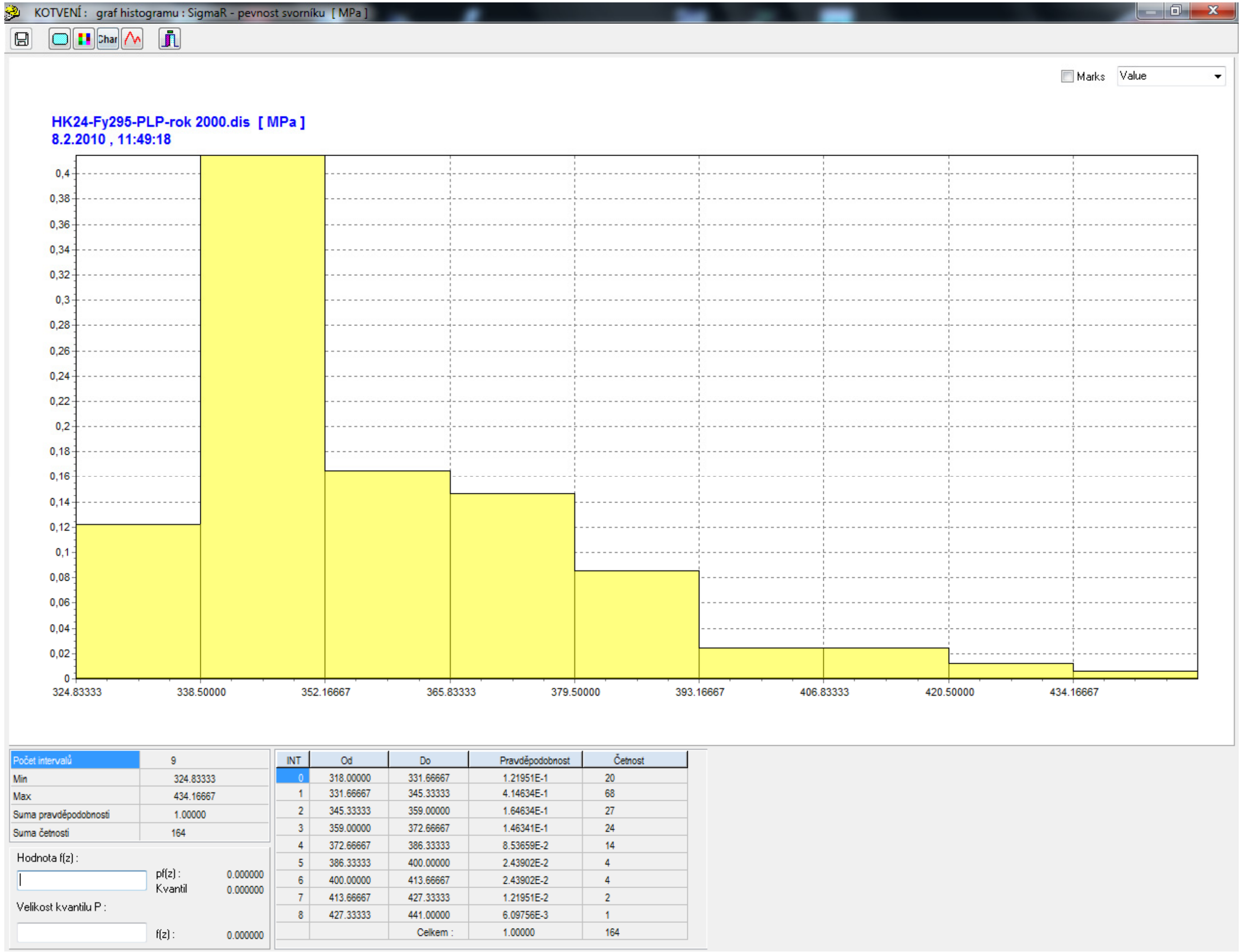
Histogram of **bolt loading** Q $[\mathrm{kN/m}]$

Software utility for probabilistic design

Results: Loading capacity of bolts Q_{sv}

$$Q_{sv} = \frac{n \cdot q_{sv}}{d_s} = \frac{n \cdot \pi \cdot (d_1 - d_2)^2 \cdot \sigma_{sv}}{4 \cdot d_s}$$

A histogram of **loading capacity of bolts** Q_{sv} [kN]



Software utility for probabilistic design

Reliability assessment of anchor reinforcement

$$RF = Q_{sv} - Q$$

A histogram of the reliability function RF with the resultant failure probability P_f