VSB TECHNICAL UNIVERSITY OF OSTRAVA | FACULTY OF CIVIL ENGINEERING | DEPARTMENT OF STRUCTURAL MECHANICS

Topic 8:

DOProC method (Direct Optimized Probabilistic Calculation)

- Theoretical background
- Optimizing techniques
- ProbCalc software
- Examples

Calculation of Probability of Failure

Reliability analysis leads to estimation of the failure probability:

$$P_f = \int_{D_f} f(X_1, X_2, \ldots, X_n)\, \mathrm{d}X_1, \mathrm{d}X_2, \ldots, \mathrm{d}X_n$$

where D_f is **failure area** and $f(X_1, X_2, \ldots, X_n)$ **failure function** of n **random variables** $X_1, X_2, \ldots, X_n$ defined by their probability distributions.

General solution of failure probability P_f based on explicit integral calculation is very difficult.

Probabilistic Methods

Simulation methods

Simple simulation Monte Carlo,

Stratified simulation techniques:

Latin Hypercube Sampling – LHS,
Stratified Sampling – SC.

Advanced simulation methods:

Importance Sampling – IS,
Adaptive Sampling – AS,
Axis Orthogonal Importance Sampling,
Directional Sampling – DS,
Line Sampling – LS,
Design Point Sampling,
Subset Simulations,
Descriptive Sampling, Slice Sampling.

Approximation methods

- First (Second) Order Reliability Method - FORM (SORM),
- Response Surface Method – RSM,
- Perturbation techniques – e.g. Stochastic Finite Element Method (SFEM),
- Artificial Neural Network – ANN.

Pure numerical methods

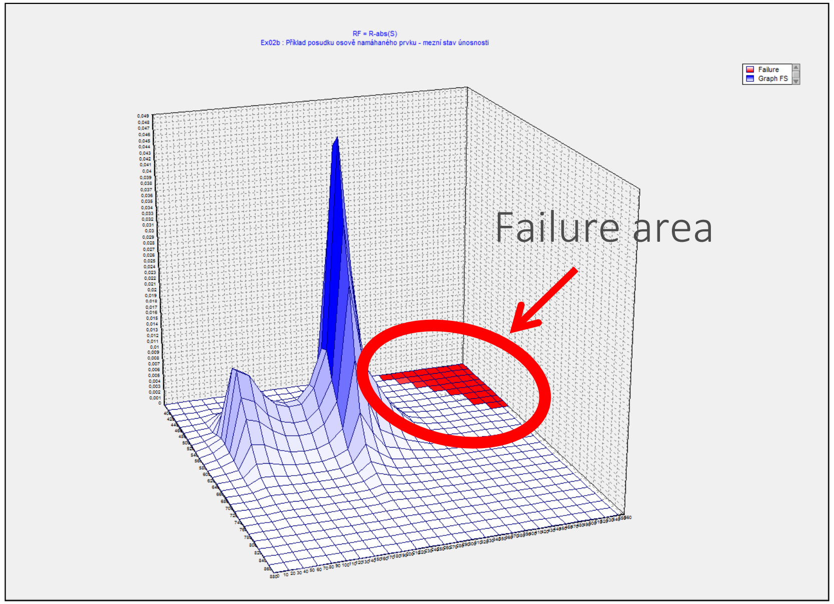
(without simulations and approximations)

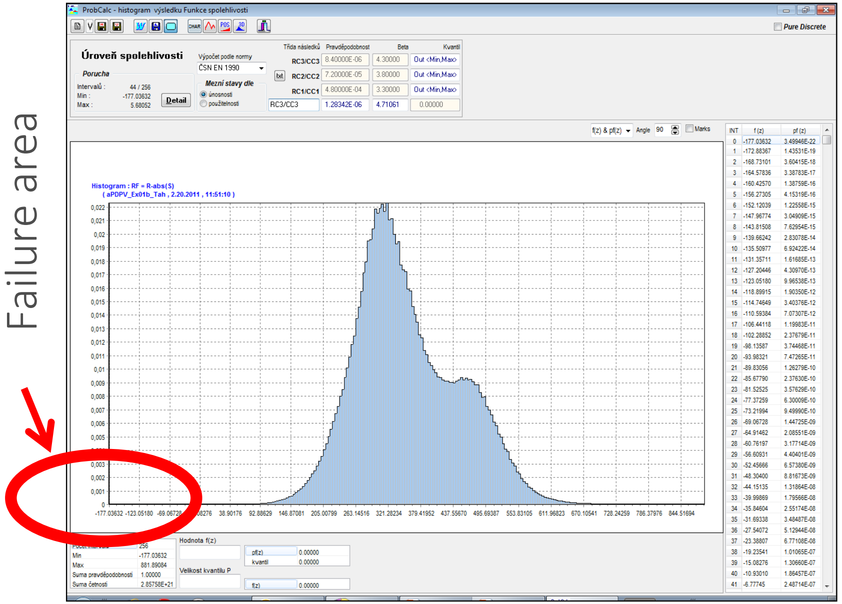
- Point Estimate Method – PEM,
- Direct Optimized Probabilistic Calculation – DOProC .

Overview e.g.:
Krejsa & Králik (2015)

Direct Optimized Probabilistic Calculation

The method can be used to **reliability assessments** of the structures or to the other probabilistic calculations.



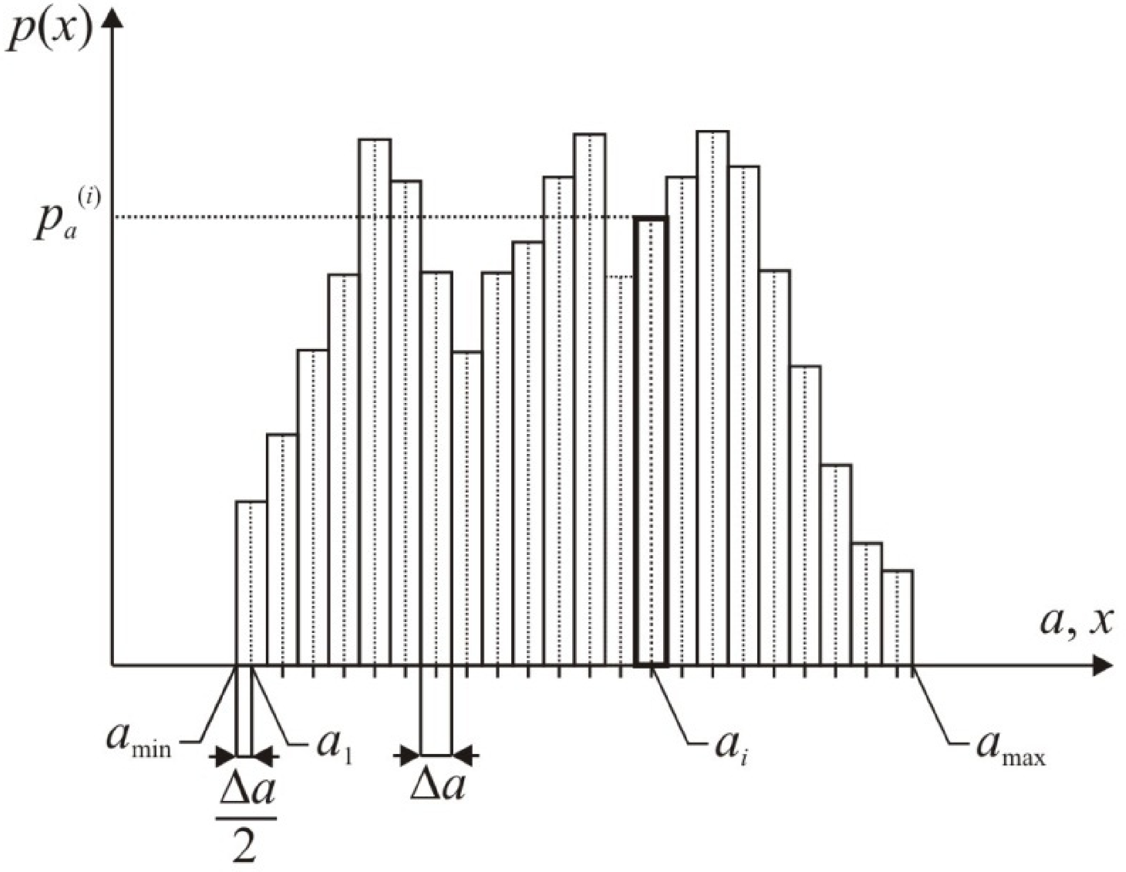


The reliability function (computation model) can be expressed analytically or using dynamic libraries using numerical methods.

Essential of DOProC Characteristics

- Direct Optimized Probabilistic Calculation – DOProC, should be effectively used for the **assessment of structural reliabilities** and/or for other probabilistic calculations.
- Input random variables (load, geometry, material properties, imperfections) are expressed by the **empirical** (non-parametric) or **parametric distributions** in histograms.
- Reliability function under analysis can be expressed analytically in text mode or using DLL library.
- Inaccuracy of calculation is done only by **discretization of input and output random variables** and by numerical error.
- The **number of intervals** (classes) of each histogram is extremely important for the number of needed numerical operations and required computing time.
- The number of numerical operations can be reduced using **optimizing techniques**.

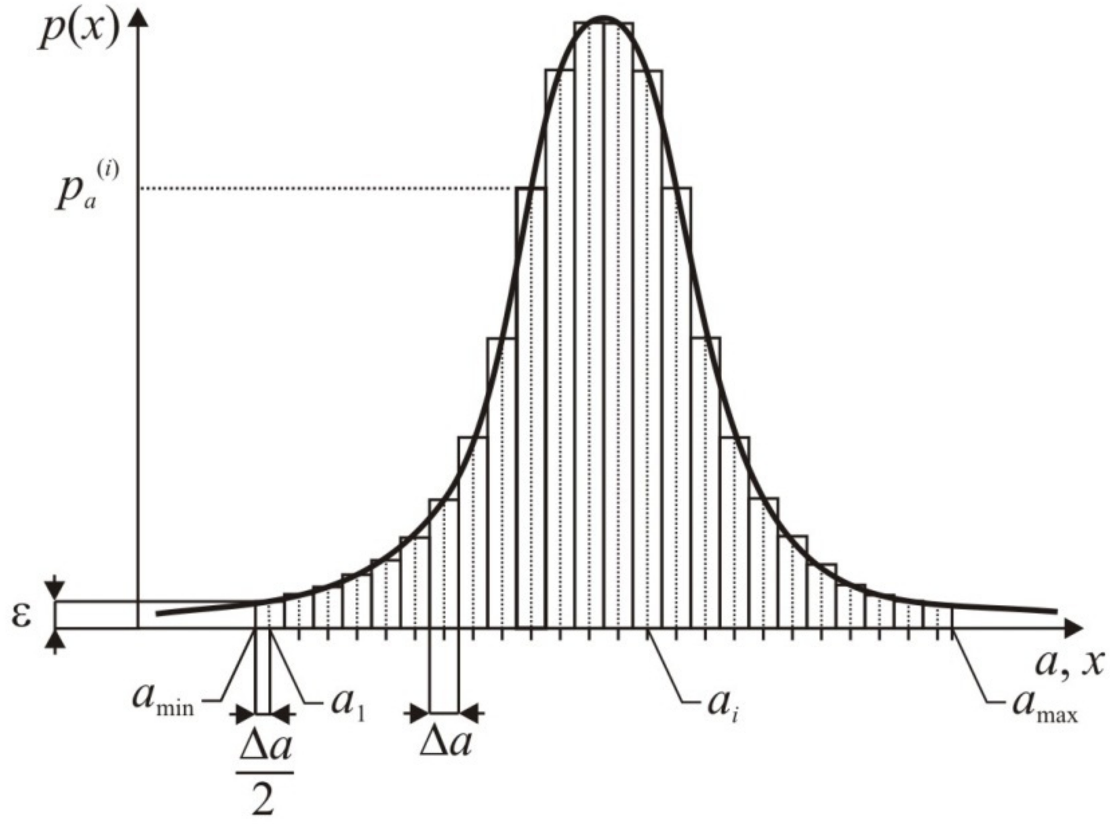
Histogram of Random Variable



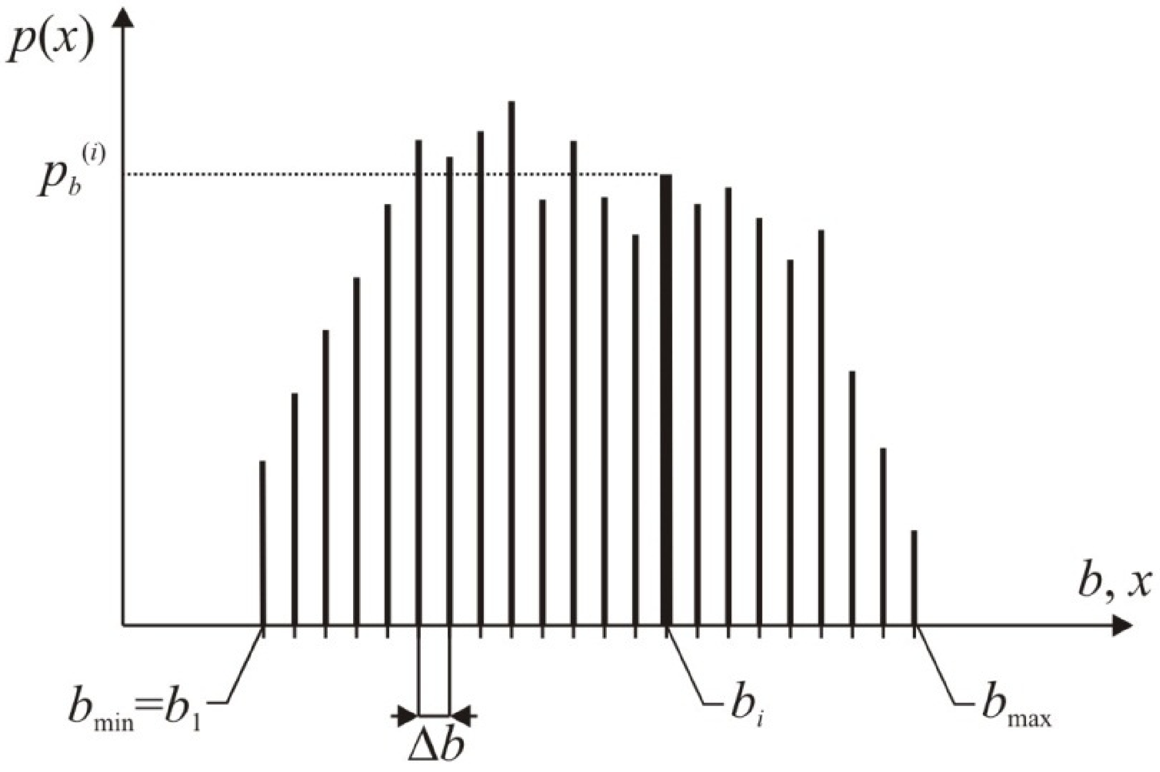
Histogram of discretized continuous random variable with non-parametric (empirical) probability distribution

Histogram of Random Variable

Histogram of discretized continuous random variable with parametric probability distribution



Histogram of Random Variable



Histogram of pure discrete random variable

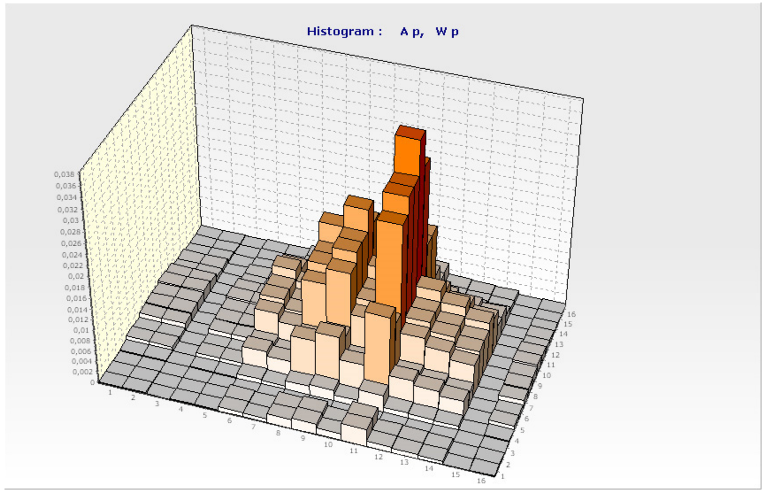
Structure of the Data File / Histogram Definition

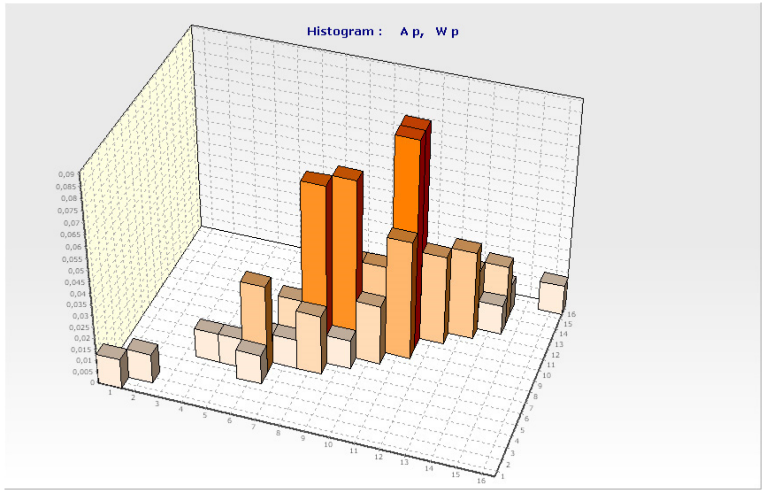
A **text file** with the extension ***.dis** (distribution), which contains data in the following form:

```
[Description] (1st section of the data file)
Identification= Optional data file description
Type= Pure Discrete | Discrete | Continuous (Histogram
type of random variable)

[Parameters] (2nd section of the data file)
Min= Minimum value of a random variable
Max= Maximum value of a random variable
Bins= Total number of classes in the histogram
Total= Sum of the frequencies in all classes

[Bins] (3rd section of the data file)
frequency in 1st class
frequency in 2nd class
etc. ...
```

Statistically Dependent Input Variables

- Some of input variables are **statistically dependent** however, e.g. cross-section characteristics, strength and material properties etc.
- Statistically independent random variables are entered into probabilistic calculation using **double** or **triple histograms**.





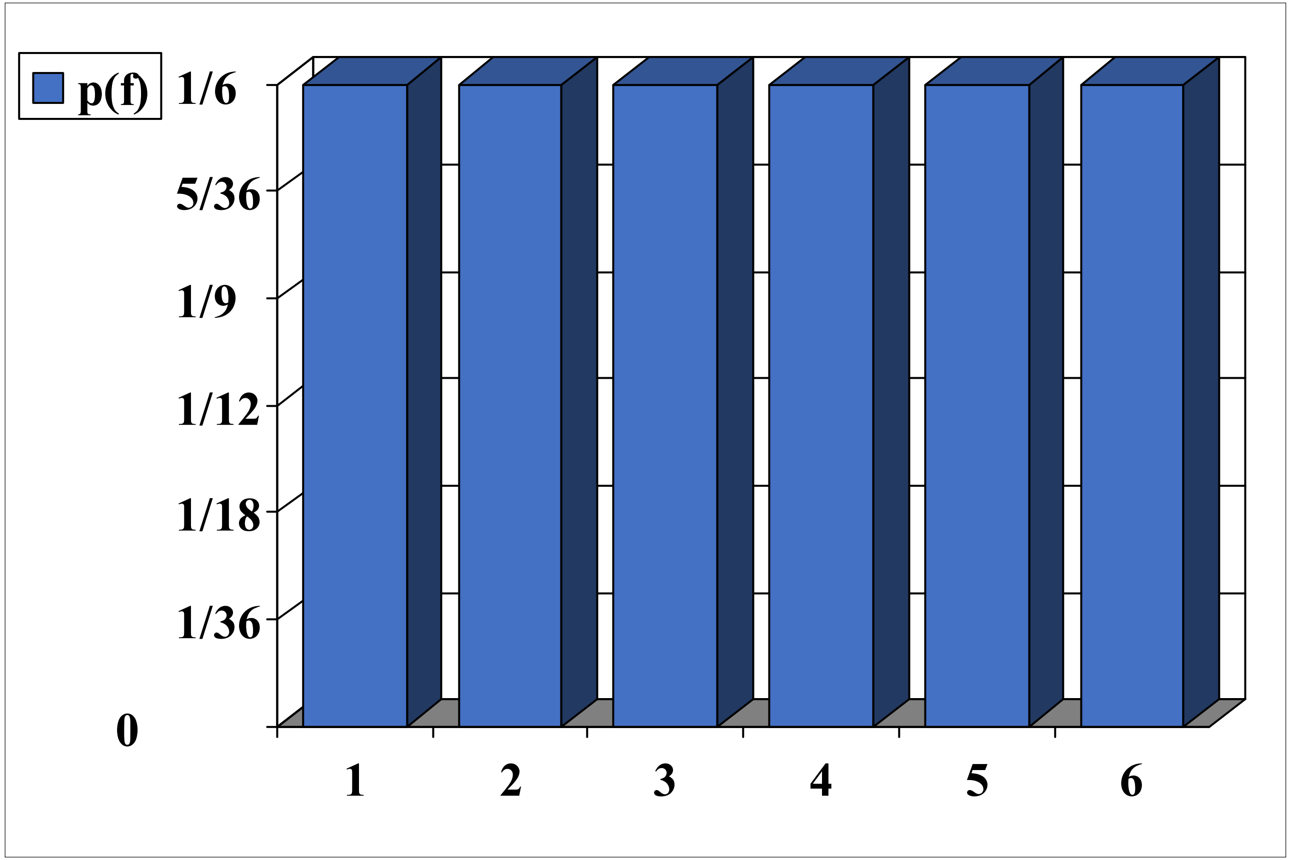
Desktop of HistAn2D: double histogram of statistically (in)dependent random variable

Statistics of Dice Throw

Throw of a **single dice** - all outcomes are equally probable.

$$p_1 = \frac{1}{n}$$

$$p_1 = \frac{1}{6}$$

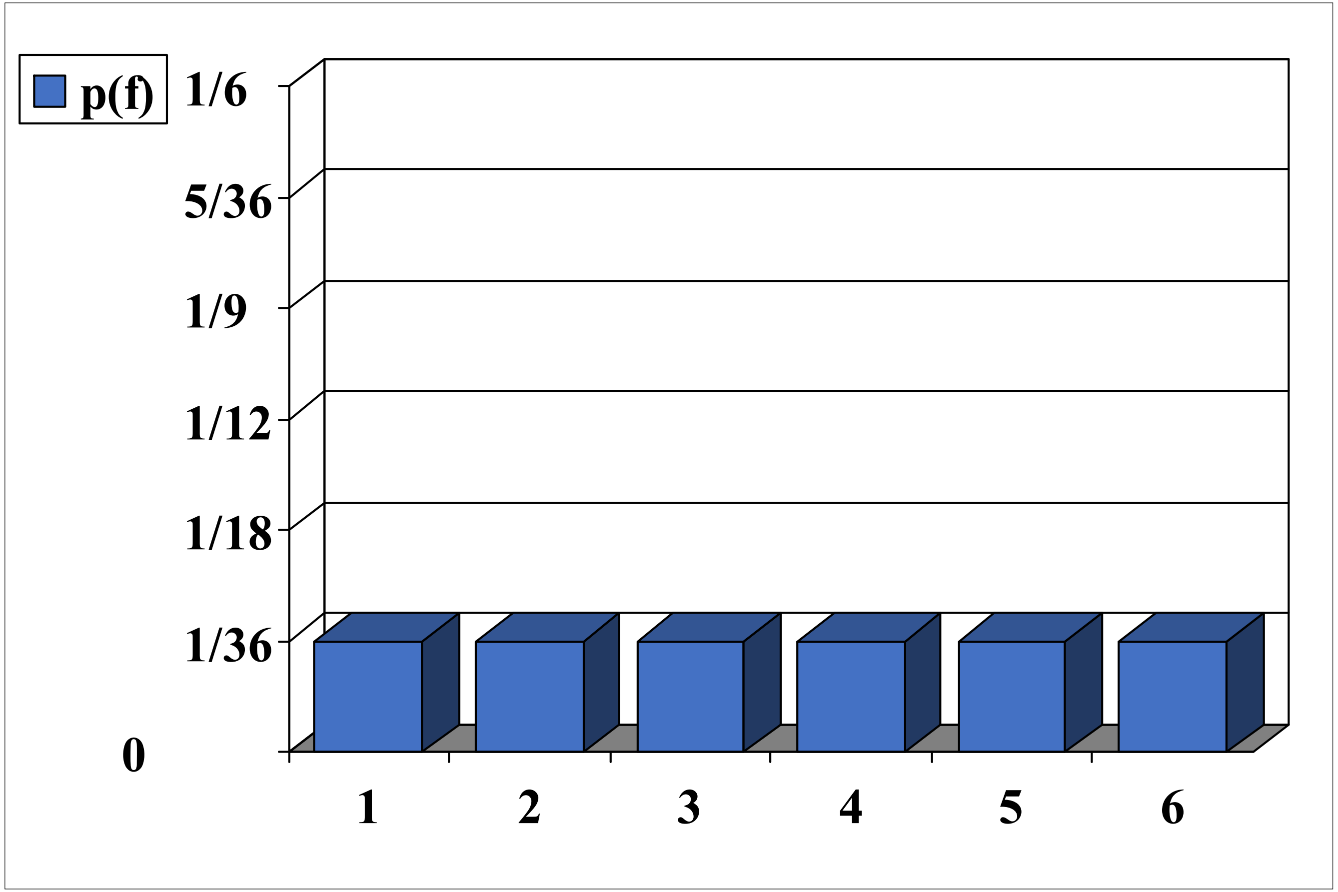


Statistics of Two Dices Throws

Probabilities of identical numbers obtained by the throw of two dices.

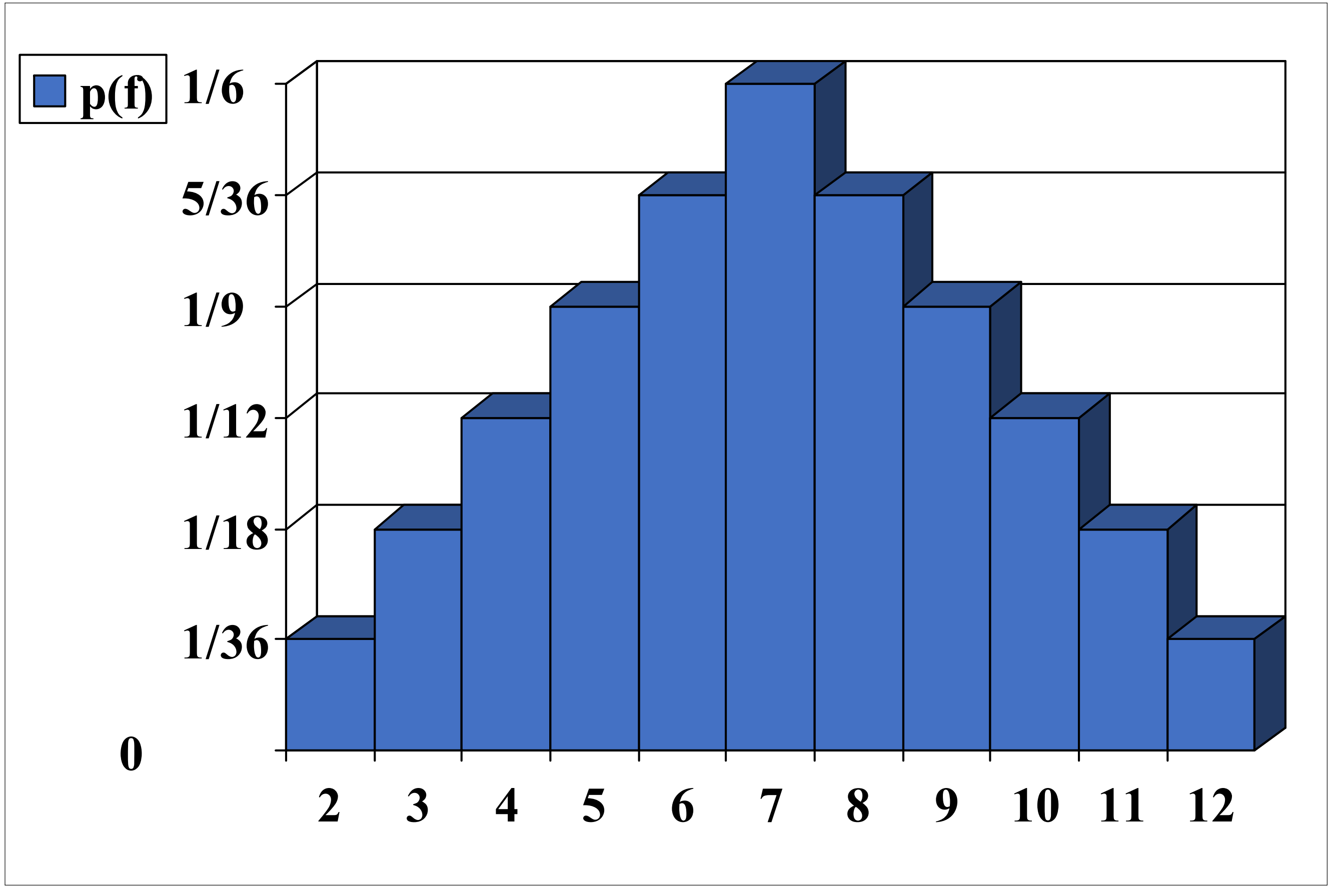
$$p = p_1 \cdot p_2$$

$$p_1 = \frac{1}{36}$$



Statistics of Two Dices Throws

The different possibilities for the **total of the numbers on two dices**.



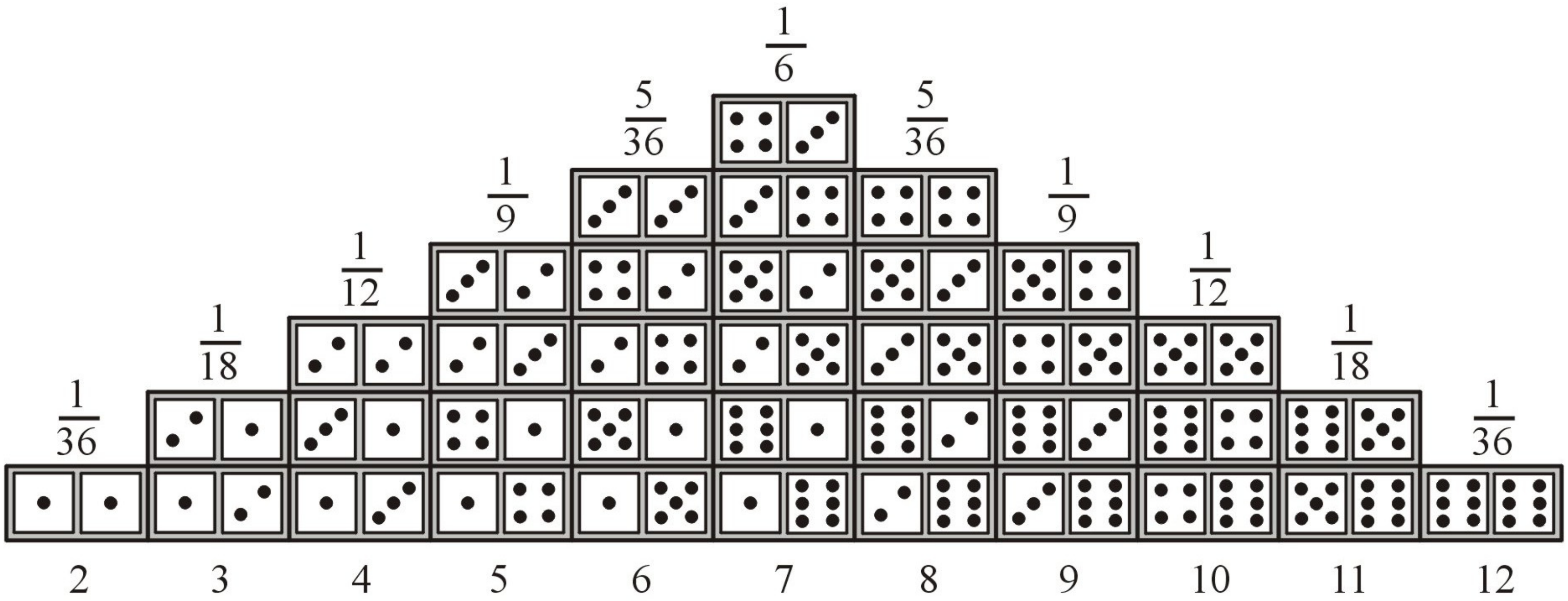
$$p(2) = \frac{1}{36}$$

$$p(3) = \frac{1}{36} + \frac{1}{36}$$

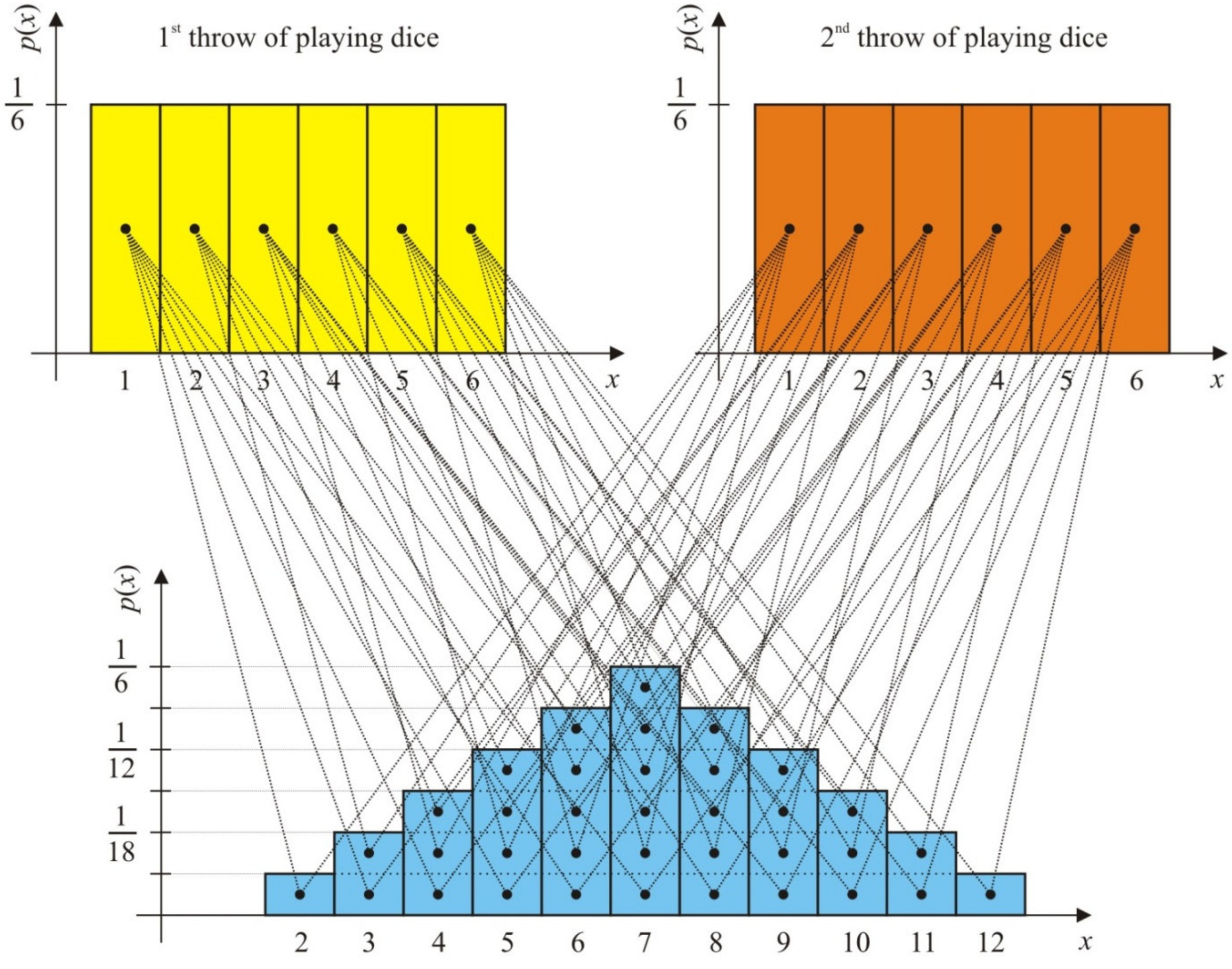
$$p(4) = \ldots$$

Statistics of Two Dices Throws

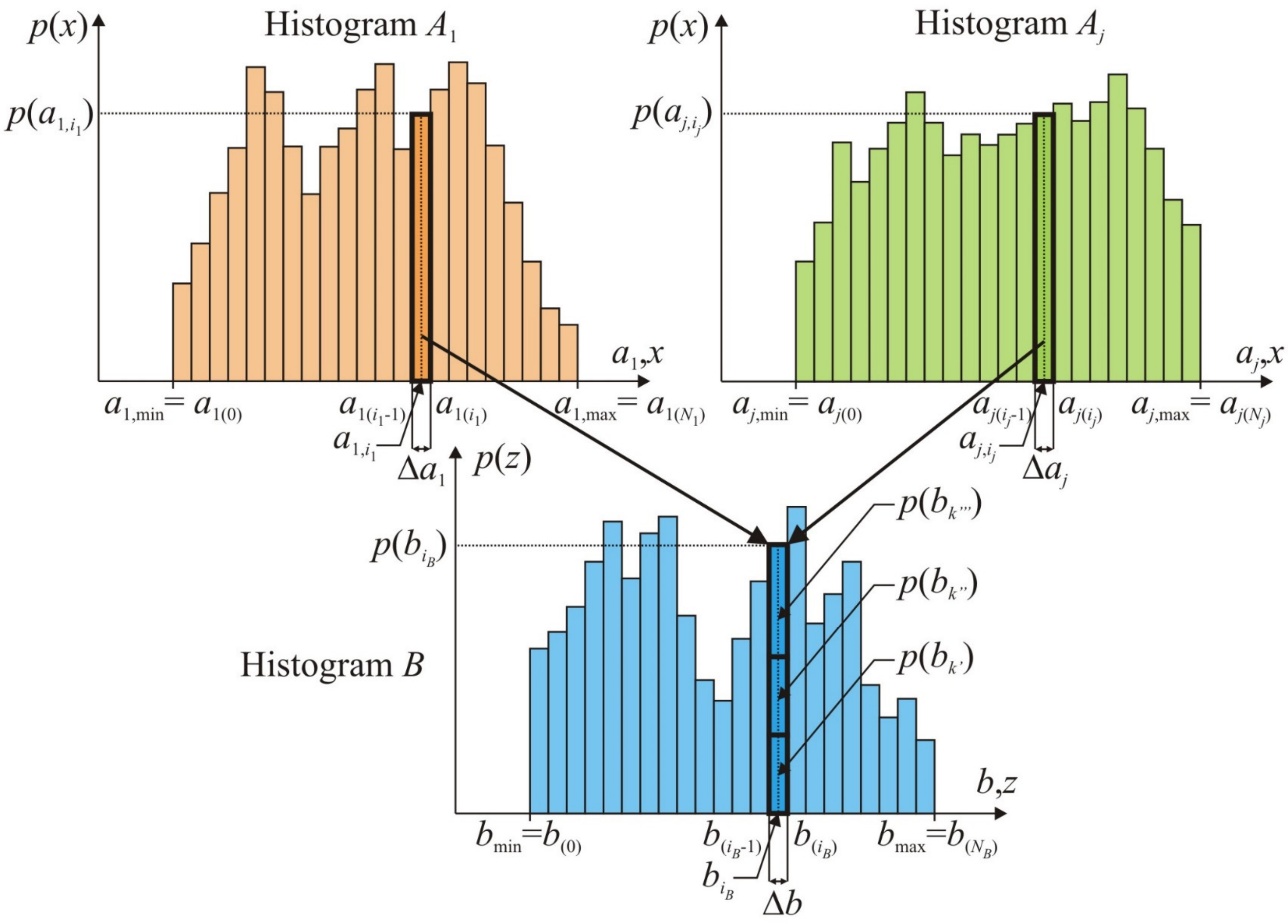
The different possibilities for the **total of the numbers on two dices** - are not equally probable - more ways to get some numbers than others.



Principle of Numerical Calculation



Basic computational algorithm



$$B = f_{(A_1, A_2, \ldots, A_j, \ldots A_n)}$$

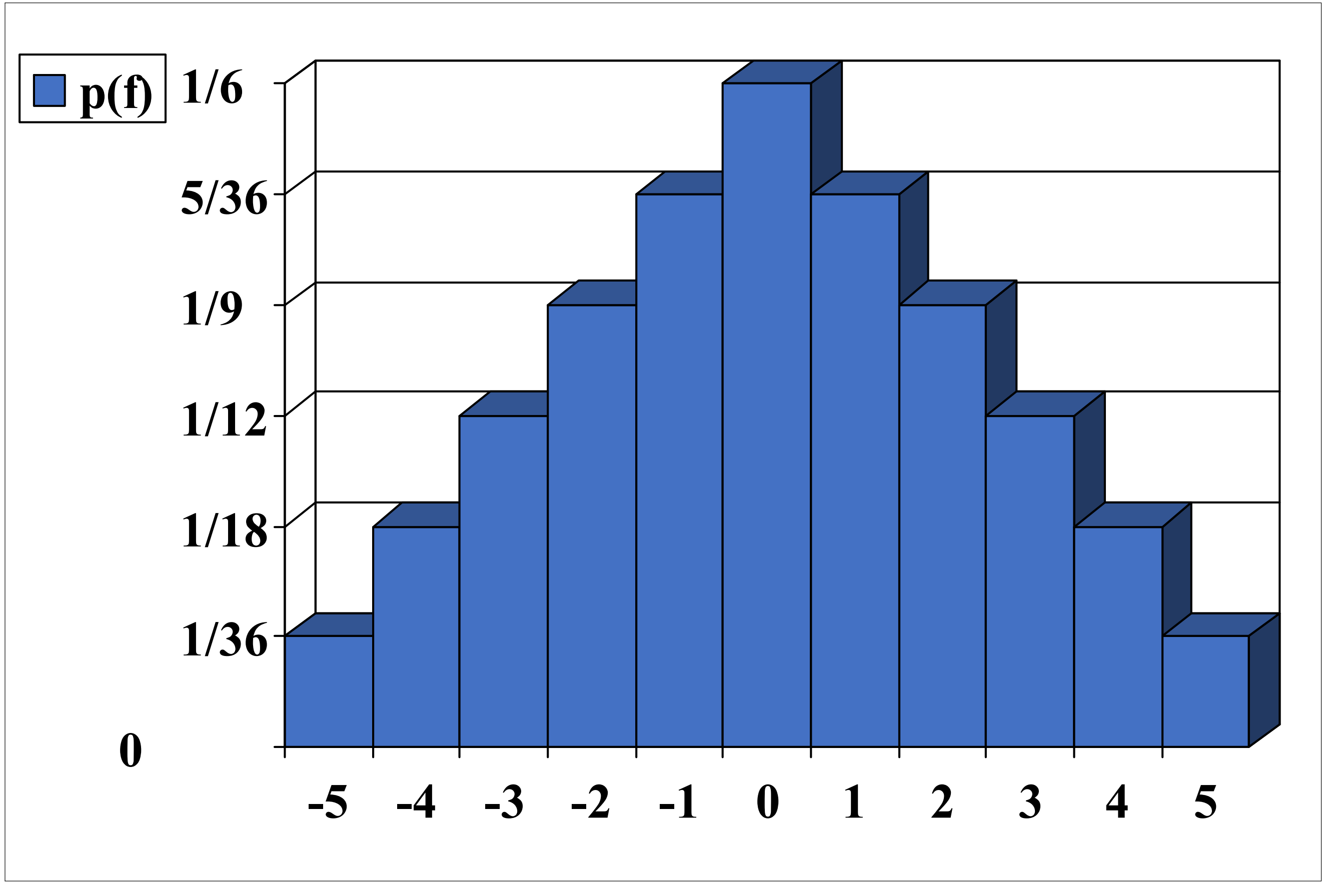
Statistics of Two Dices Throws

The different possibilities for the **algebraic difference of the numbers on two dices**.

$$p(5) = \frac{1}{36}$$

$$p(4) = \frac{1}{36} + \frac{1}{36}$$

$$p(3) = \ldots$$



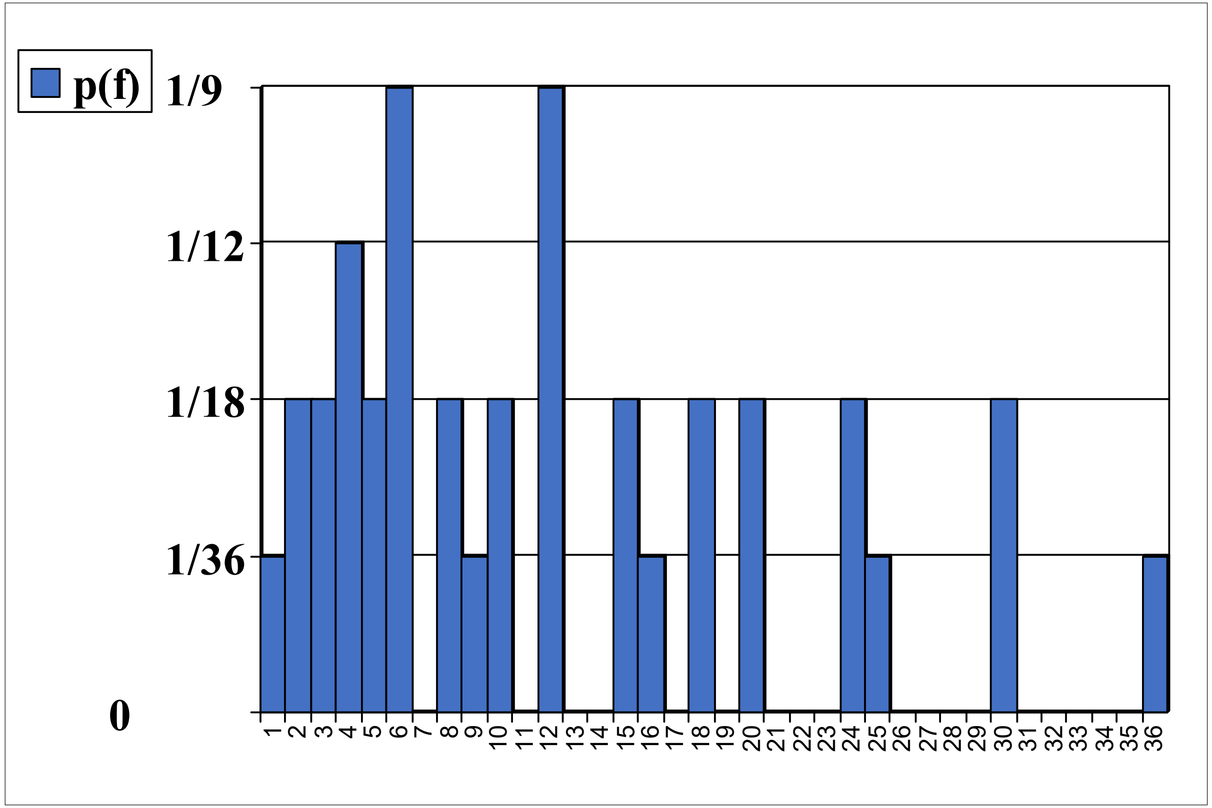
Statistics of Two Dices Throws

The different possibilities for the **arithmetic product of the numbers on two dices**.

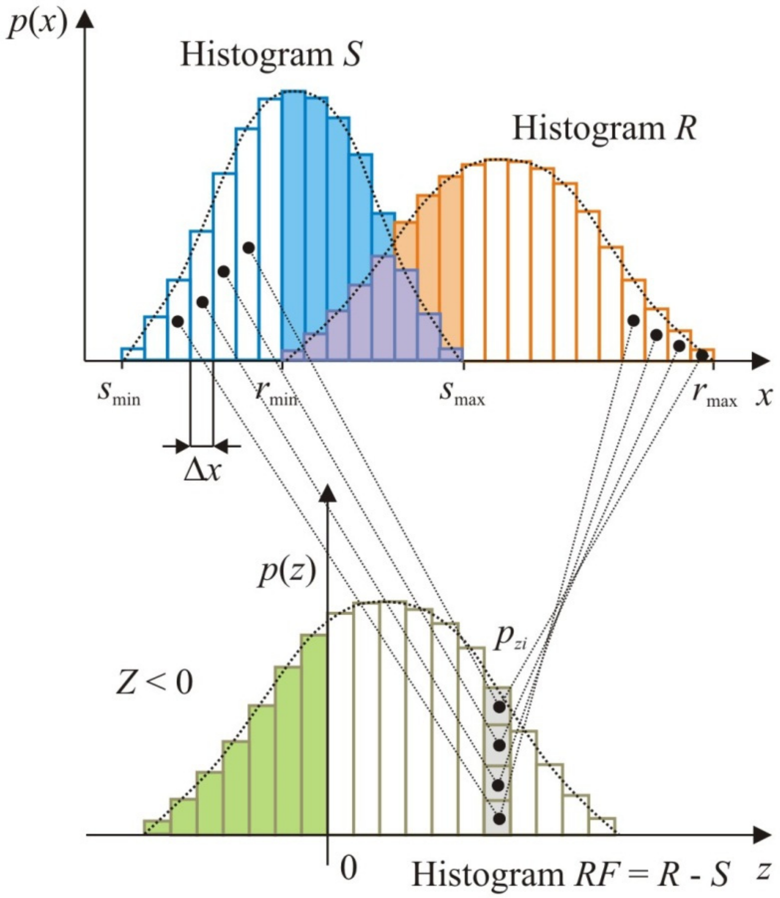
$$p(1) = \frac{1}{36}$$

$$p(2) = \frac{1}{36} + \frac{1}{36}$$

$$p(3) = \ldots$$



Principle of Numerical Calculation



Calculation of all pair value combinations S_i and R_i, for resulting histogram Z.

Probability of failure P_f corresponds to probability for $Z < 0$.

This approach was originally used for DOProC, similar to the Monte Carlo method.

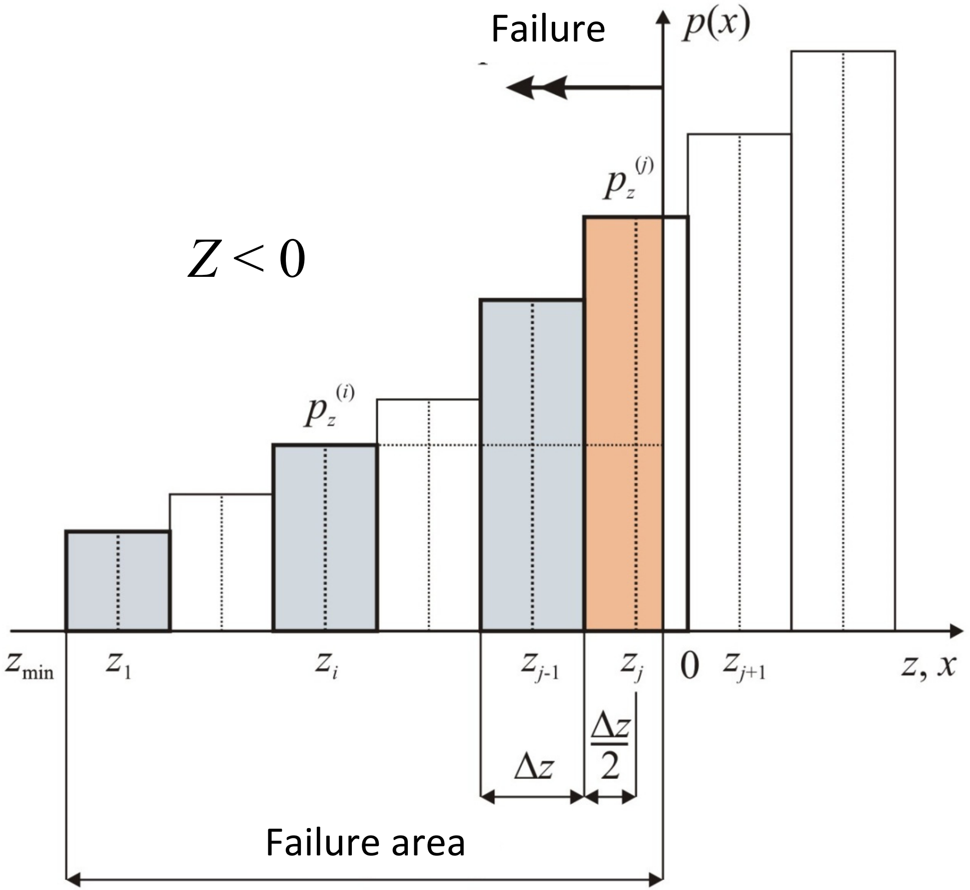
Principle of Numerical Calculation

Scheme of **probability of failure** P_f calculation using bounded histogram of **reliability function** Z.

Histogram Z included n subintervals with width Δz.

$$p_f = \sum_{i=1}^{j-1} p_z^{(i)} + p_z^{(j)} \cdot \left(1 - \frac{z_j + \frac{\Delta z}{2}}{\Delta z}\right) = $$

$$= \sum_{i=1}^{j-1} p_z^{(i)} + p_z^{(j)} \cdot \left(\frac{1}{2} - \frac{z_j}{\Delta z}\right)$$



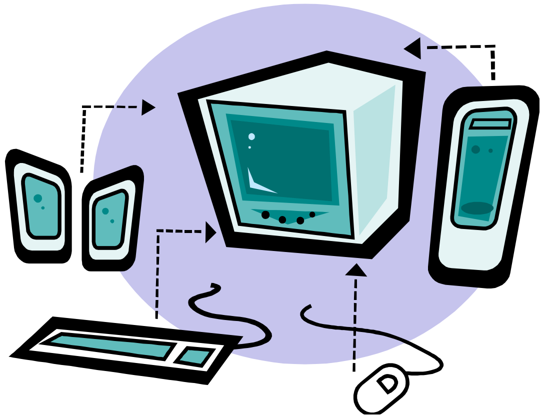
Basic Computational Algorithm

The **computational complexity** of the basic computational algorithm of DOProC method is especially given by:

- The number of **random input variables** $i = 1 \cdots N$,
- The **number of classes** (subintervals) n_i in histogram for each random input variables,
- **Difficulty of solved tasks** (calculation model),
- Probabilistic calculation algorithm (the way how is defined in the computational model):
  - in text mode,
  - in machine code (dynamic link libraries).

Optimizing Techniques in DOProC

- **Grouping of input random variables**, which can be expressed by the common histogram.
- **Interval optimizing** - decreasing the number of intervals in input variable histograms (sensitive analysis).
- **Zonal optimizing** - each histogram is divided into areas (zones) depending on their share in the failure.
- **Trend optimization** – using correct or incorrect trend of input variable on the result.
- **Grouping of partial calculations results**.
- **Parallelization** of the calculation – calculation is proceeded on number of processors.
- Combination of the mentioned optimizing techniques.



Grouping of Input Random Variables

Let be $B = A_1 + A_2 + A_3 + A_4 + \cdots + A_N$, whereas in each histogram are n classes (e.g. $n = 256, N = 10$).

All allowable combinations are $P_0 = n^N = 256^{10} = 1.20893 \cdot 10^{24}$.

The same result is possible to get step-by-step counting of both histograms. Then is $P_0^* = (N-1) \cdot n^2 = 9 \cdot 256^2 = 589{,}824$ and ratio:

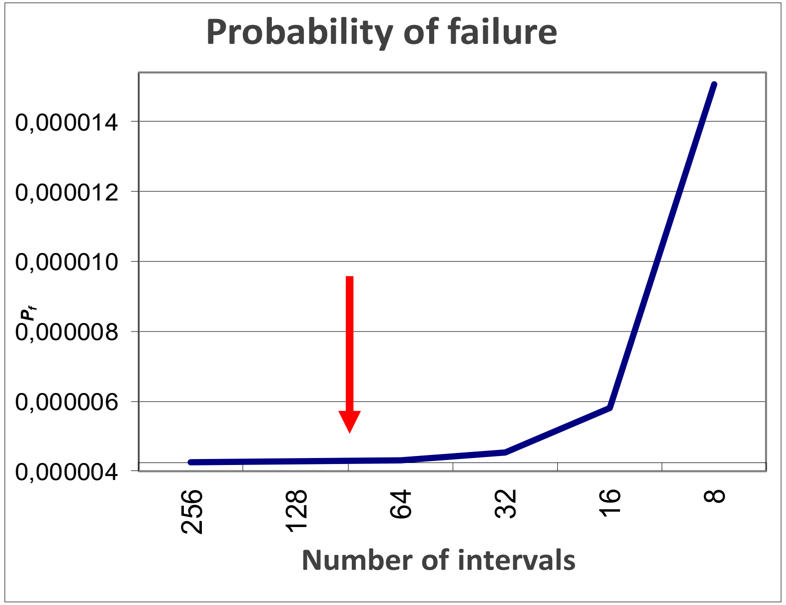
$$\frac{P_0^*}{P_0} = (N-1) \cdot n^{(N-2)} = 9 \cdot 256^8 = 4.87891 \cdot 10^{-19} .$$

If the creation of common histograms is correct – **grouping of input random variables is very rational procedure**.

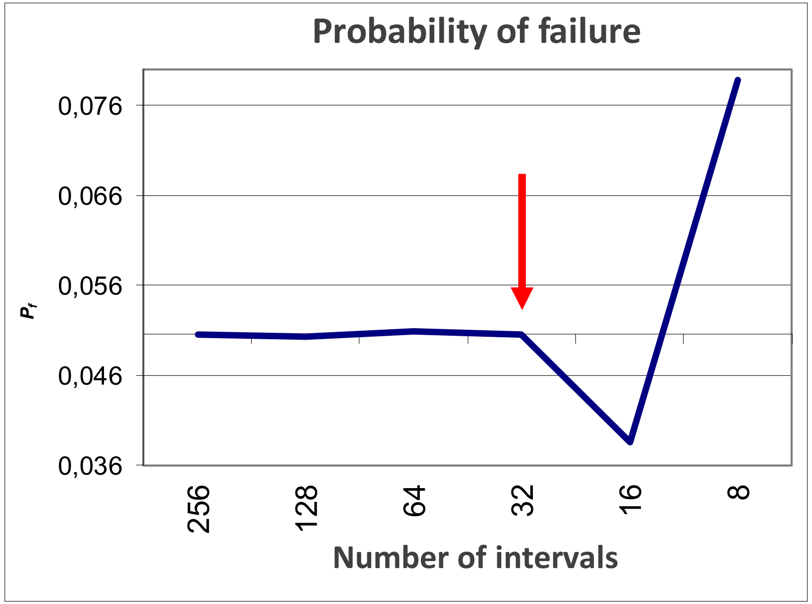
Interval Optimizing

Sense of **interval optimization** is:

- number classes minimizing in histograms,
- decreasing number of numerical operations and minimizing of computing time.

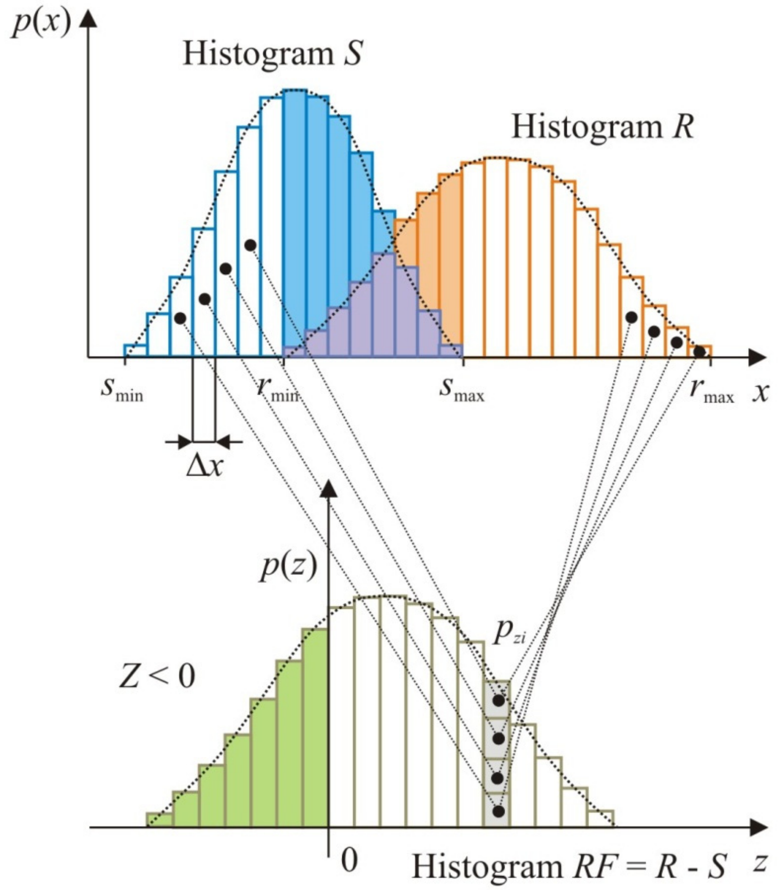


Sufficient number of classes (intervals) of histogram

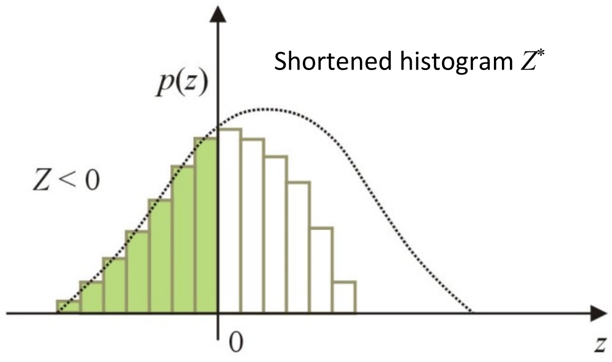


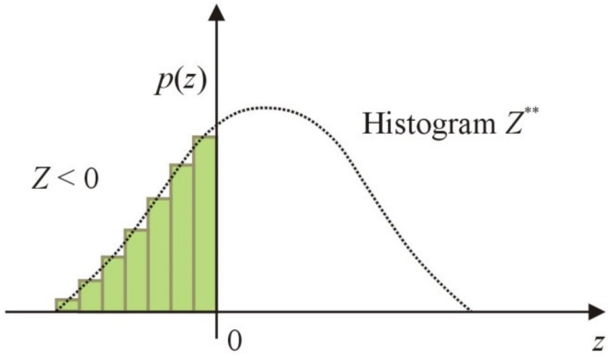
Sufficient number of classes (intervals) of histograms

Principle of Numerical Calculation



Calculation of pair value combinations S_i and R_i, for resulting histogram of random variable Z.





Zonal Analysis and Optimizing

Each histogram is divided into areas (zones – „the **zonal optimizing**“) depending on their share in the failure, whatever are the values of the other variables:

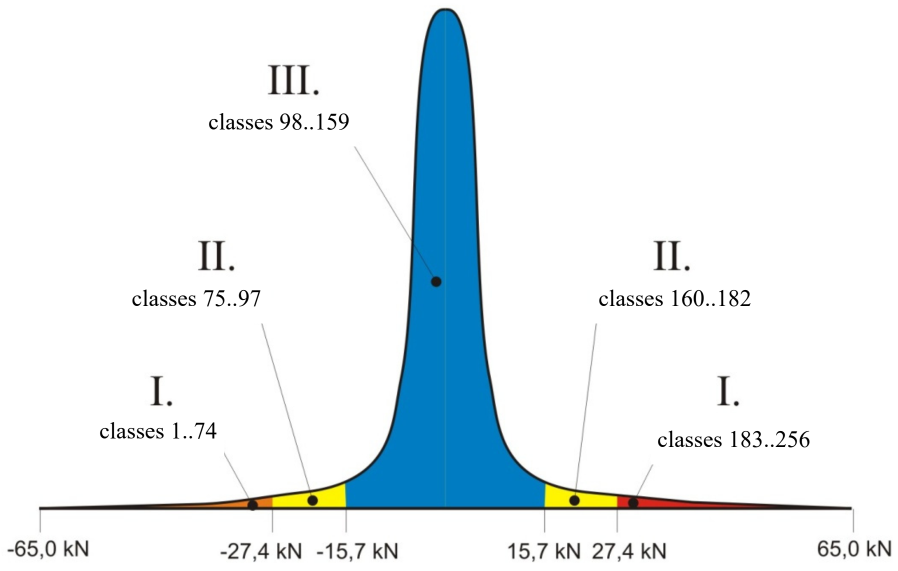
$$P_f = P_{f,1} + P_{f,2}$$

- **1st zone** – the failure occurs always
- **2nd zone** – the failure may occur depending on values of the other variables
- **3rd zone** – the failure does not occur

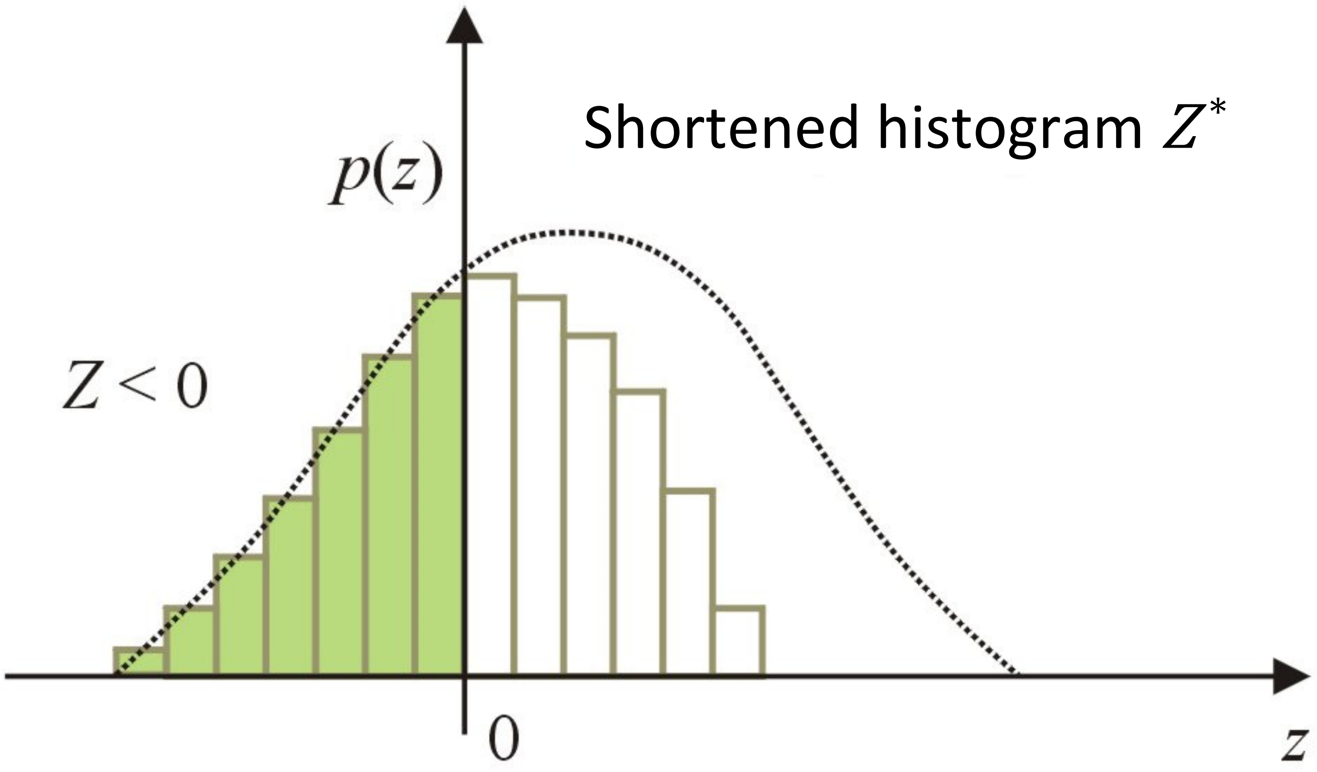
$P_f = 0$ always

$P_{f,2}$ only in some events

$P_{f,1}$ always



Zonal Analysis and Optimizing

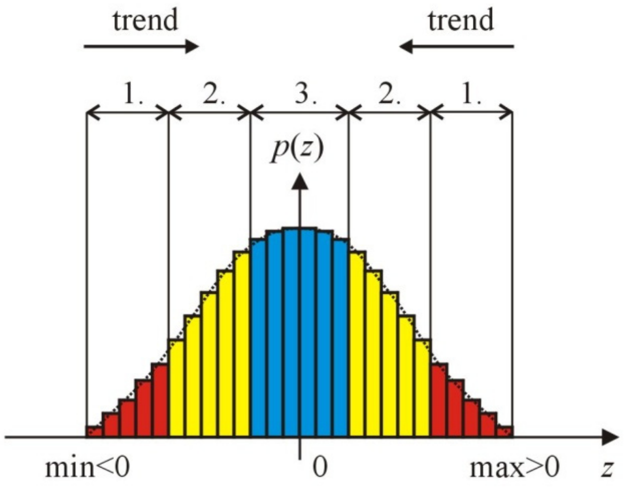


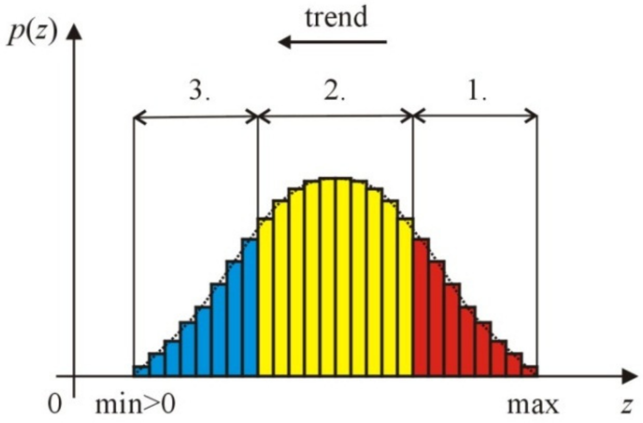
Resulting histogram of **reliability function** RF using DOProC method in action **zonal optimizing** – so-called „shortened histogram“ Z^*

Trend Analysis and Optimizing

Non-monotonous histogram:

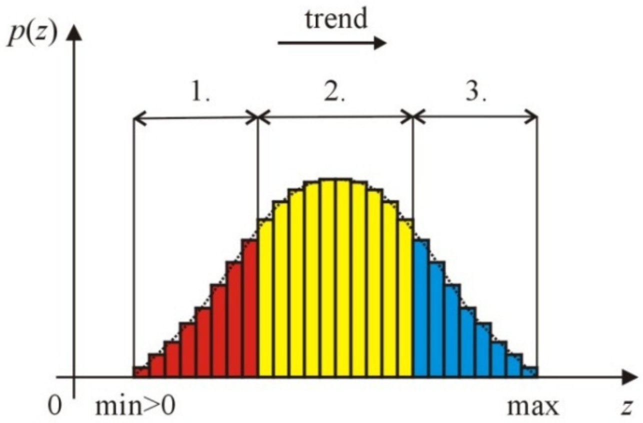
- zones in histograms are not changing only in one direction,
- histograms have two same zones at least.





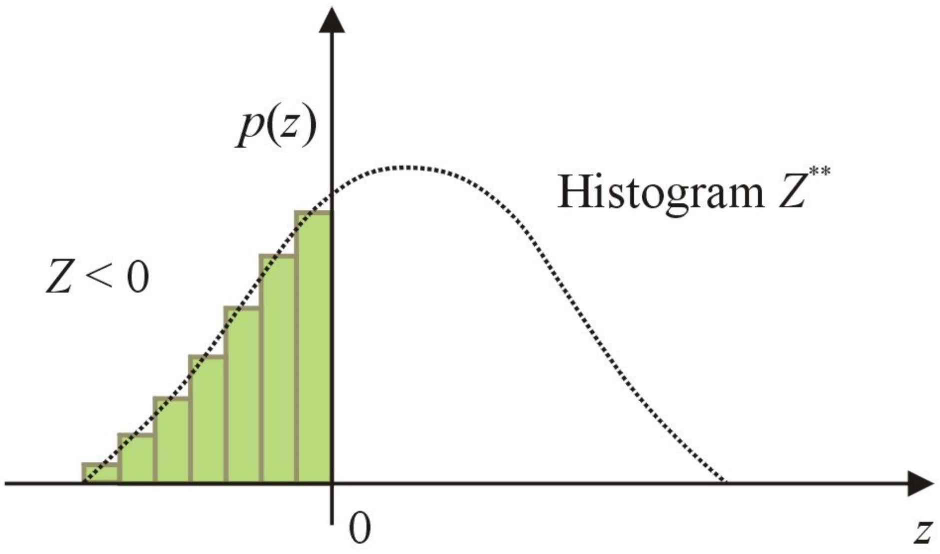
Monotonous histograms:

- zones in histograms are changing in one direction.



Trend Analysis and Optimizing

Resulting histogram of reliability function RF using DOProC method in action of **trend optimizing** – histogram Z^{**}



Calculation of failure probability P_f in case of several random variables and in application of **zonal** and **trend optimization** is numerical solution of integral:

$$P_f = \int_{D_f} f(X_1, X_2, \ldots, X_n)\, \mathrm{d}X_1, \mathrm{d}X_2, \ldots, \mathrm{d}X_n$$

Grouping of Partial Calculations Results

Is analogy of input variables grouping.

If e.g. :

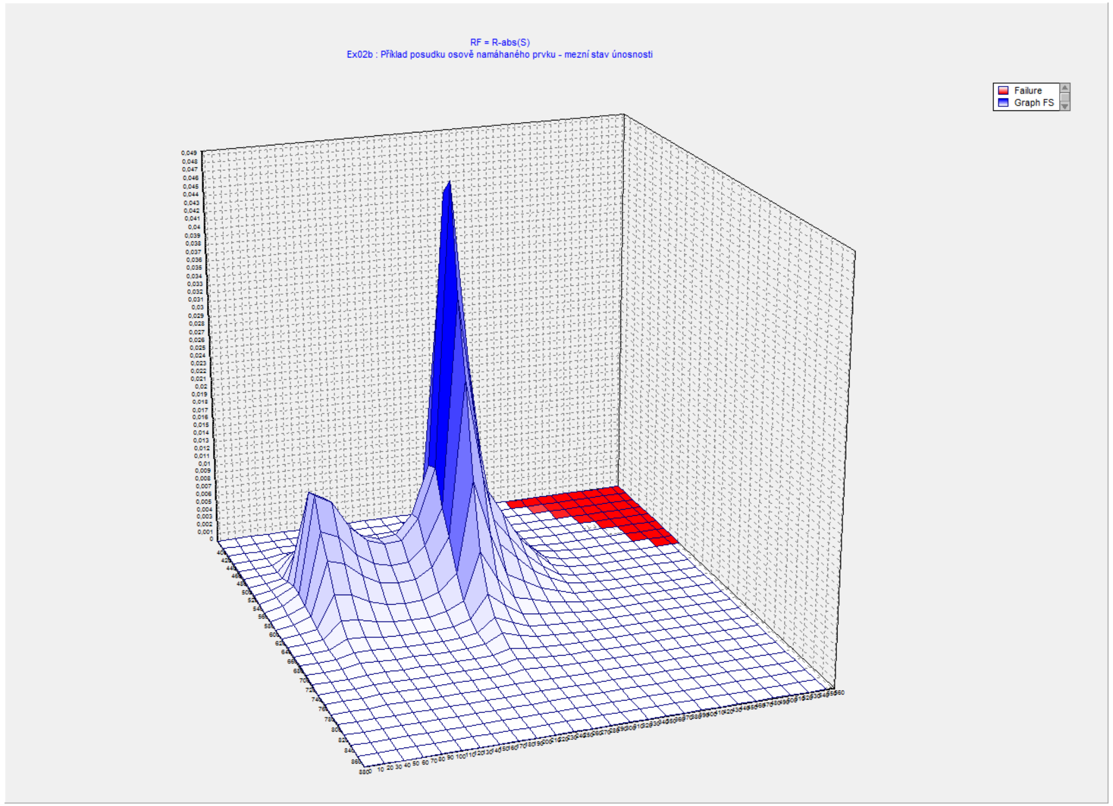
$$RF = R - f(A_1, A_2, A_3, \cdots, A_N)$$

then is often useful proceed independently calculation

$$E = f(A_1, A_2, A_3, \cdots, A_N)$$

and following

$$RF = R - E$$



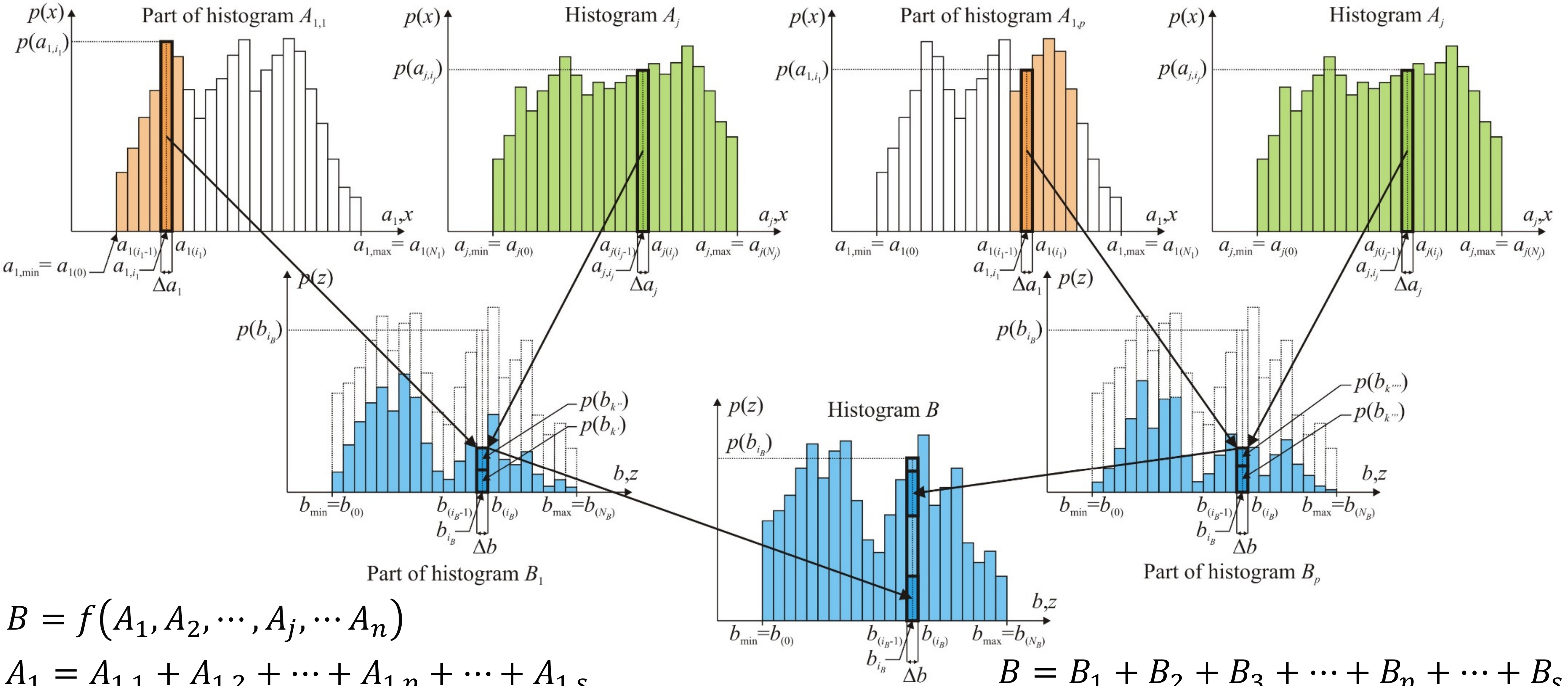
Parallelization, Combination of the Optimizing Techniques

DOProC method is able to:

- **combine** the mentioned optimizing techniques,
- **parallelize** the calculation (still tested on supercomputers).

The National Supercomputing Center
IT4 Innovations, Ostrava

Basic Computational Algorithm for Parallelization



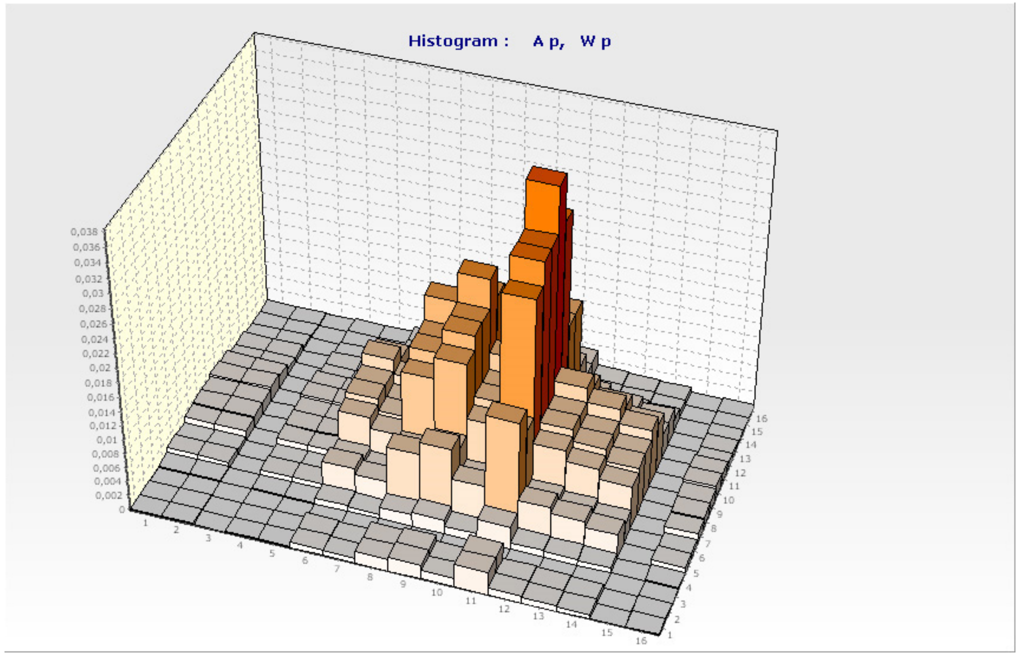
$$B = f\left(A_1, A_2, \cdots, A_j, \cdots A_n\right)$$

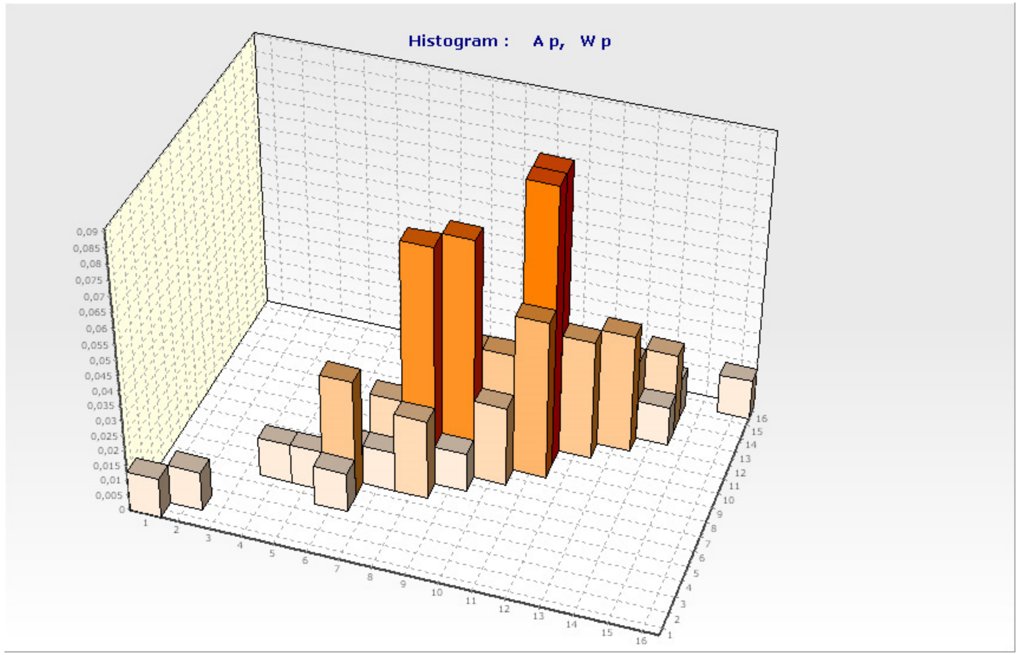
$$A_1 = A_{1,1} + A_{1,2} + \cdots + A_{1,p} + \cdots + A_{1,s}$$

$$B = B_1 + B_2 + B_3 + \cdots + B_p + \cdots + B_s$$

Statistically Dependent Input Variables

Statistically independent random variables are entered into probabilistic calculation using **double** or **triple histograms**.

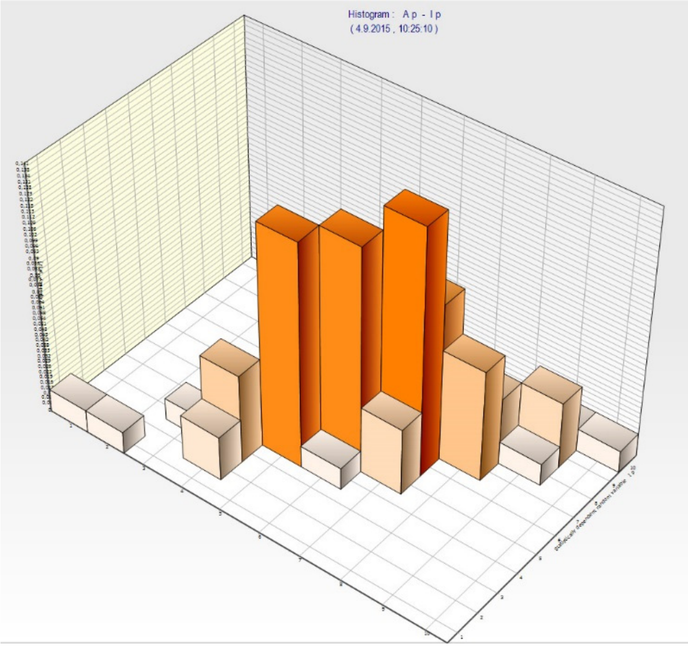




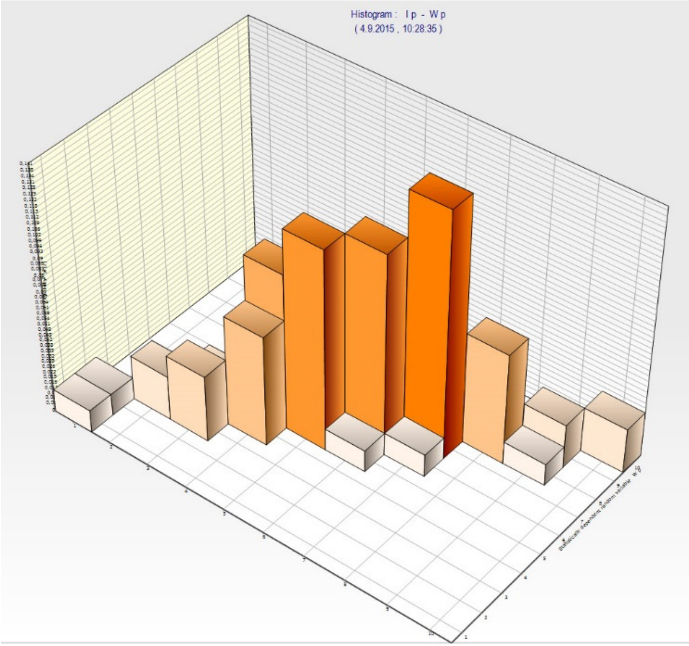
Desktop of HistAn2D: double histogram of statistically independent (left) and dependent (right) random variable

Statistically Dependent Input Variables

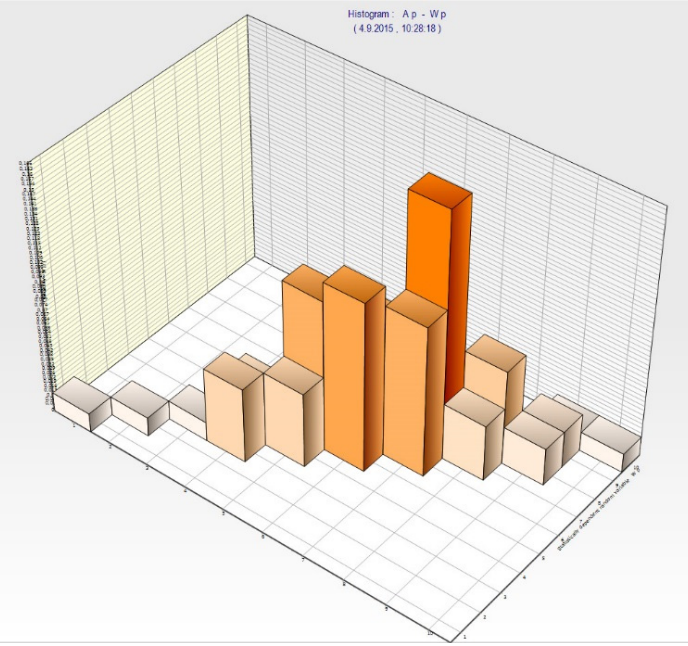
Used double histograms for statistically dependent random **cross-section properties of HE300B** profile.



$A_{var}, I_{y,var}$



$I_{y,var}, W_{y,var}$



$A_{var}, W_{y,var}$

Statistically Dependent Input Variables

Theoretical Background: In each standard histogram A, one axis includes the a_j class which is limited by $a_{\min}$ and $a_{\max}$, while the other axis shows typically the probability, p_{a_j}, of occurrence of that class, a_j.

The sum of probabilities for each class a_j in the histogram is $\sum p_{a_j} = 1$.

In the double histogram of two random variables, Z_1 and Z_2, the quantity z_1 is limited again by $z_{1,\min}$ and $z_{1,\max}$, while z_2 is limited by $z_{2,\min}$ and $z_{2,\max}$.

The values can be divided, using the step $\Delta z_{1,}$ into N_1 intervals for random quantities Z_1, or, using the step Δz_2, into N_2 intervals for the random quantities Z_2. The number of intervals is as follows:

$$N_1 = \frac{z_{1,\max} - z_{1,\min}}{\Delta z_1} \quad \text{and} \quad N_2 = \frac{z_{2,\max} - z_{2,\min}}{\Delta z_2}.$$

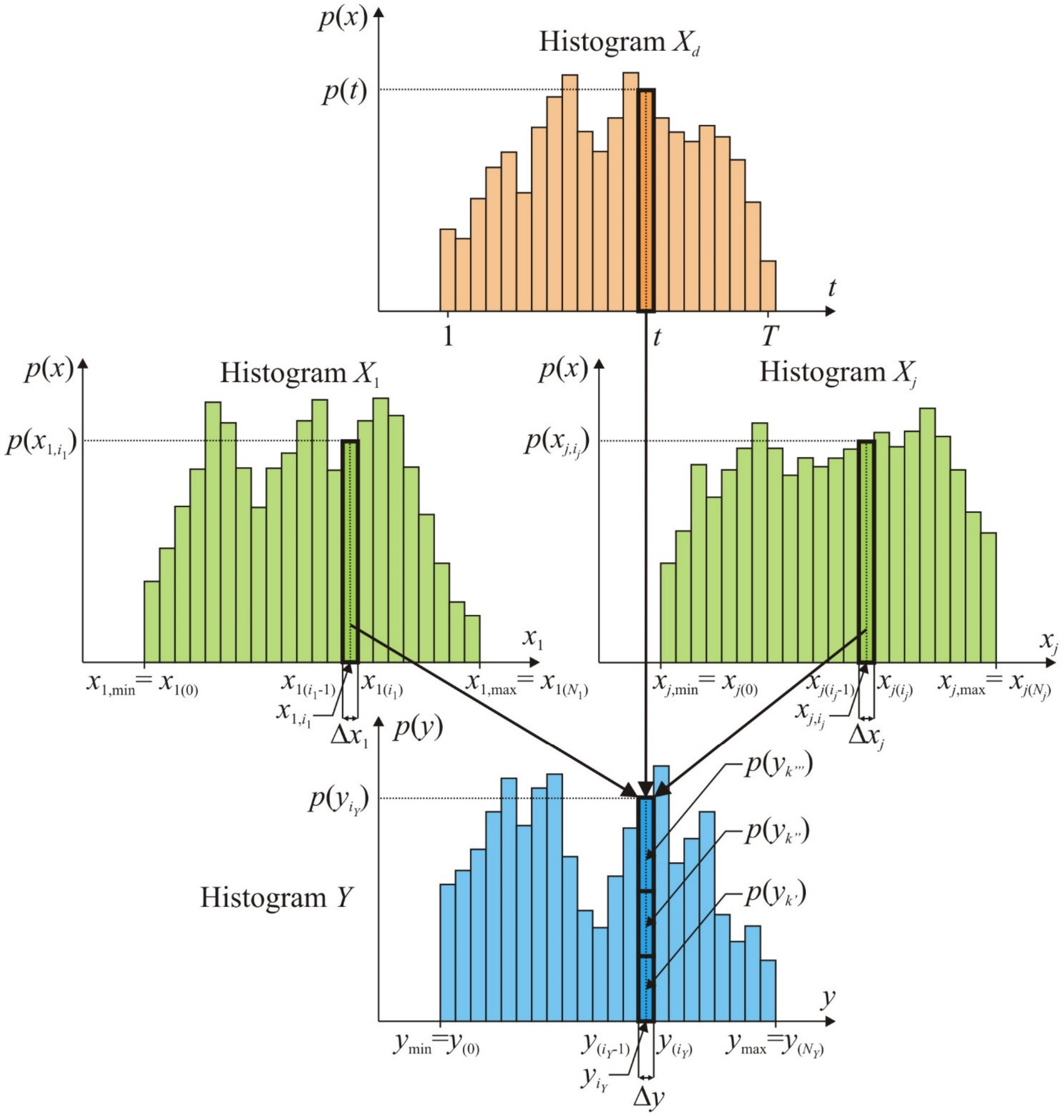
Statistically Dependent Input Variables

Theoretical Background: If the input variable z_1 is in the j^{th} class of $z_{1,j}$ in theory, z_2 could acquire following values: $z_{2,1}, z_{2,2}, \cdots, z_{2,j}, \cdots, z_{2,N_2}$. This means, it can acquire N_2 values.

The double histogram of the random quantities z_1 and z_2 can contain $N_1 \cdot N_2$ classes. This means, each class is determined by two values, $z_{1,j}$ and $z_{2,j}$, and by the probability of occurrence of that class, $p_{z_{1,j},z_{2,j}}$. Again: $\sum p_{z_{1,j},z_{2,j}} = 1$.

The number of classes with the non-zero probability can reach the product of $N_1 \cdot N_2$. If the random quantities are dependent, the number of classes in the histogram with the non-zero probability can be considerably lower than the product $N_1 \cdot N_2$.

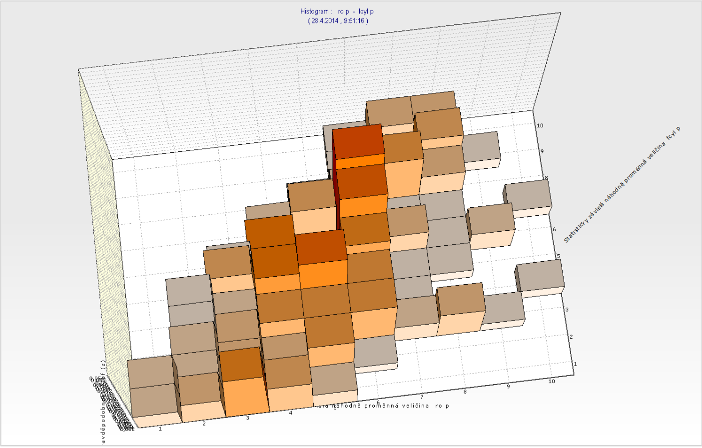
Basic computational algorithm with statistically dependent input variables



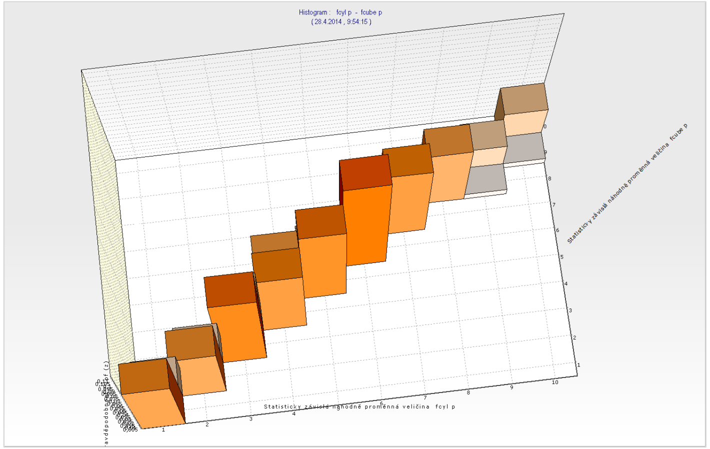
$$Y = f\left(X_1, \cdots X_i, \cdots X_n, X_{1_d}, \cdots X_{j_d}, \cdots X_{m_d}\right)$$

Software: HistAn2D and HistAn3D

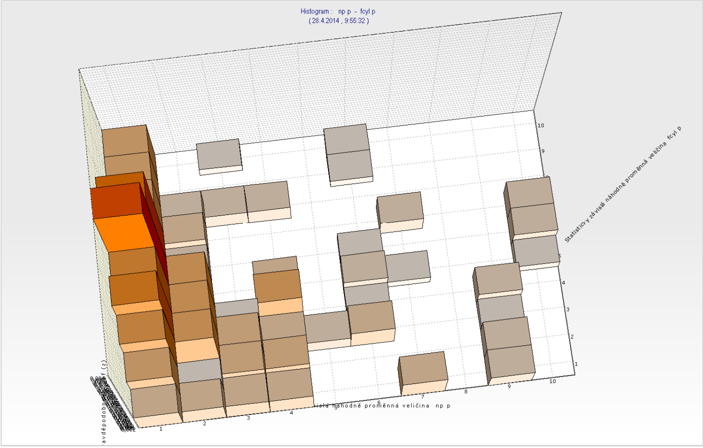
Statistically independent random variables are entered into probabilistic calculation using ProbCalc software



bulk density vs. compressive strength
the correlation 60.8% to 62.2%



cube vs. cylinder compressive strength
the correlation 99.8% to 100.0%

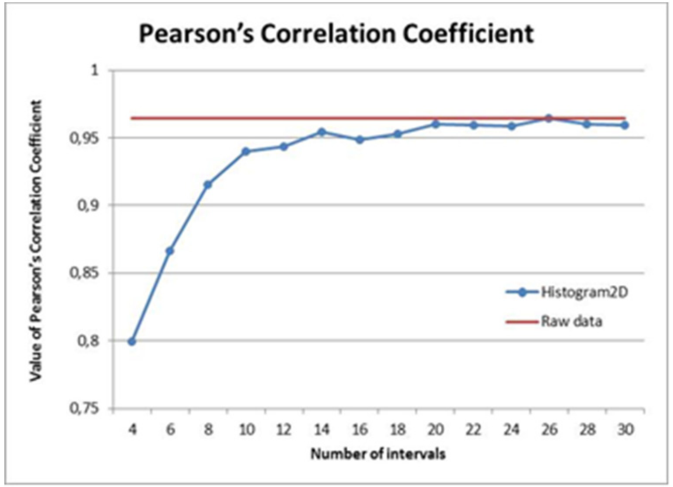


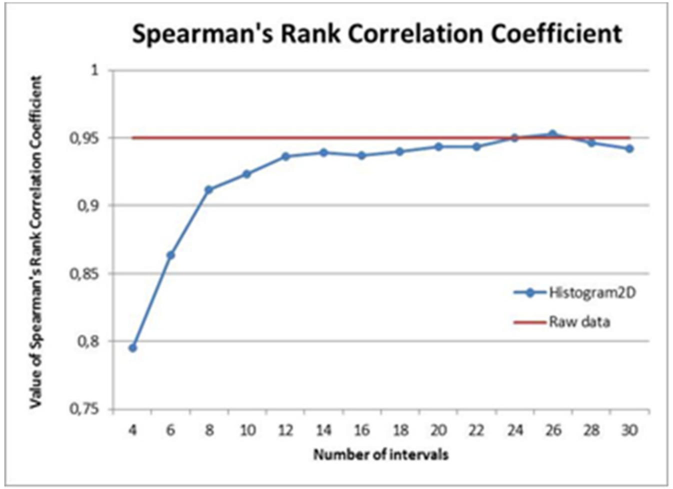
compressive strength of concrete vs. floor in the building
the correlation -21.1% to -25.8%

Software: HistAn2D and HistAn3D

Correlation coefficients of a **double histogram of the statistically dependent quantities** with different numbers of intervals (Pearson's correlation coefficient for raw data is 0.9645; Spearman correlation coefficient for raw data is 0.9499)

Number of intervals in a double histogram	Pearson's correlation coefficient	Spearman's rank correlation coefficient	Number of intervals in a double histogram	Pearson's correlation coefficient	Spearman's rank correlation coefficient
$4^2 = 16$	0.79985097	0.79507798	$18^2 = 324$	0.95267109	0.94023800
$6^2 = 36$	0.86661900	0.86360377	$20^2 = 400$	0.96046634	0.94378886
$8^2 = 64$	0.91530000	0.91194405	$22^2 = 484$	0.95940904	0.94355084
$10^2 = 100$	0.93984931	0.92352904	$24^2 = 576$	0.95903334	0.94989866
$12^2 = 144$	0.94381175	0.93613068	$26^2 = 676$	0.96464064	0.95260826
$14^2 = 196$	0.95443331	0.93939308	$28^2 = 784$	0.96017017	0.94660574
$16^2 = 256$	0.94876401	0.93694950	$30^2 = 900$	0.95938019	0.94245225



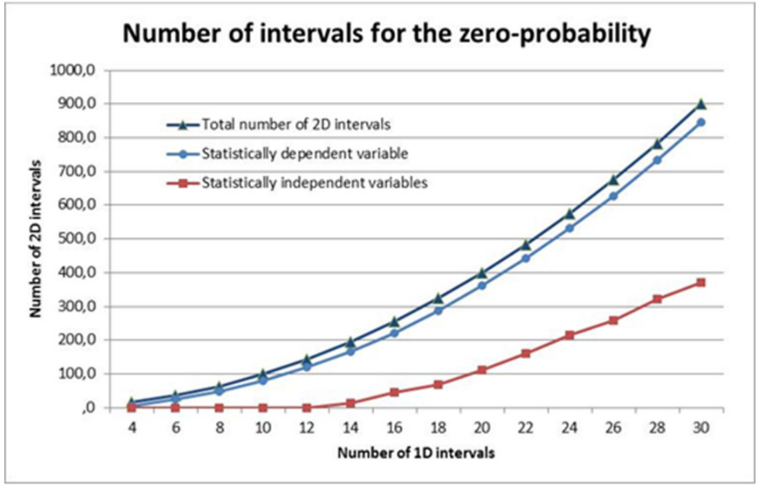


Pearson's correlation coefficient (up) and **Spearman's rank correlation coefficient** (bottom) of a double histogram vs. number of intervals

Software: HistAn2D and HistAn3D

Number of intervals in a double histogram	Number of zero-probability intervals		Number of intervals in a double histogram	Number of zero-probability intervals	
	Statistically dependent quantities	Statistically independent quantities		Statistically dependent quantities	Statistically independent quantities
$4^2 = 16$	6	0	$18^2 = 324$	288	69
$6^2 = 36$	24	0	$20^2 = 400$	361	112
$8^2 = 64$	48	0	$22^2 = 484$	443	160
$10^2 = 100$	80	0	$24^2 = 576$	531	216
$12^2 = 144$	119	0	$26^2 = 676$	627	258
$14^2 = 196$	166	14	$28^2 = 784$	735	322
$16^2 = 256$	222	46	$30^2 = 900$	847	372

The **number of classes** for double histograms **with zero probability** vs. the number of intervals chosen during creation of the histograms from the primary data



Number of intervals for the zero-probability in double histogram

Software: HistAn2D and HistAn3D

Numerical correlation index – can characterize the dependence between random variables not only for the linear relationship between two variables, but also for nonlinear dependence, or even for more than two random variables:

$$I_k = \frac{T_M - T_C}{T_M}$$

where T_M is the number of all classes in double or triple histogram (for optimal number of intevals and raw data), T_c is the number of non-zero probability classes in double or triple histogram.

For **statistically dependent variables**:

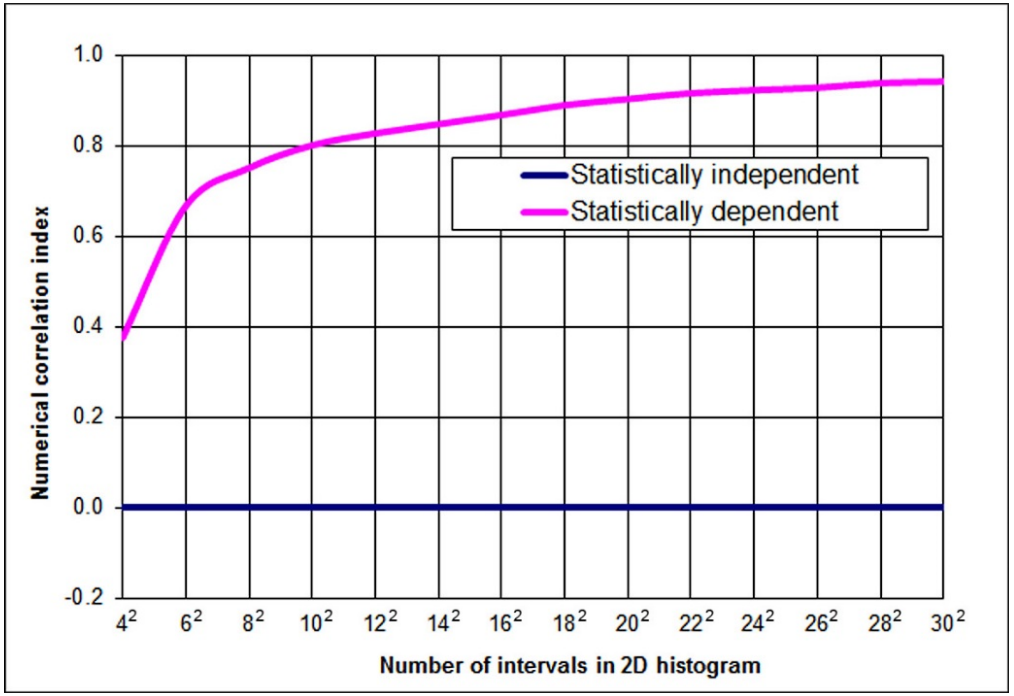
Correction for insufficient number of data:

where $n_1, n_2, n_3, \cdots, n_t$ are the numbers of intervals in histograms, $p_1, p_2, p_3, \cdots, p_t$ are the numbers of intervals without raw data.

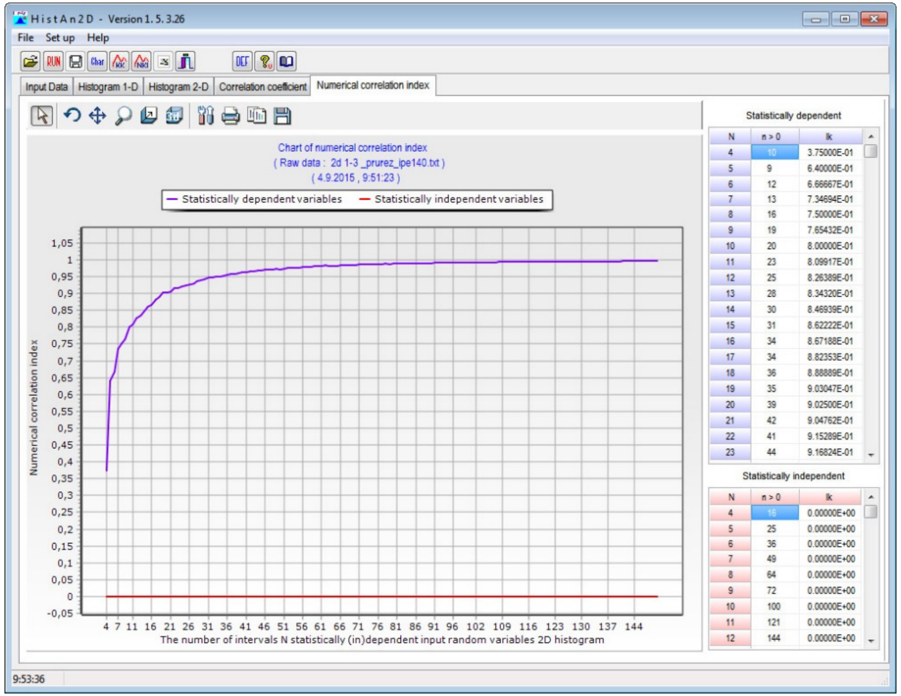
2 dependent variables: $T_M = (n_1 - p_1) \cdot (n_2 - p_2)$

t dependent variables: $T_M = (n_1 - p_1) \cdot (n_2 - p_2) \cdot (n_3 - p_3) \cdot \cdots \cdot (n_t - p_t)$

Software: HistAn2D and HistAn3D



The **numerical correlation index** for two random variables - cross-sectional area A and cross-section modulus W_y



The calculation of **numerical correlation index** in HistAn2D software for variable number of intervals in double histogram

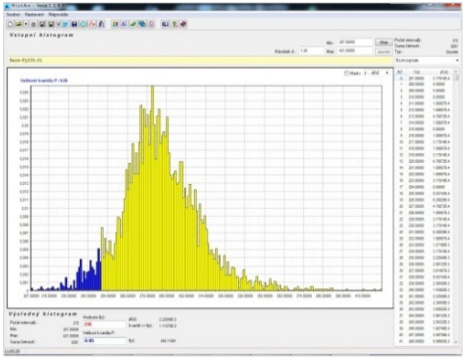
Program System ProbCalc

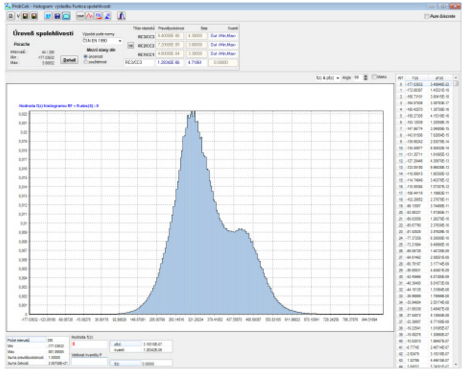
DOProC method was implemented in developed software utilities:

HistAn, **HistAn2D** and **HistAn3D** - utilities for analysis of bounded histograms,

HistOp - tool for basic arithmetic operations with two histograms,

ProbCalc - served for probabilistic structural reliability assessment and for the other probabilistic problems. Calculation model can be defined in text model or using DLL library. All of optimizing techniques were implemented.

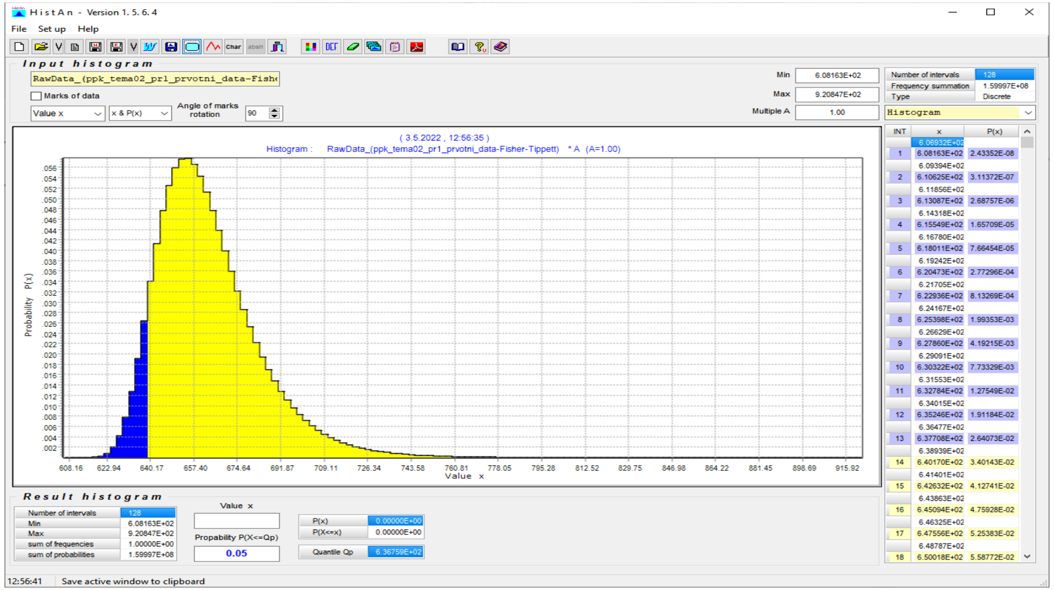




HistAn Software Tool

Program for more detailed **analysis of input histograms**:

- **Minimum** and **maximum values** of a random variable
- **Number of histogram classes** (intervals) and frequencies defined in them
- **Simple probabilistic calculations** with histograms (determination of p-quantile and probability of exceedingly the determined value of a random variable)
- Determining the **combination of several input histograms**
- Creation of **histograms with parametric distribution**
- Processing of **measured raw data**

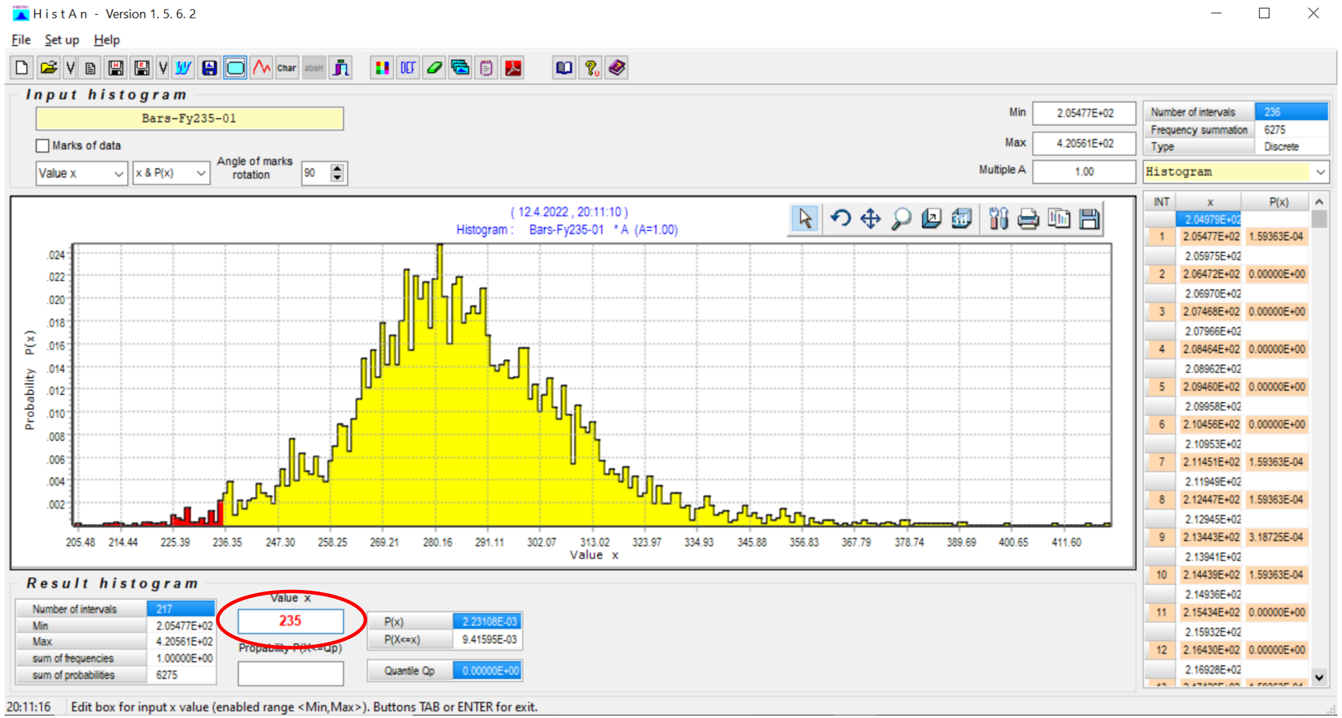


Desktop of the **HistAn software tool**:
Use of **Parametric distributions**

HistAn Software Tool

Detailed analysis of input histogram of the **yield stress of the steel S235**:

- Calculation of probability of exceedingly the determined value of the yield stress (value of random variable $x = 235\ \mathrm{MPa}$, resulting probability is $P(X \leq x) = 9.41595 \cdot 10^{-3}$)

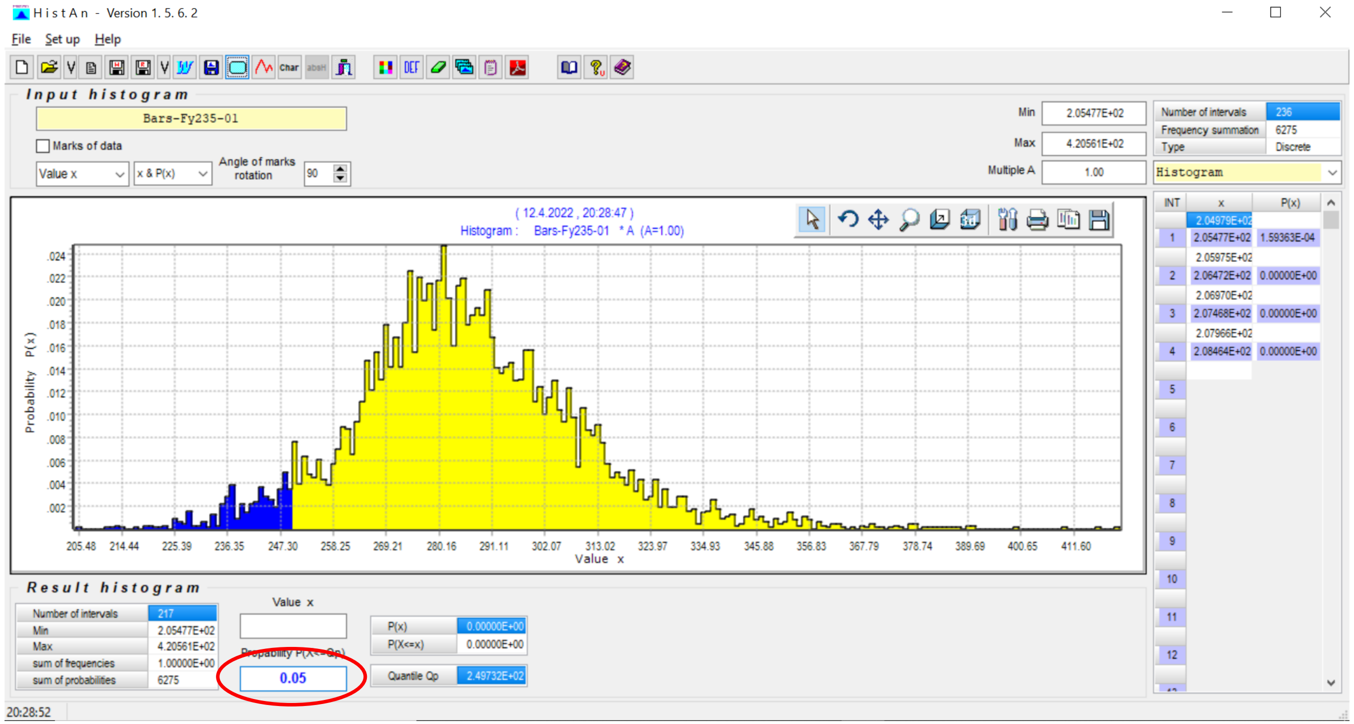


Desktop of the **HistAn software tool**

HistAn Software Tool

Detailed analysis of input histogram of the **yield stress of the steel S235**:

- Calculation of **five percent quantile $x_{0.05}$** of the yield stress (value of the specified probability $P(X \leq x_{0.05}) = 0.05$, resulting quantile $\mathbf{x_{0.05} = 249.732\ MPa}$)

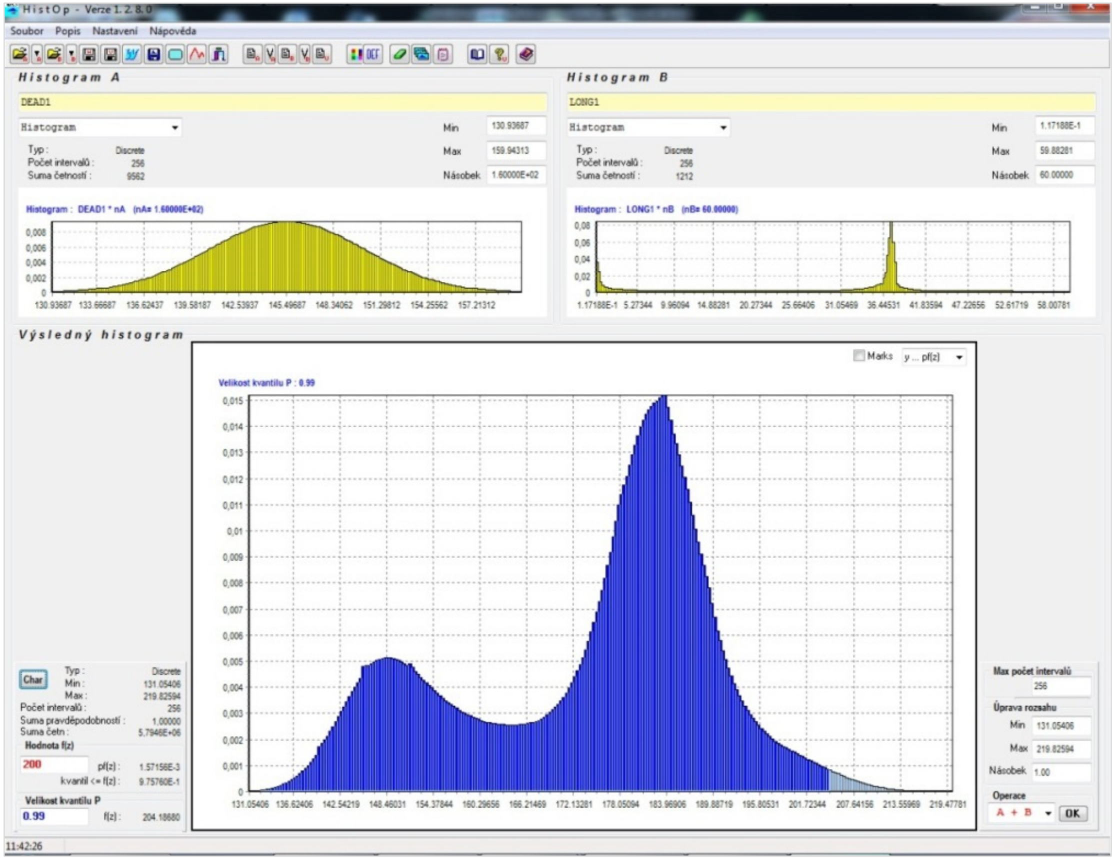


Desktop of the **HistAn software tool**

Software: HistOp

With its utilization can perform **basic arithmetic operations** with histograms A and B:

- **Total**,
- **Differential**,
- **Product**,
- **Ratio**,
- **Second power** of A histogram,
- **Absolute value** of A histogram.



Software: ProbCalc

Grouping of input random variables

Reliability function

Calculator

Command line

Analytical definition of calculation model

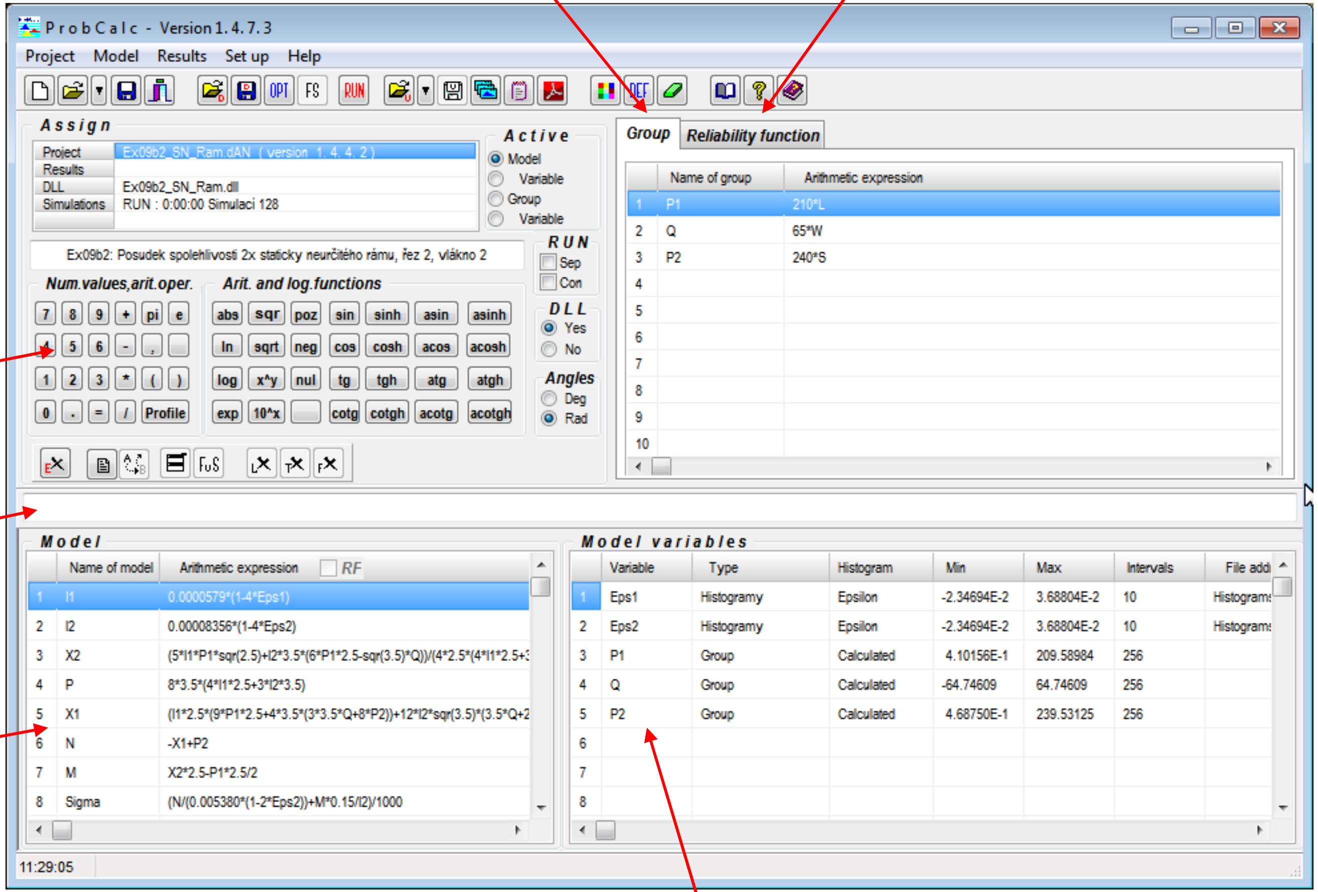
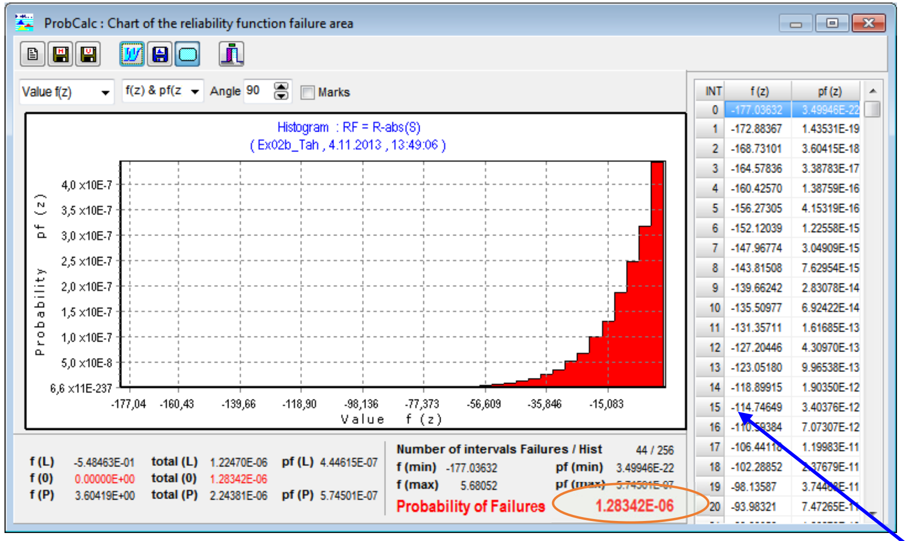
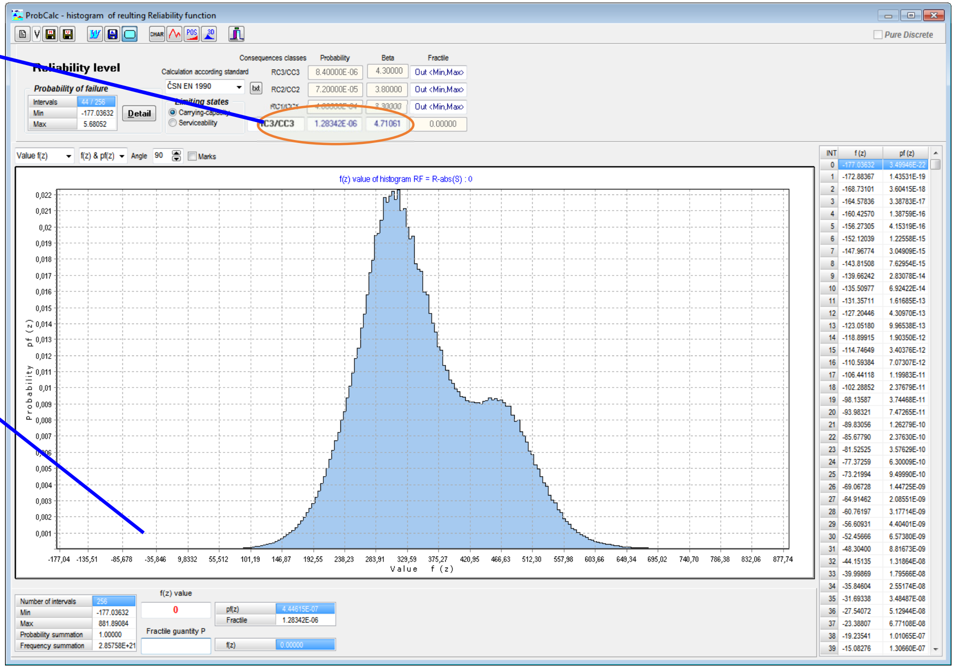


Table of input random variables

Software: ProbCalc



Desktop of **ProbCalc** under probabilistic reliability assessment: histogram of **reliability function** RF and resulting **probability of failure** P_f



Probability of failure

$$P_f = 1.28 \cdot 10^{-6}$$

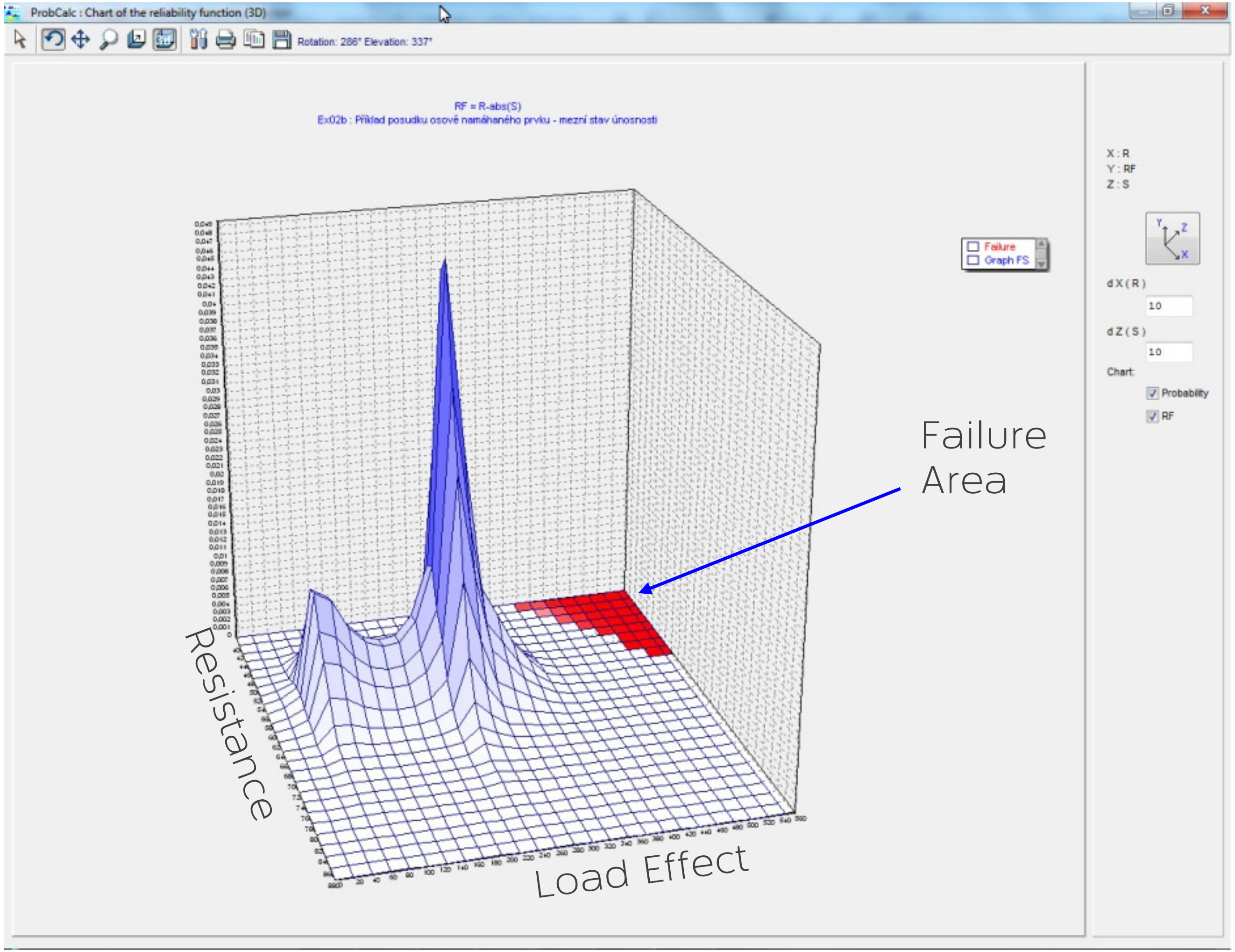
meets requirements of EN 1990

for **consequences class RC3/CC3**

with design probability

$$P_d = 8.4 \cdot 10^{-6}$$

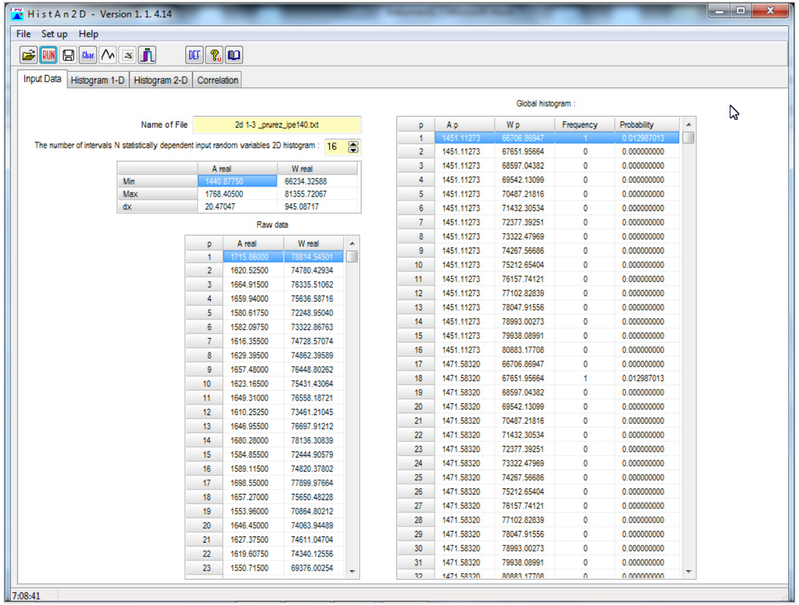
Software: ProbCalc

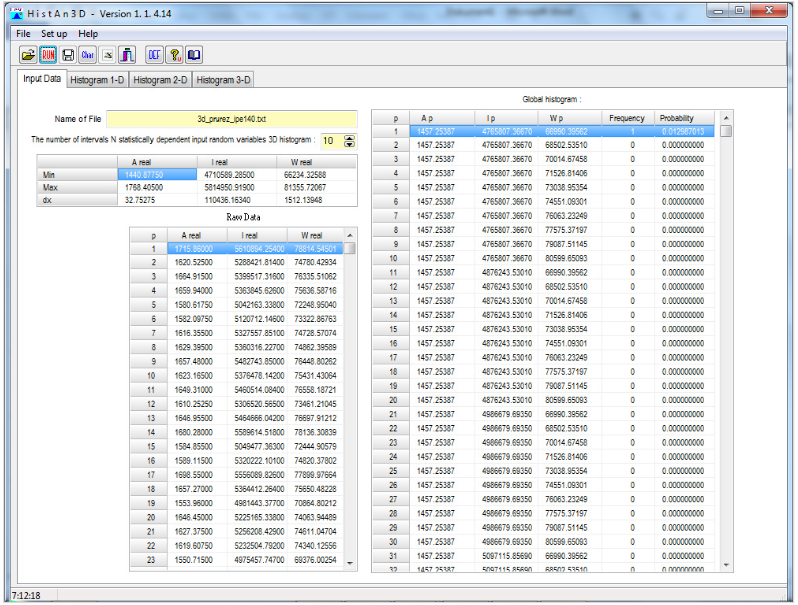


3D chart of **reliability function** RF

Software: HistAn2D and HistAn3D

Special software applications **HistAn2D** (left) and **HistAn3D** (right) were developed for creation of the **double** and **triple histograms** which describe the **statistical dependence** between two or three random variables.

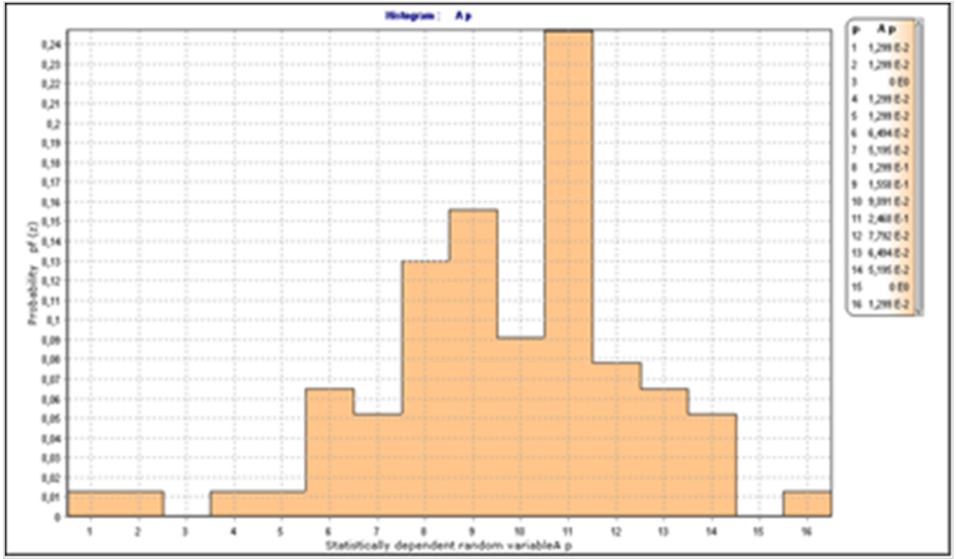


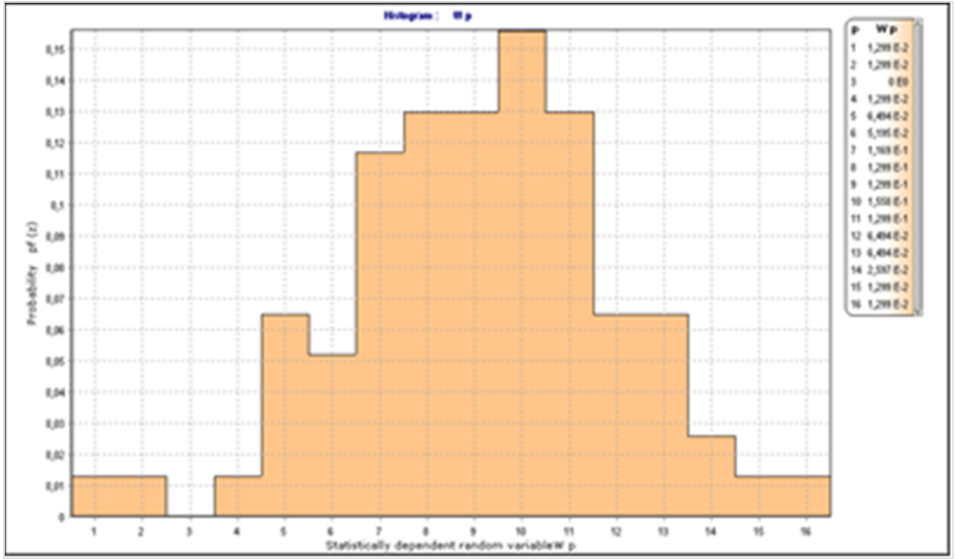


Desktop of **HistAn2D** (left) and **HistAn3D** (right):
raw data of rolled shape IPE 140 cross-section properties under analyses

Software: HistAn2D and HistAn3D

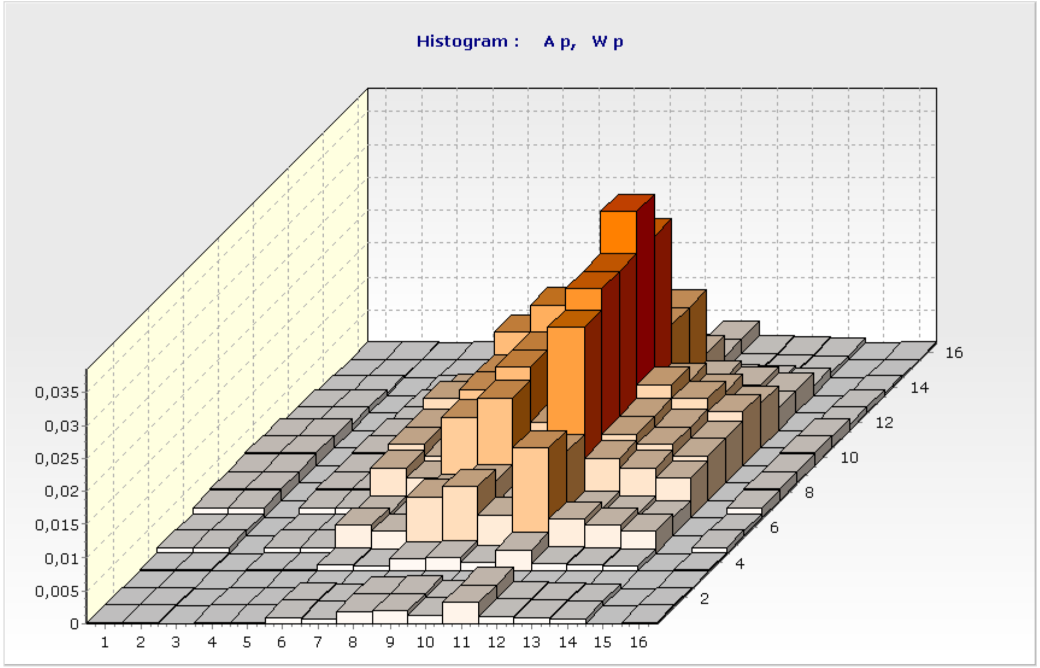
Using the software, it is possible to view for each random variable a simple histogram with non-parametric (empirical) distribution of probability as well as a multidimensional histogram which describes the statistical dependence between the quantities.



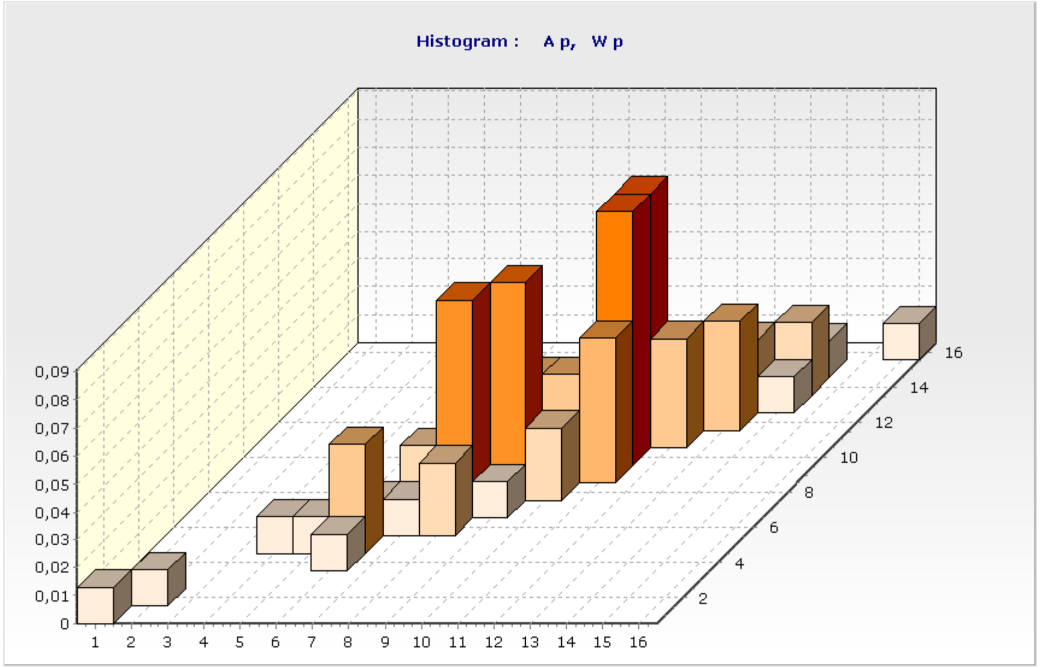


Histograms with **non-parametric** (empirical) **distribution of probability**:
Histogram of the IPE140 cross-section area A (left) and cross-section modulus W_y (right)

Software: HistAn2D and HistAn3D

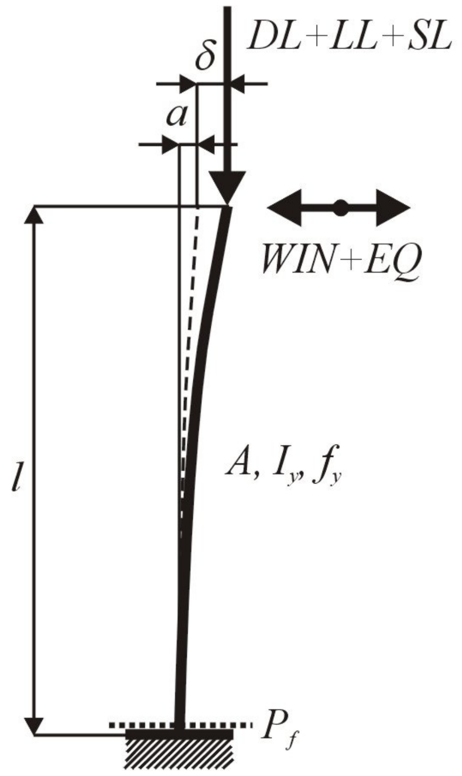


Double histogram for two **statistically independent** random quantities – cross-section area A and cross-section modulus W_y



Double histogram for two **statistically dependent** random quantities – cross-section area A and cross-section modulus W_y

Example 1, Reliability Assessment



Scheme of the structure under assessment

Reliability assessment of the column

l ... 6 m
profile HEB 300, **steel S235**, E ... $2.1 \cdot 10^{11}$ Pa
imperfections: a ... ± 30 mm

Load	Type	Extremal value [kN]
D	Dead	350
L	Long Lasting	75
S	Short Lasting	75
W	Wind	40
EQ	Earthquake	$\frac{1}{20} \cdot (D+L+S) = \frac{500}{20} = 25$

Example 1, Reliability Assessment

Ultimate limit state

$$RF = R - E$$

R … structural resistance – yield stress f_y
E … load effect – stress in outer fibres σ

Serviceability limit state

$$RF = \delta_{tol} - |\delta|$$

δ_{tol} … structural resistance – allowed deformation (35 mm)
δ … load effect – maximal horizontal deformations

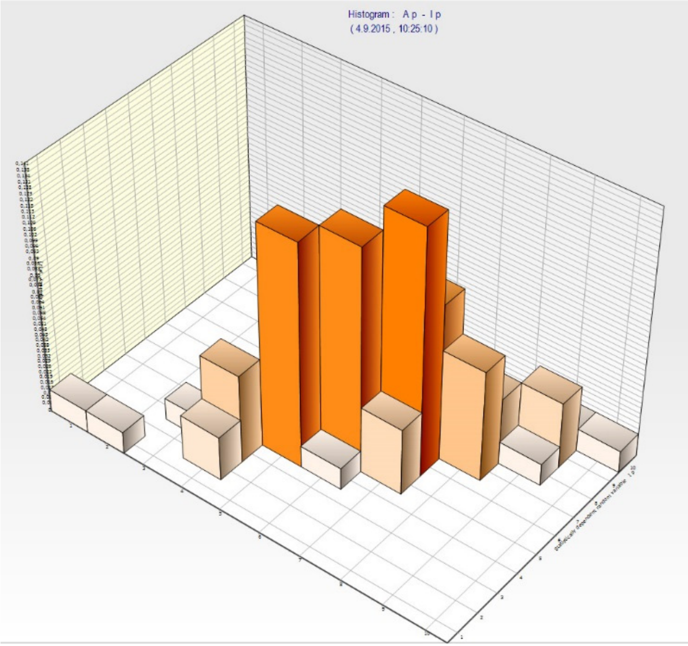
Random input variables:

- 5 load components,
- cross-section variability,
- initial imperfection in column,
- yield stress f_y.

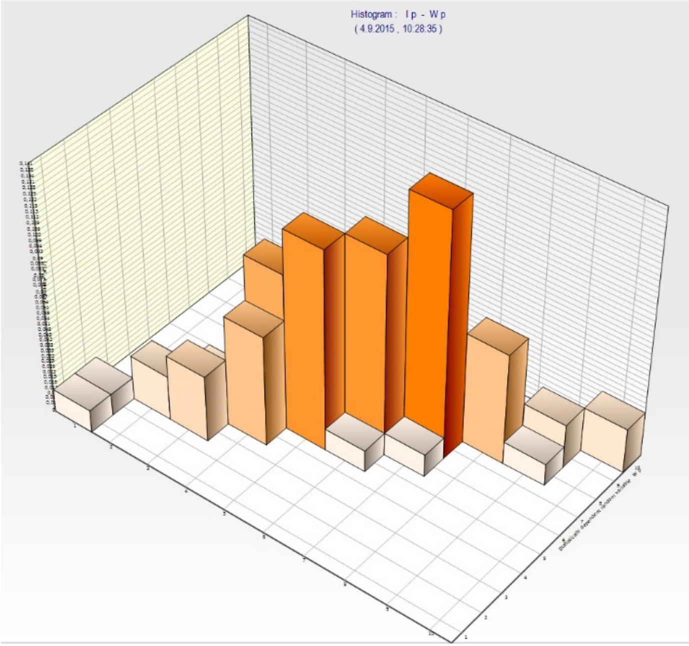
8 random input variables in total

Example 1, Statistically Dependent Input Variables

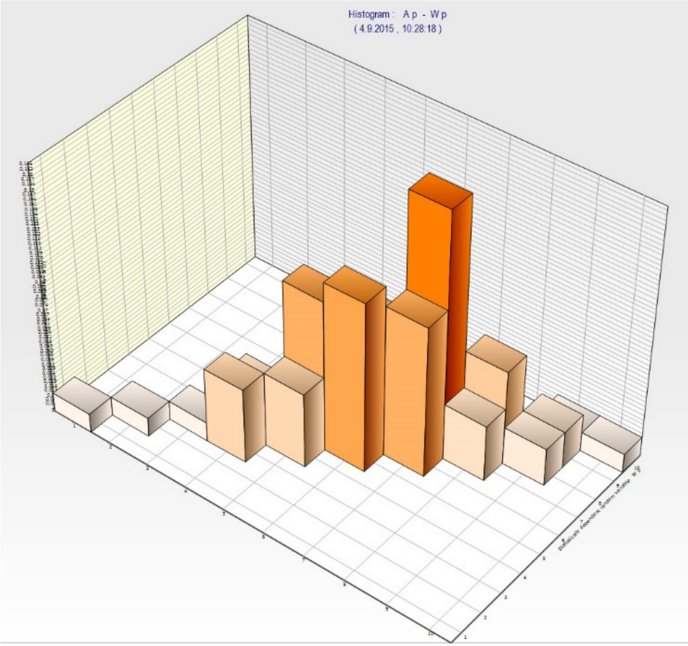
Used double histograms for statistically dependent random **cross-section properties of HE300B** profile.



$A_{var}, I_{y,var}$



$I_{y,var}, W_{y,var}$



$A_{var}, W_{y,var}$

Example 1, Description of Input Variables

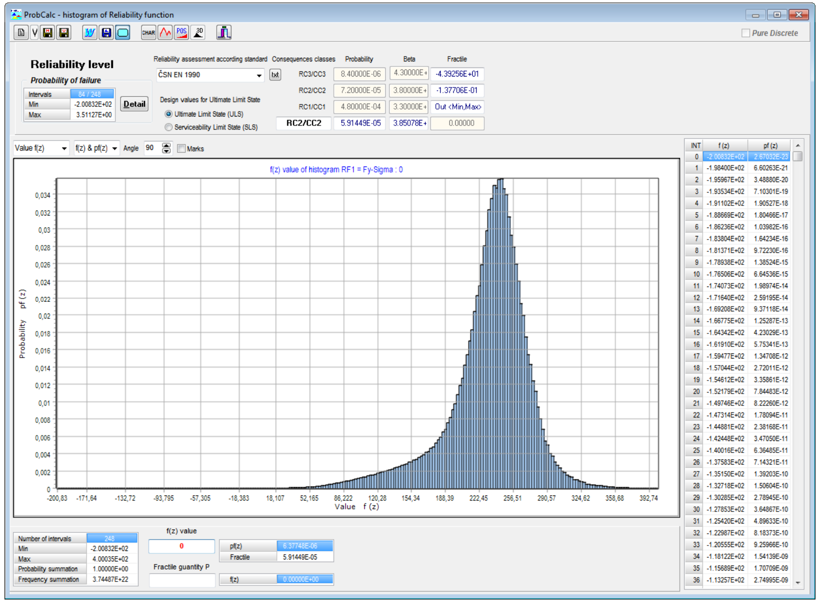
Input variable	Minimum	Maximum	N_j	Histogram
Column height l	6 m	-	-	-
Yield stress f_y	200 MPa	435 MPa	217	FY235-01
Dead load *DL*	260 kN	320 kN	256	DEAD1*
Long-lasting load *LL*	0 kN	120 kN	256	LONG1*
Short-lasting load *SL*	0 kN	75 kN	256	SHORT1*
Wind load *WIN*	-45 kN	45 kN	256	WIND1*
Earthquake *EQ*	-30 kN	30 kN	256	EARTH*
Geometric imperfections *Imp*	-30 mm	30 mm	16	IMP016
Variability of cross section properties A, W_y and I_y	-	-	10^3	3DHE300B**
Cross-sectional area A	13076 mm^2	16048 mm^2	10	1DHE300BA
Cross section modulus W_y	$1.44 \cdot 10^6$ mm^3	$1.77 \cdot 10^6$ mm^3	10	1DHE300BW
Moment of inertia I_y	$2.19 \cdot 10^8$ mm^4	$2.70 \cdot 10^8$ mm^4	10	1DHE300BI

* Histograms are taken from (Marek et al. 1995).

** 3D histogram was used for calculation considering the statistical dependence of cross section properties A, W_y, and I_y. Histograms 1DHE300BA, 1DHE300BW and 1DHE300BI are based on this, as well.

Example 1, Results

Histograms of reliability function RF, ultimate limit state



Statistically independent cross-section parameters

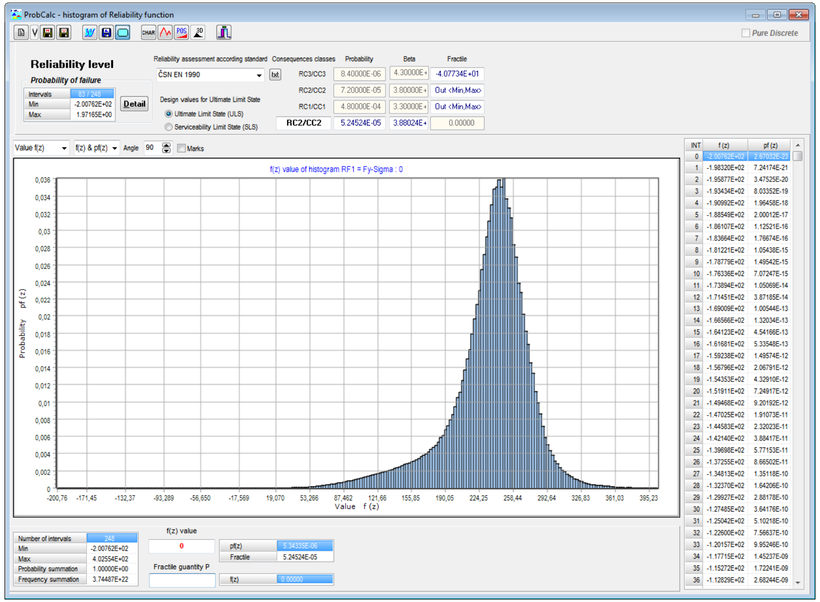
Failure probability $P_f = 5.133 \cdot 10^{-5}$ (RC2/CC2)

Time of calculation 3:20 min.

Statistically dependent cross-section parameters

Failure probability $P_f = 5.247 \cdot 10^{-5}$ (RC2/CC2)

Time of calculation 9 sec.



Example 1, Analysis of the results

Optimization used	Calculation time	p_f	RC/CC	Calculation steps
Without optimization	>>24 hours		not performed	$4.13554 \cdot 10^{18}$
Grouping of output quantities	>>24 hours		not performed	$1.75235 \cdot 10^{16}$
Grouping of input quantities	>>24 hours		not performed	$2.27541 \cdot 10^{11}$
Grouping of input variables, zone optimization	>>24 hours		not performed	$1.83501 \cdot 10^{11}$
Grouping of input variables, interval optimization	2:33:22 hours	$5.6736 \cdot 10^{-5}$	RC2/CC2	$4.59571 \cdot 10^{9}$
Grouping of input variables, interval and zone optimization	2:17:29 hours	$5.5559 \cdot 10^{-5}$	RC2/CC2	$3.38479 \cdot 10^{9}$
Grouping of input variables, interval, zone and the trend optimization	1:20:43 hours	$5.5559 \cdot 10^{-5}$	RC2/CC2	$2.04303 \cdot 10^{9}$
Grouping of input and output variables	37:05 min.	$5.1330 \cdot 10^{-5}$	RC2/CC2	$1.04858 \cdot 10^{9}$
Grouping of input and output variables, zone optimization	28:29 min.	$5.2469 \cdot 10^{-5}$	RC2/CC2	$8.22473 \cdot 10^{8}$
Grouping of input and output variables, parallelization (2 cores)	9:06 min.	$5.1330 \cdot 10^{-5}$	RC2/CC2	$1.04858 \cdot 10^{9}/2$
Grouping of input and output variables, interval optimization	4:30 min.	$5.0480 \cdot 10^{-5}$	RC2/CC2	$1.35032 \cdot 10^{8}$
Grouping of input and output variables, zone and interval optimization	3:35 min.	$4.8711 \cdot 10^{-5}$	RC2/CC2	$1.06021 \cdot 10^{8}$
Grouping of input and output variables, parallelization (8 cores)	3:20 min.	$5.1330 \cdot 10^{-5}$	RC2/CC2	$1.04858 \cdot 10^{9}/8$

Note: Calculations were performed using a DLL library on a PC with the following specifications: an Intel(R) Core(TM) i7-2600 CPU @ 3.40 GHz, MS Windows 7/64-bit/SP1; ProbCalc v.1.5.3.

Analysis of the results for probabilistic reliability assessment of individual types of **optimization steps** used considering **statistical independence** of input random variables

Example 1, Analysis of the Results

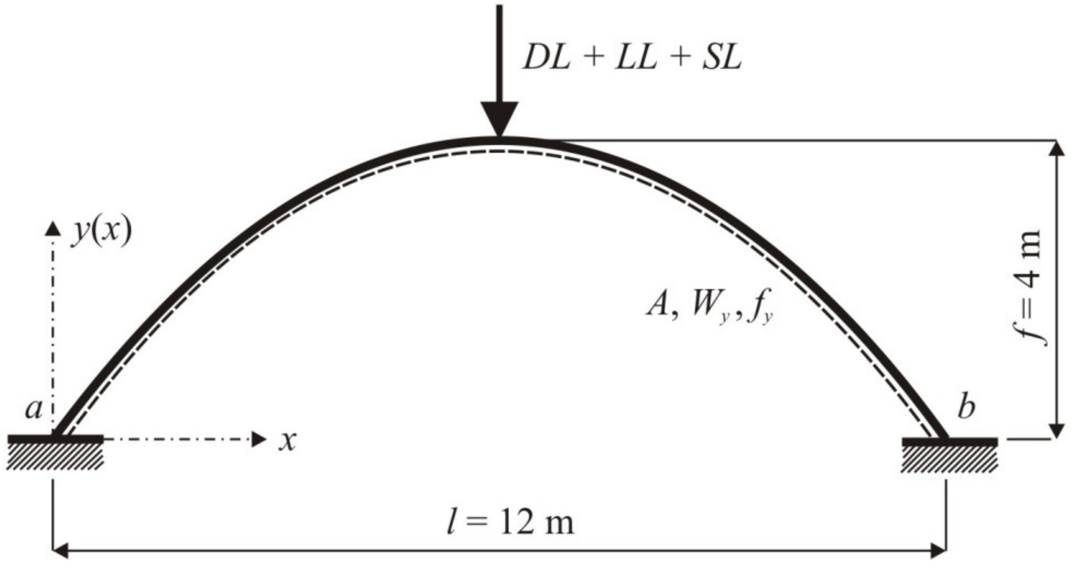
Optimization used	Calculation time	p_f	RC/CC	Calculation steps
Without optimization	>>24 hours		not performed	$9.50648 \cdot 10^{16}$
Grouping of output quantities	>>24 hours		not performed	$5.43227 \cdot 10^{14}$
Grouping of input quantities	3:52:03 hours	$5.2442 \cdot 10^{-5}$	RC2/CC2	$5.68852 \cdot 10^{9}$
Grouping of input and output variables	1:09 min.	$5.2467 \cdot 10^{-5}$	RC2/CC2	$3.25059 \cdot 10^{7}$
Grouping of input and output variables, parallelization (2 cores)	19 sec.	$5.2467 \cdot 10^{-5}$	RC2/CC2	$3.25059 \cdot 10^{7}/2$
Grouping of input and output variables, parallelization (8 cores)	9 sec.	$5.2467 \cdot 10^{-5}$	RC2/CC2	$3.25059 \cdot 10^{7}/8$

Note: Calculations were performed using a DLL library on a PC with the following specifications: an Intel(R) Core(TM) i7-2600 CPU @ 3.40 GHz, MS Windows 7/64-bit/SP1; ProbCalc v.1.5.3.

Analysis of the results for probabilistic reliability assessment of individual types of **optimization steps** used considering **statistical dependence** of input random variables

Example 2

Static scheme of the elemental structure of a **parabolic arch** fixed in both ends and loaded with combination of three single loads



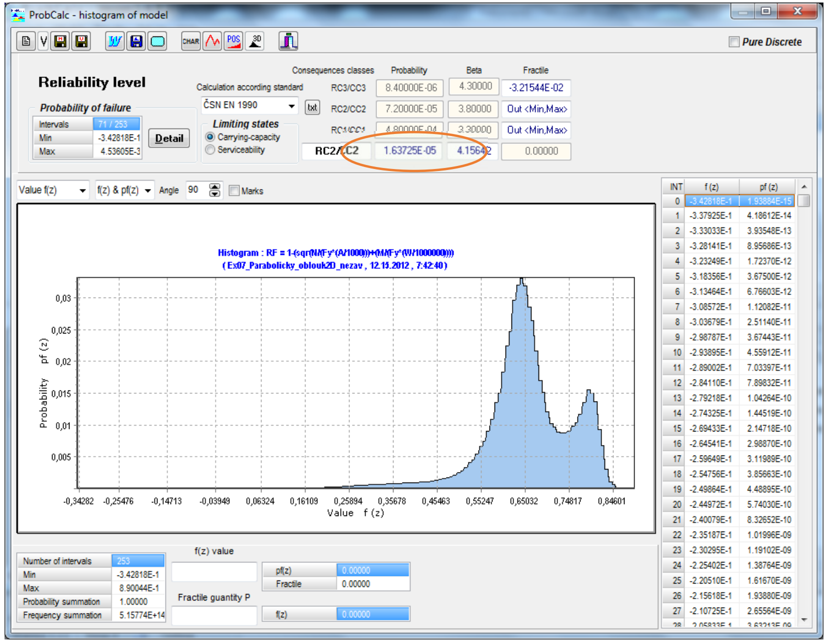
The reliability assessment has been made using the interaction formula:

$$\left(\frac{N_{Ed}}{N_{Rd}}\right)^2 + \frac{M_{Ed}}{M_{Rd}} \leq 1$$

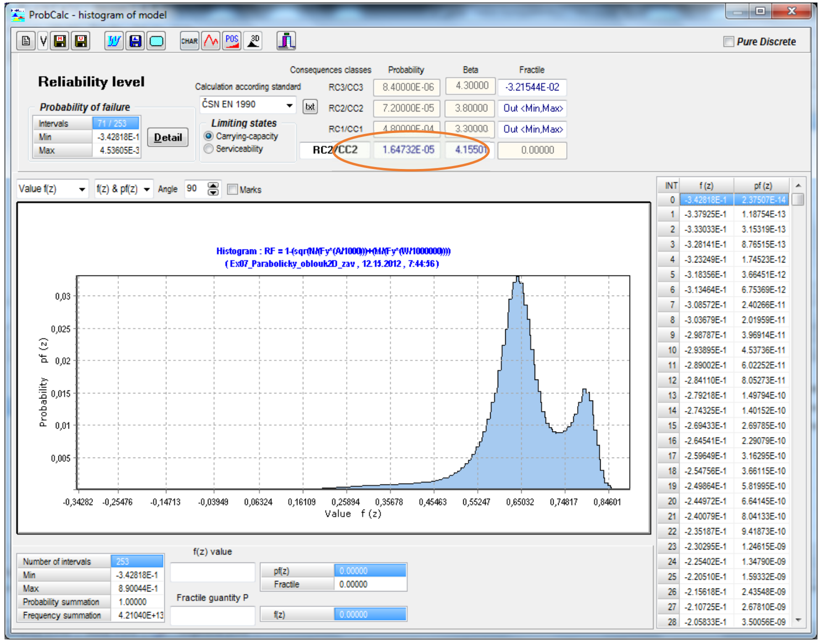
The failure probability P_f was determined using the reliability function RF:

$$P_f = P(RF < 0) = P\left(1 - \left[\left(\frac{N_{Ed}}{N_{Rd}}\right)^2 + \frac{M_{Ed}}{M_{Rd}}\right] < 0\right)$$

Example 2, Results



Histogram of reliability function RF, for the probabilistic calculation with **statistically independent cross-section parameters** of the cross-section area A and cross-section modulus W_y, failure probability $P_f = 1.637 \cdot 10^{-5}$.



Histogram of reliability function RF, for the probabilistic calculation with statistically dependent cross-section parameters of the cross-section area A and cross-section modulus W_y, failure probability $1.647 \cdot 10^{-5}$.

Usage of DOProC method

- Probabilistic assessment of load combinations,
- Probabilistic reliability assessment of cross-sections and systems of statically (in)determined load-bearing constructions,
- Probabilistic approach to assessment of mass concrete and fibrous concrete mixtures,
- **Reliability assessment of arch supports in underground and mining workings**,
- Reliability assessment of load-bearing constructions under impact loads,
- **Probabilistic calculation of fatigue damage prediction in cyclically loaded steel structures**.