Topic 7:

Latin Hypercube Sampling (LHS)

- Theoretical background
- Freet software
- Examples

Probabilistic Methods

Simulation methods

Simple simulation Monte Carlo,

Stratified simulation techniques:

Latin Hypercube Sampling – LHS,
Stratified Sampling – SC.

Advanced simulation methods:

Importance Sampling – IS,
Adaptive Sampling – AS,
Axis Orthogonal Importance Sampling,
Directional Sampling – DS,
Line Sampling – LS,
Design Point Sampling,
Subset Simulations,
Descriptive Sampling, Slice Sampling.

Approximation methods

- First (Second) Order Reliability Method - FORM (SORM),
- Response Surface Method – RSM,
- Perturbation techniques – e.g. Stochastic Finite Element Method (SFEM),
- Artificial Neural Network – ANN.

Pure numerical methods

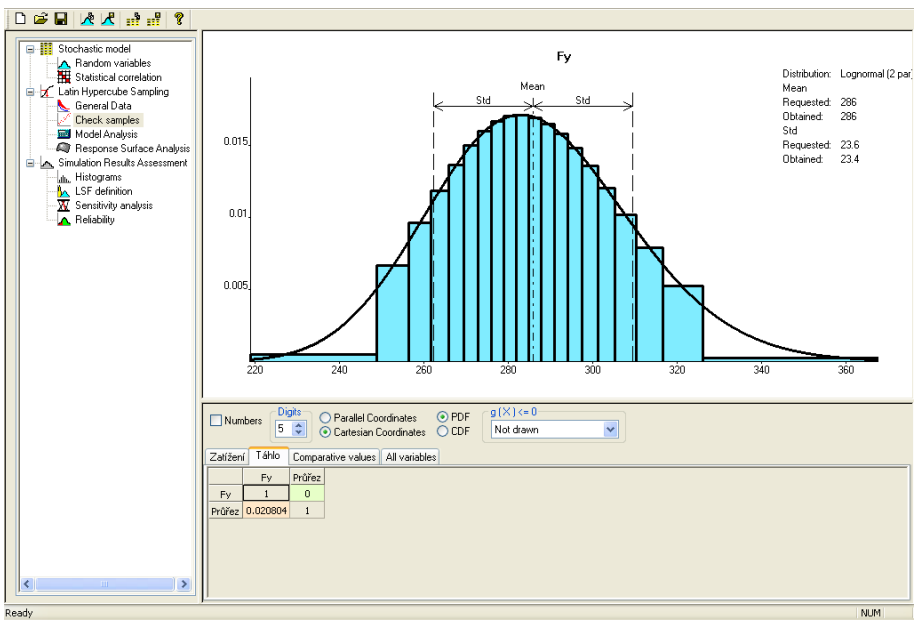
(without simulations and approximations)

- Point Estimate Method – PEM,
- Direct Optimized Probabilistic Calculation – DOProC .

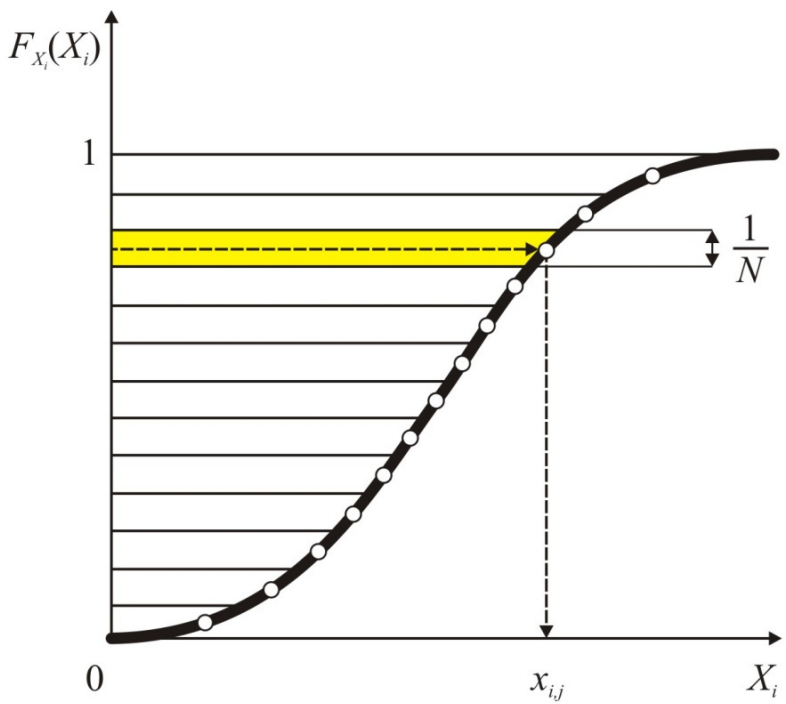
Overview e.g.:
Krejsa & Králik (2015)

Reliability Assessment Using LHS

Latin Hypercube Sampling (LHS): Applied e.g., in the **Freet** program



Freet desktop: panel of input variables entering

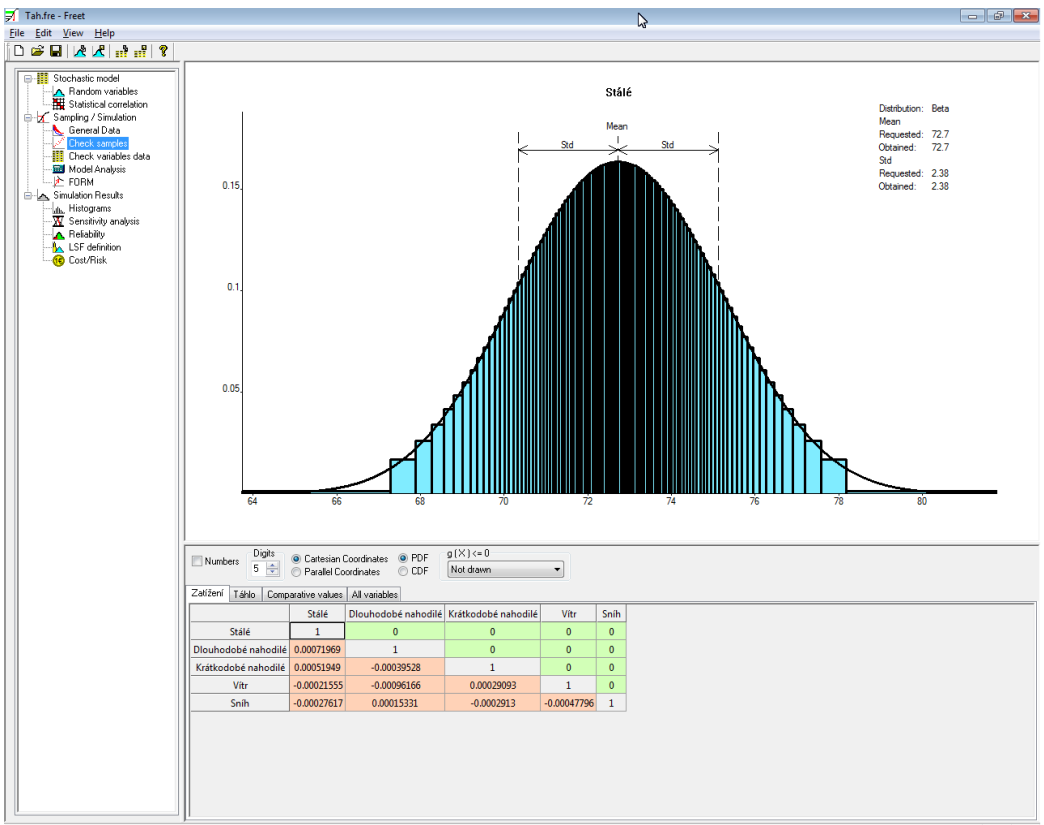


Principal of LHS: the division of the distribution function domain

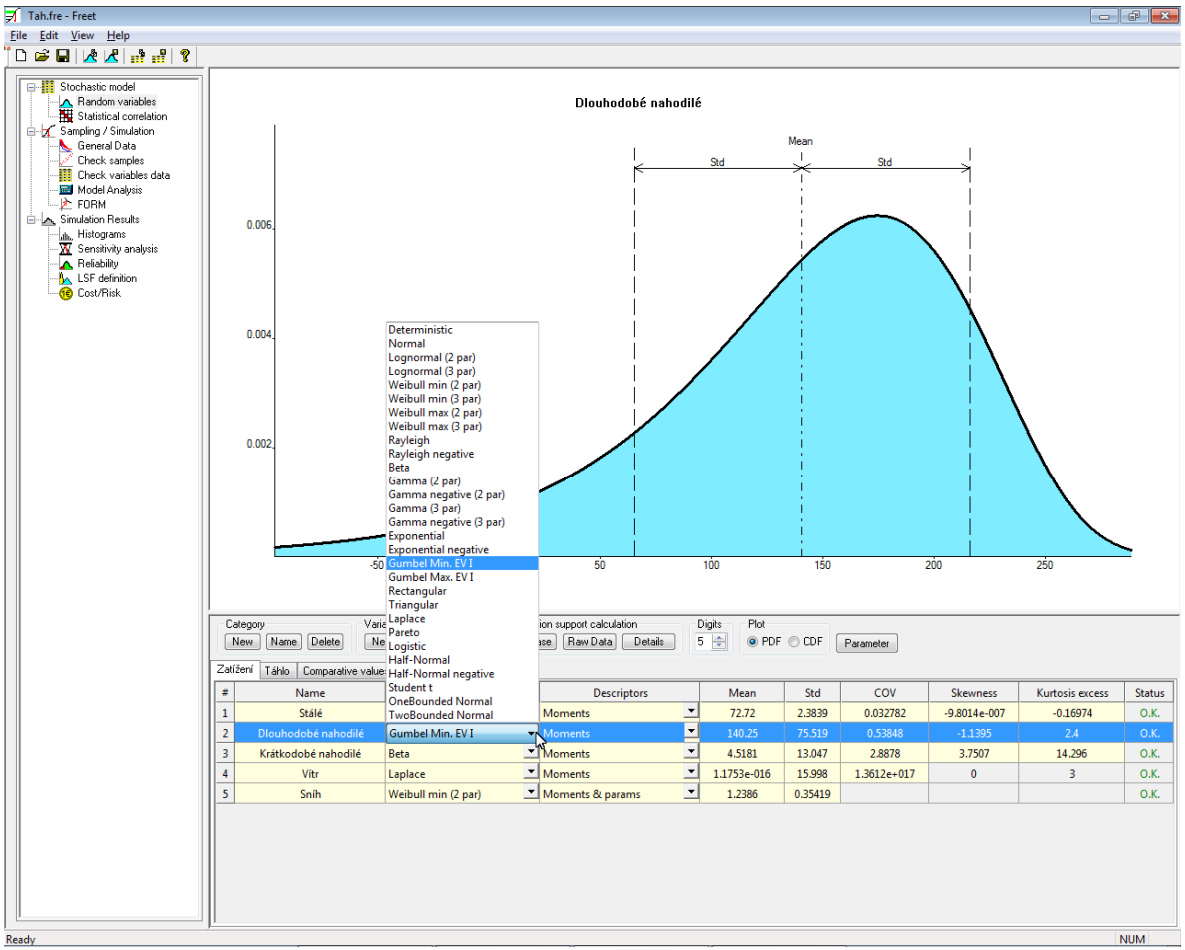
Freet (Feasible Reliability Engineering Tool)

Freet: Probabilistic multi-purpose software for statistical, sensitivity and reliability analysis.

- Developed at the Institute of Structural mechanics, Faculty of Civil Engineering, Brno University of Technology.
- Version 1.5, Demo version is available to download on **http://www.freet.cz** .



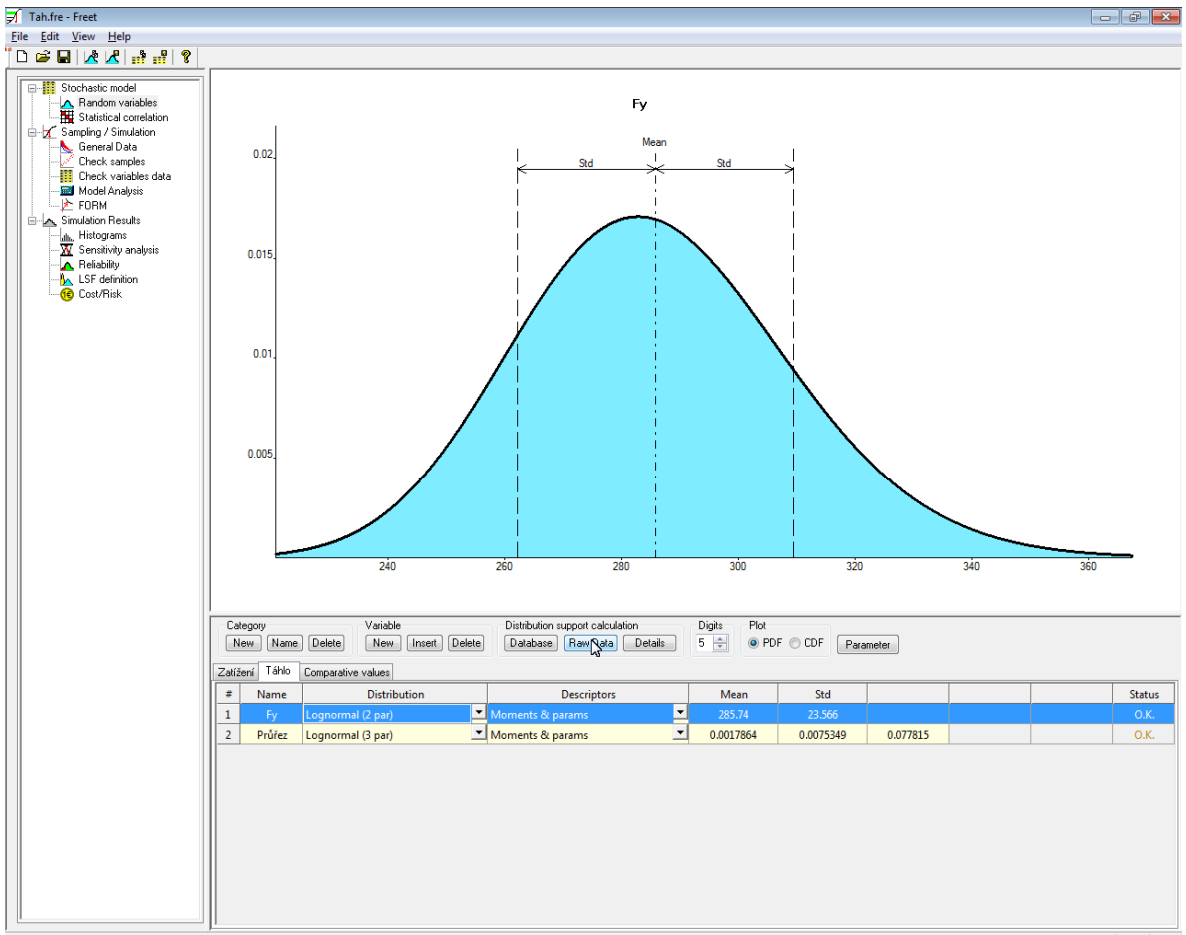
Freet: Entering of Input Values

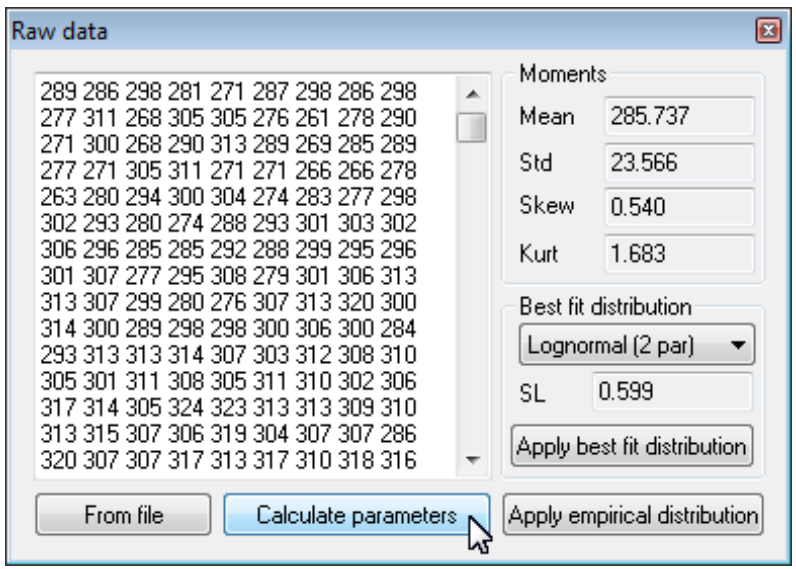


Freet: input random variables with parametric probability distribution.

Choice of parametric distributions from the database and entering specific values of statistical moments of the random variables.

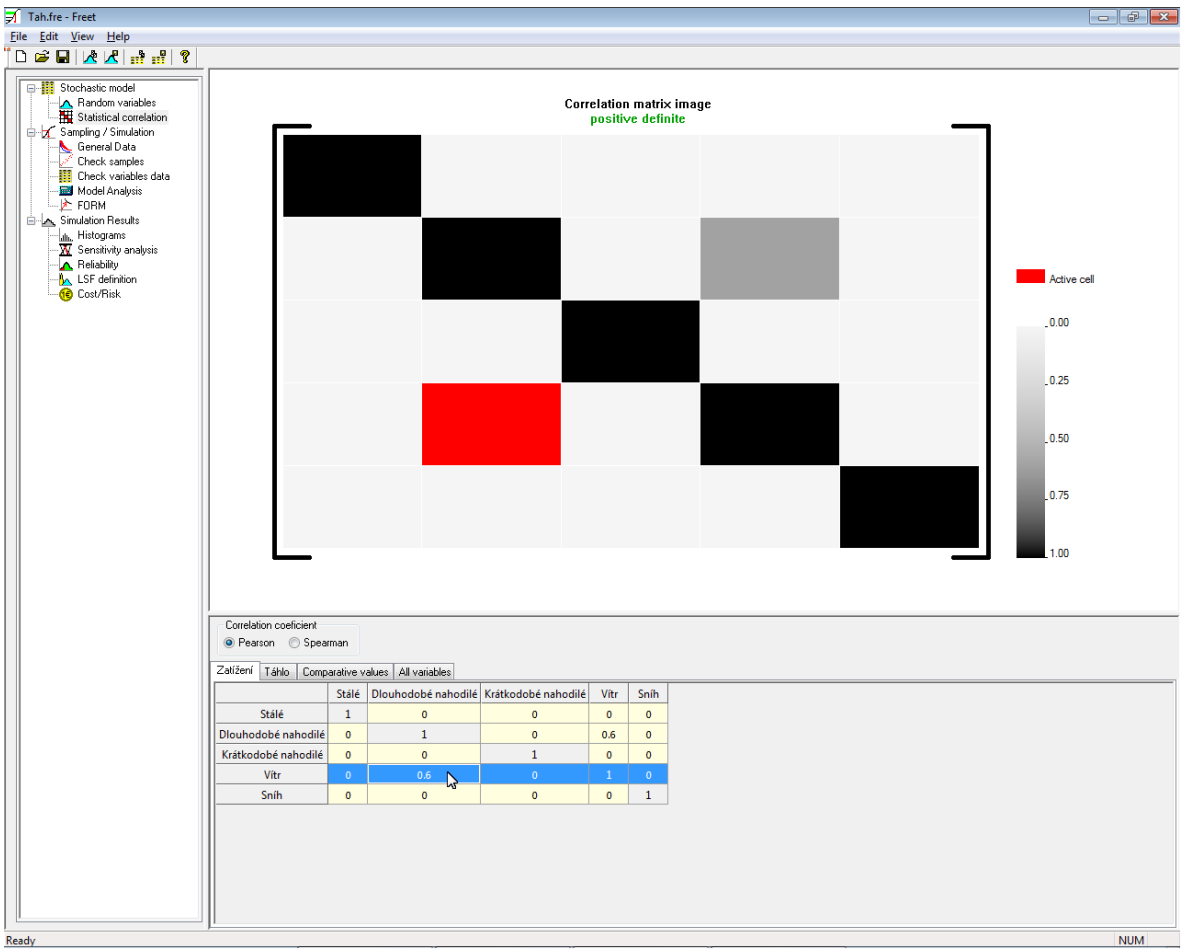
Freet: Measurements Data Utilization





Selecting the appropriate parametric distribution for the specified measurements.

Freet: Correlation Matrix Entering



	Stálé	Dlouhodobé nahodilé	Krátkodobé nahodilé	Vítr	Sníh
Stálé	1	0	0	0	0
Dlouhodobé nahodilé	0	1	0	0.6	0
Krátkodobé nahodilé	0	0	1	0	0
Vítr	0	0.6	0	1	0
Sníh	0	0	0	0	1

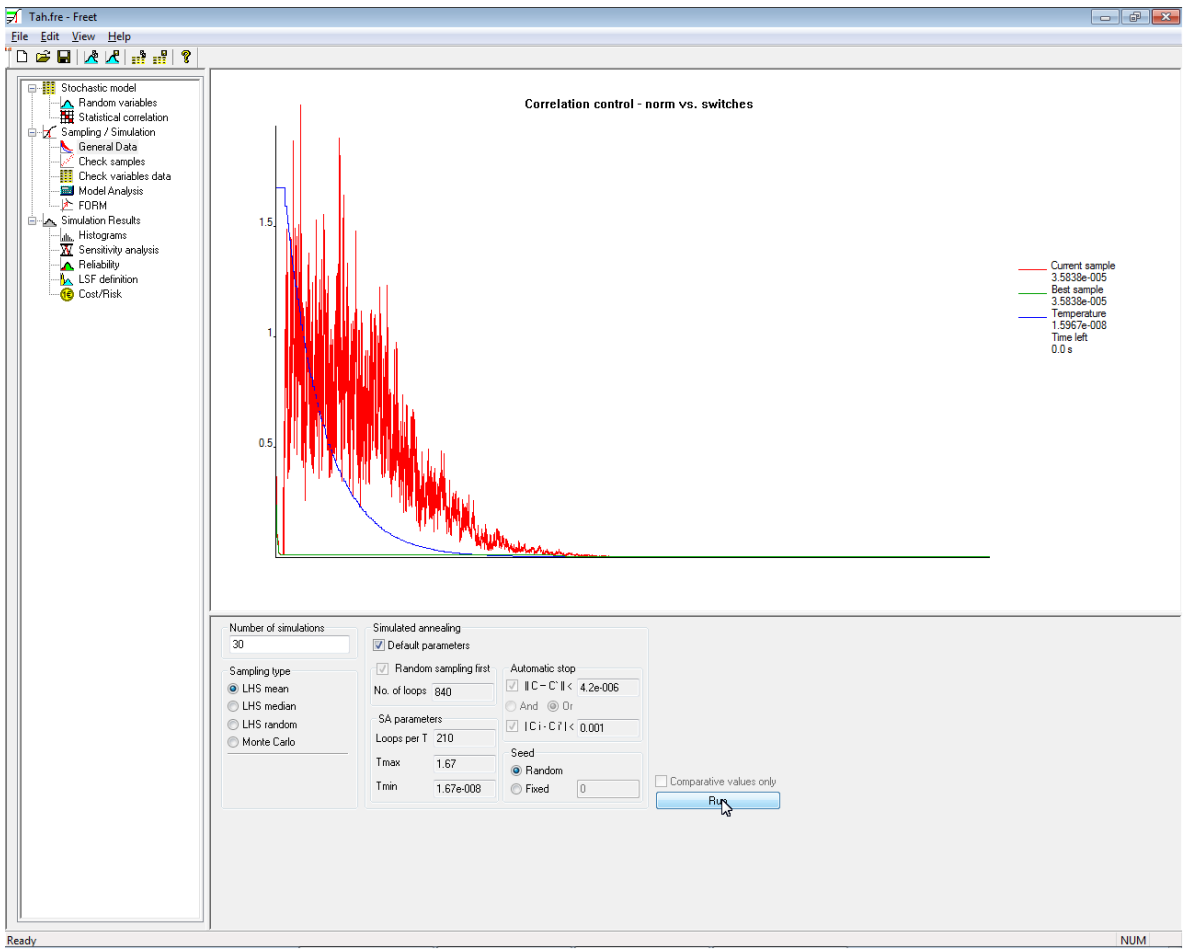
The **correlation coefficients**:

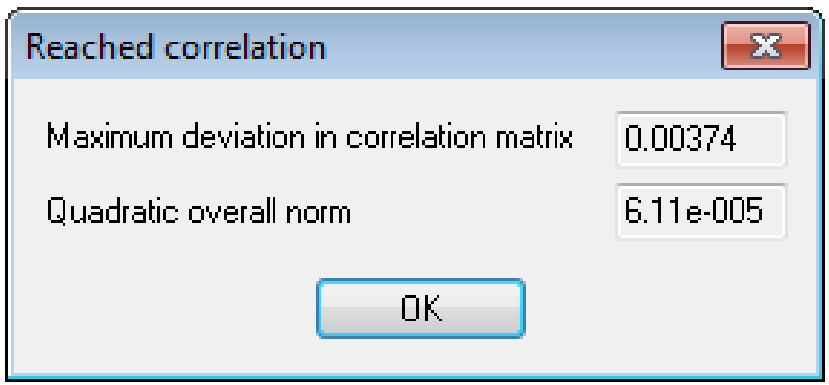
$= 0$ … **statistical independence**

$\neq 0, \geq -1, \leq 1$

… **statistical dependence**

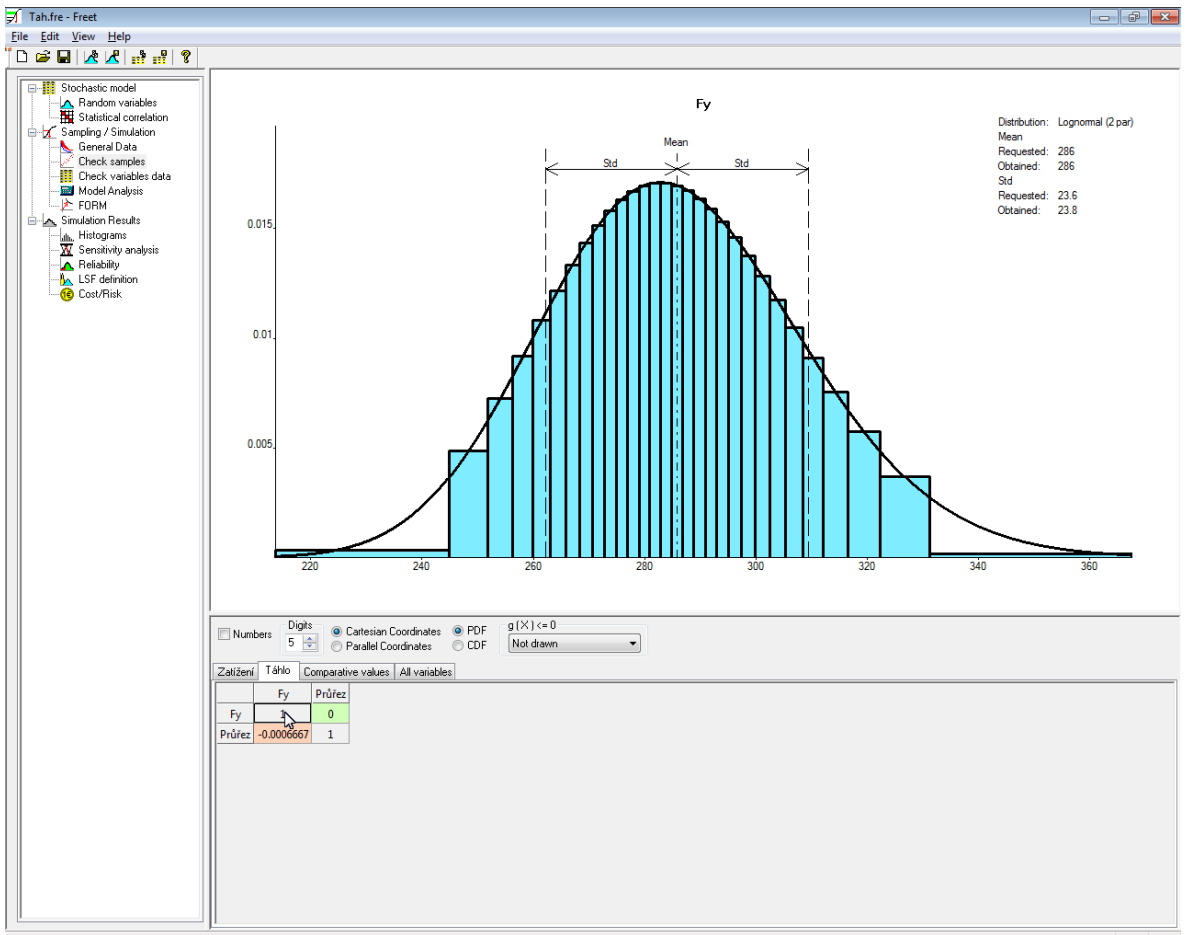
Freet: Generating of Simulations





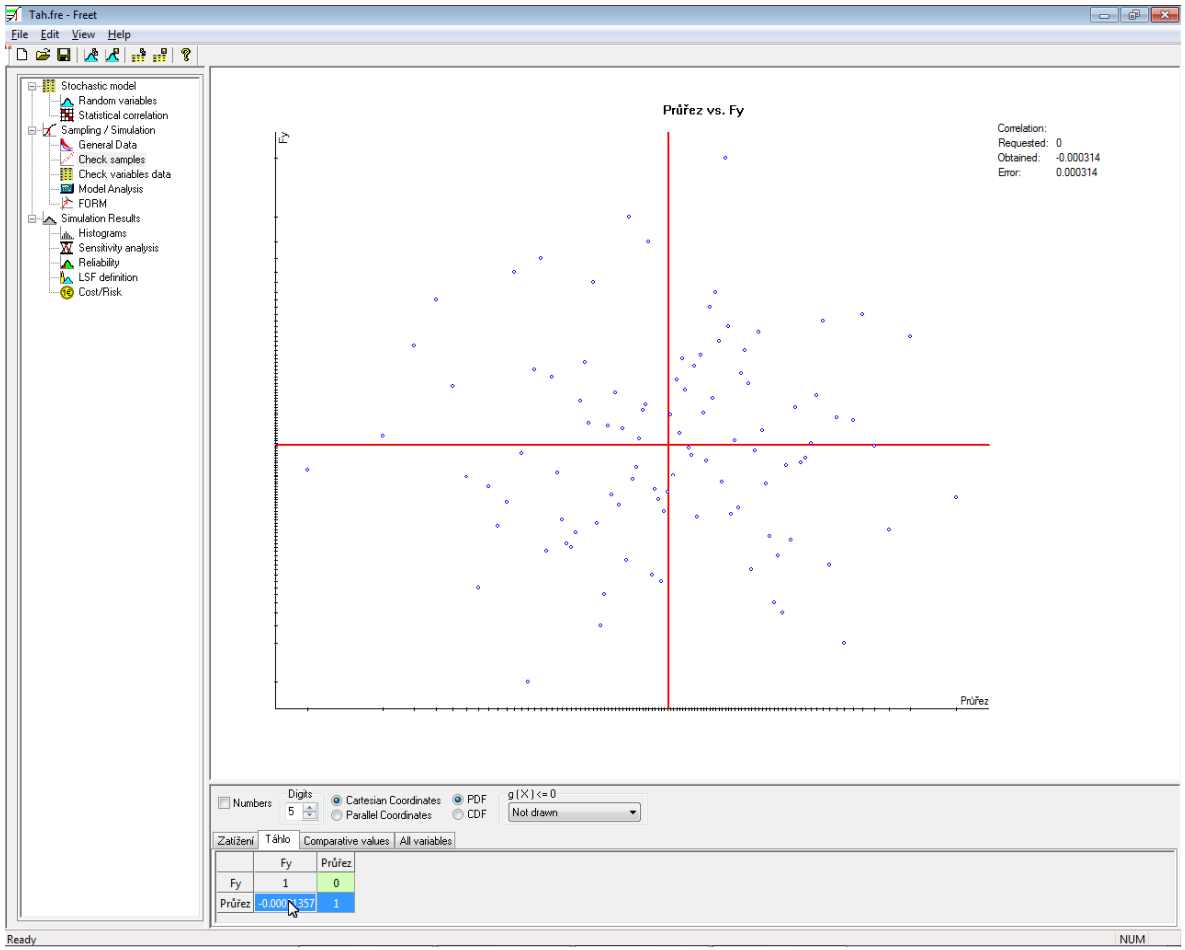
Freet: Iterative reordering the contents of the **table of random permutations** using method of **Simulated annealing**

Freet: Generating of Simulations



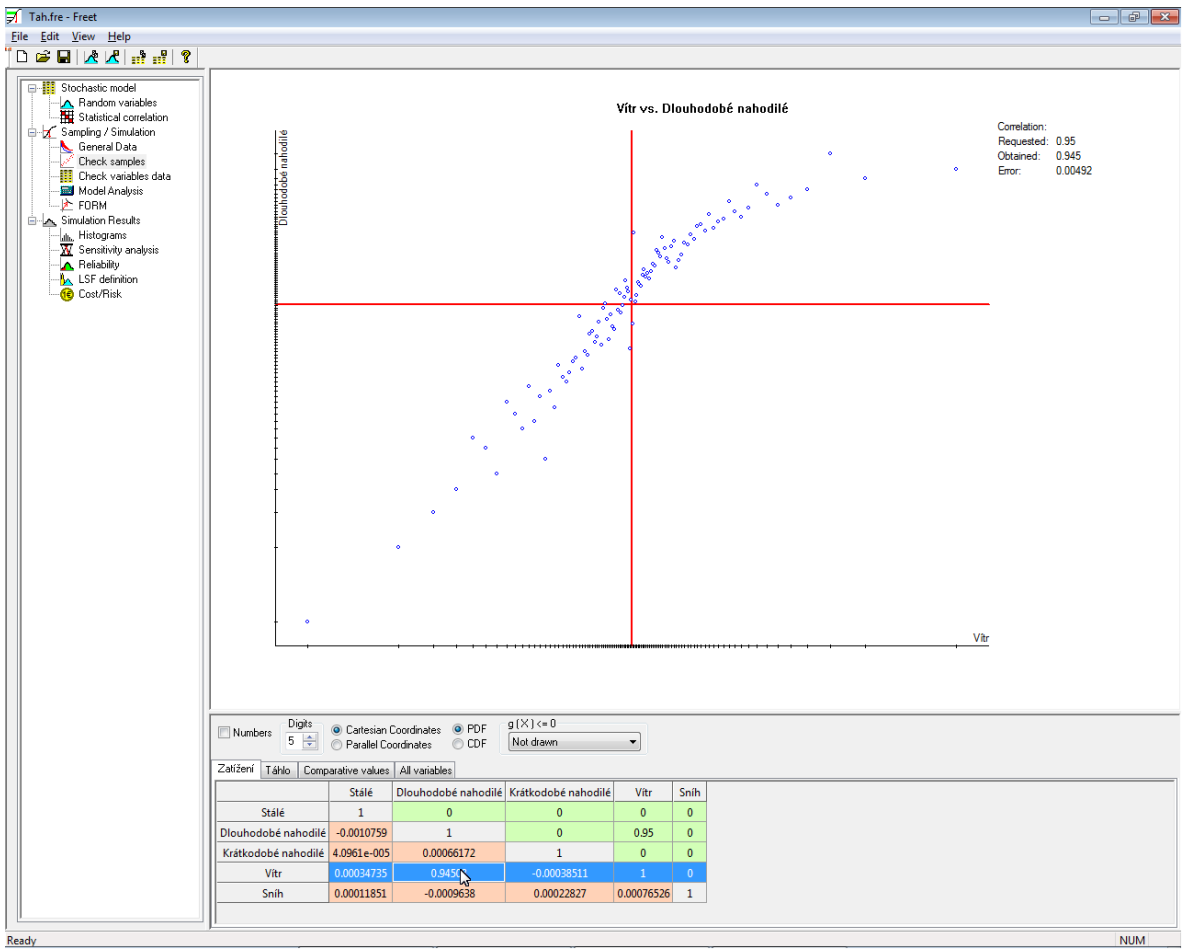
Freet: The division of each probability distribution on N intervals with the same probability.

Freet: Generating of Simulations



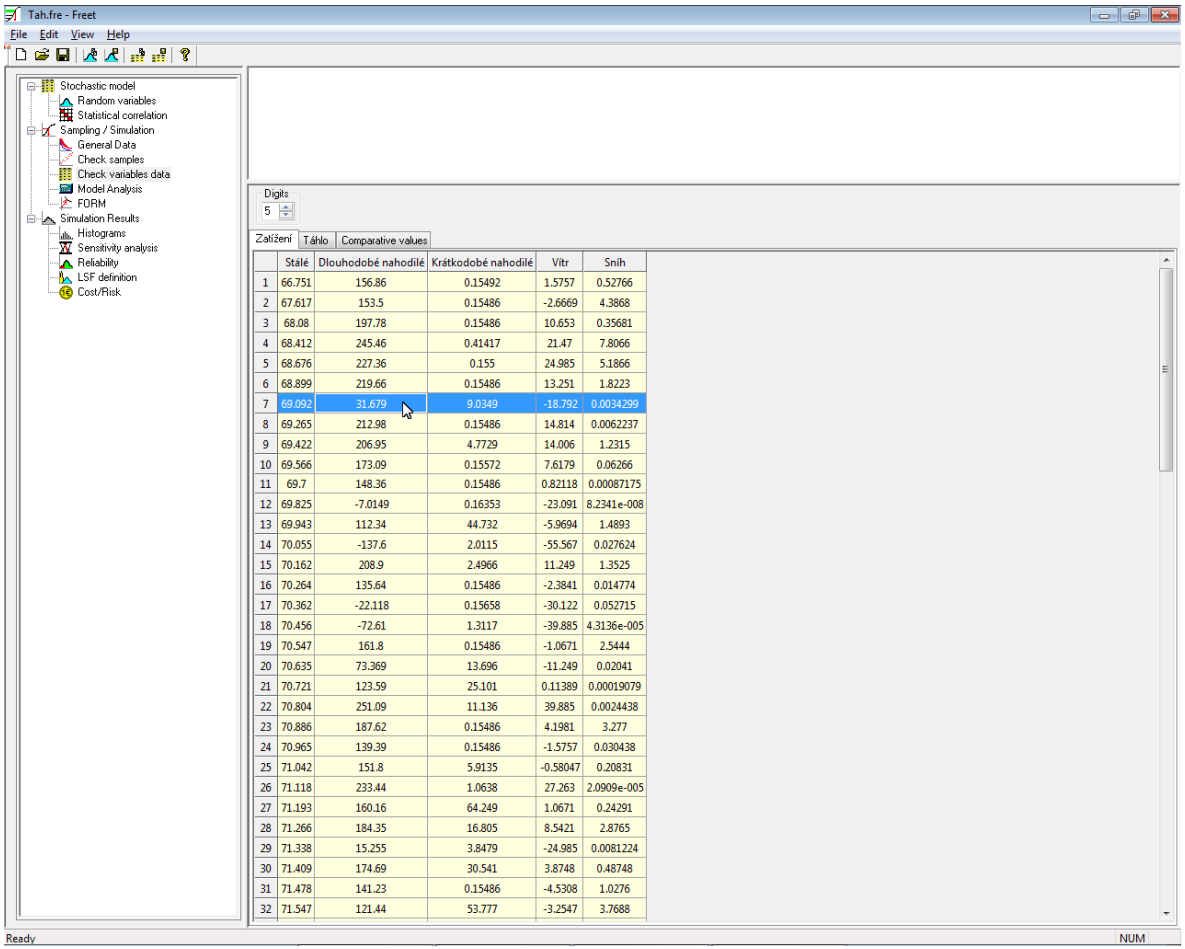
Freet: Sample of generated simulations of two random variables that are **statistically independent**.

Freet: Generating of Simulations



Freet: Sample of generated simulations of two random variables that are **statistically dependent** (95 %)

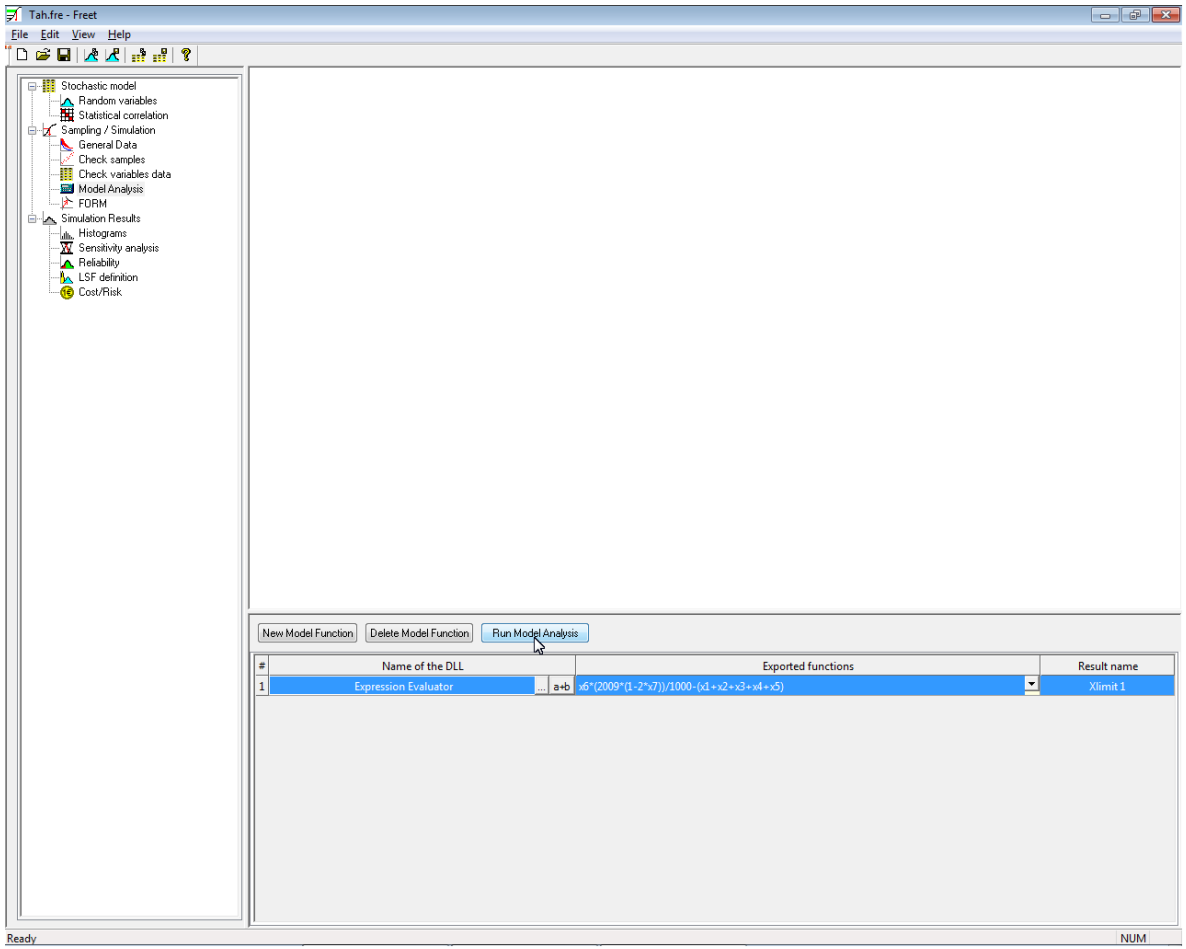
Freet: Generating of Simulations

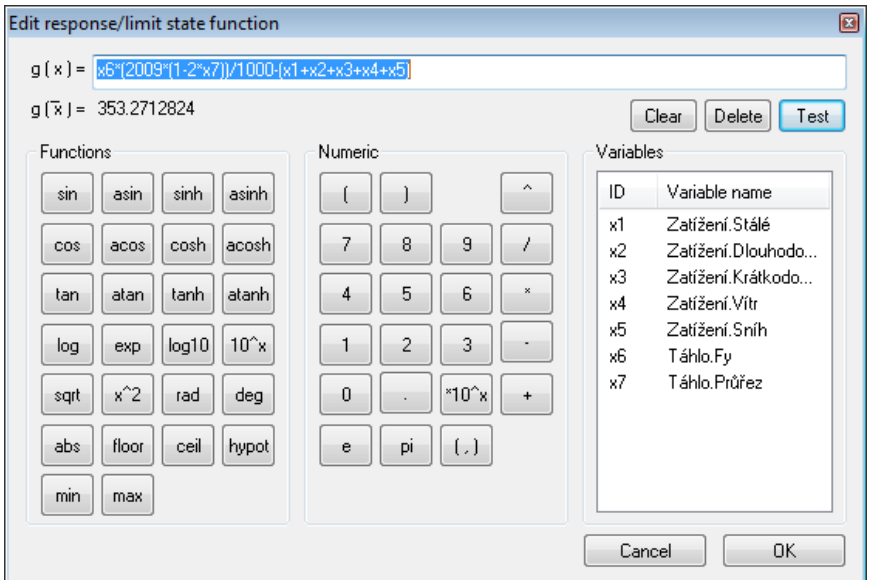


	Stálé	Dlouhodobé nahodilé	Krátkodobé nahodilé	Vítr	Sníh
1	66.751	156.86	0.15492	1.5757	0.52766
2	67.617	153.5	0.15486	-2.6669	4.3868
3	68.08	197.78	0.15486	10.653	0.35681
4	68.412	245.46	0.41417	21.47	7.8066
5	68.676	227.36	0.155	24.985	5.1866
6	68.899	219.66	0.15486	13.251	1.8223
7	69.092	31.679	9.0349	-18.792	0.0034299
8	69.265	212.98	0.15486	14.814	0.0062237
9	69.422	206.95	4.7729	14.006	1.2315
10	69.566	173.09	0.15572	7.6179	0.06266
11	69.7	148.36	0.15486	0.82118	0.00087175
12	69.825	-7.0149	0.16353	-23.091	8.2341e-008
13	69.943	112.34	44.732	-5.9694	1.4893
14	70.055	-137.6	2.0115	-55.567	0.027624
15	70.162	208.9	2.4966	11.249	1.3525
16	70.264	135.64	0.15486	-2.3841	0.014774
17	70.362	-22.118	0.15658	-30.122	0.052715
18	70.456	-72.61	1.3117	-39.885	4.3136e-005
19	70.547	161.8	0.15486	-1.0671	2.5444
20	70.635	73.369	13.696	-11.249	0.02041
21	70.721	123.59	25.101	0.11389	0.00019079
22	70.804	251.09	11.136	39.885	0.0024438
23	70.886	187.62	0.15486	4.1981	3.277
24	70.965	139.39	0.15486	-1.5757	0.030438
25	71.042	151.8	5.9135	-0.58047	0.20831
26	71.118	233.44	1.0638	27.263	2.0909e-005
27	71.193	160.16	64.249	1.0671	0.24291
28	71.266	184.35	16.805	8.5421	2.8765
29	71.338	15.255	3.8479	-24.985	0.0081224
30	71.409	174.69	30.541	3.8748	0.48748
31	71.478	141.23	0.15486	-4.5308	1.0276
32	71.547	121.44	53.777	-3.2547	3.7688

Freet: Table of generated and re-arranged random permutations.

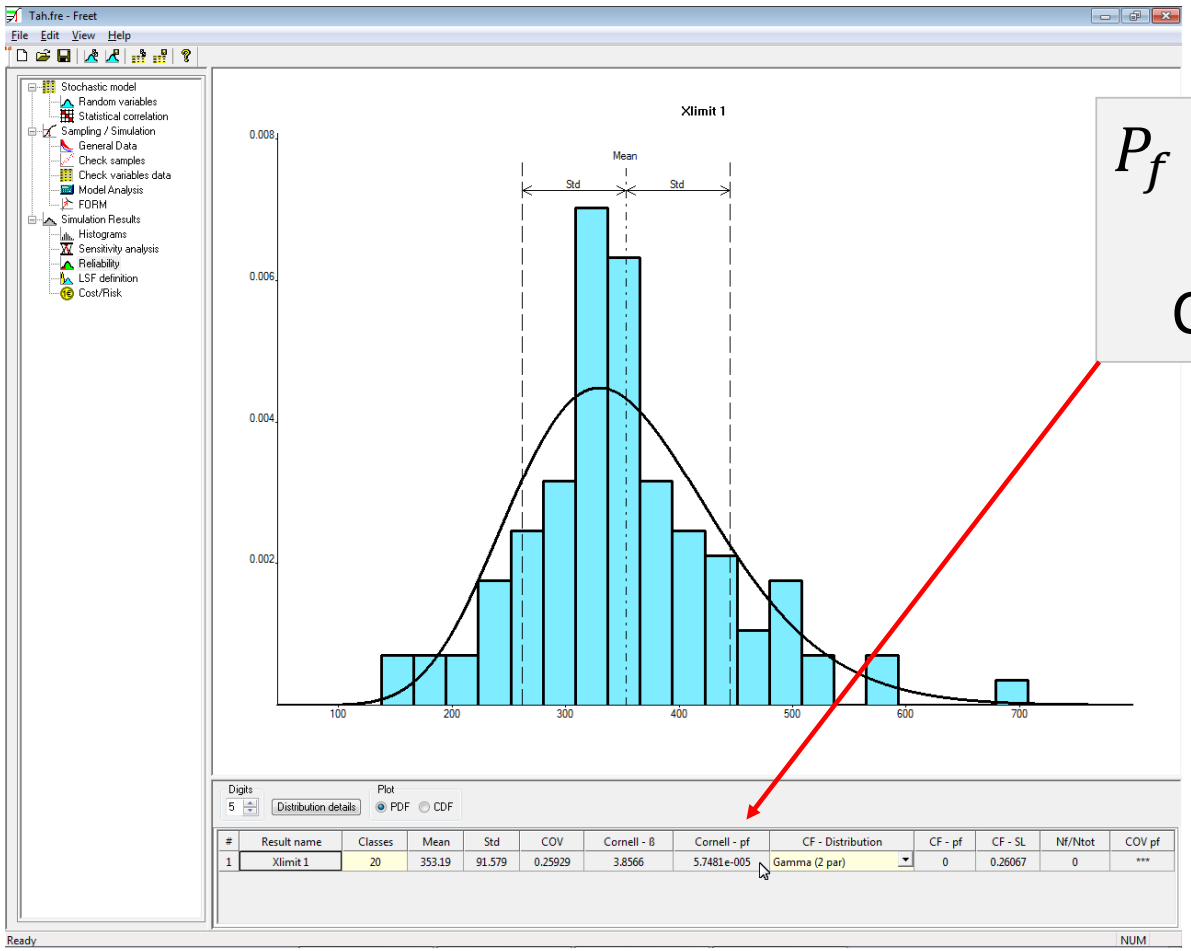
Freet: Definition of the Computing Model





Freet: Definition of the computing model and substituting the generated permutations in this model.

Freet: Estimation of the Probability of Failure



$P_f = 5.75 \cdot 10^{-5} < P_d = 7.2 \cdot 10^{-5}$

supporting element is OK.

Class of consequences **RC2/CC2**.

Freet: The resulting estimation of the reliability function's probability distribution, estimation of the probability of failure P_f

Example 1, Reliability Assessment

Expression and idealization of the structure under actual static or dynamic loads in space and time using mathematics-physical relationships determining the stress, strain, acceleration etc. from a time dependent load variable.

E.g.:

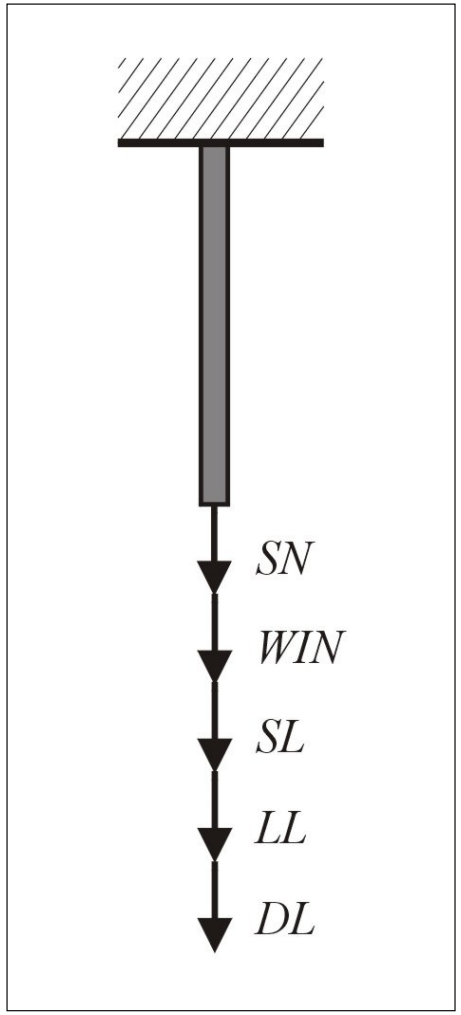
Reliability function RF: $$RF = R - \text{abs}(E)$$

Structural resistance R (axial load capacity N_{Rd}) :

$$R = N_{Rd} = A_{var} \cdot f_y$$

Load effect E (axial force N_{Ed}):

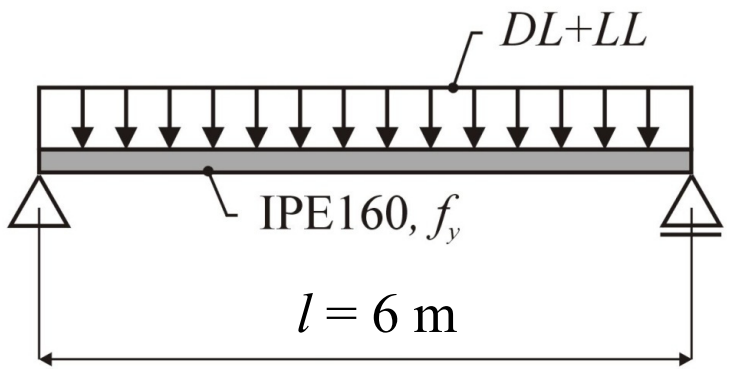
$$E = N_{Ed} = 80 \cdot DL + 293.5 \cdot LL + 80 \cdot SL + 70 \cdot WIN + 40 \cdot SN$$



Example 2, Reliability Assessment

Mathematical model of probabilistic calculation:

Reliability function: $RF = R - E$



Structural resistance (ultimate bending moment):

$$R = M_{Rd} = W_{y,var} \cdot f_y$$

Load effect (maximal bending moment):

$$E = M_{Ed} = \frac{1}{8} \cdot (2.1 \cdot DL + 3.5 \cdot LL) \cdot l^2$$

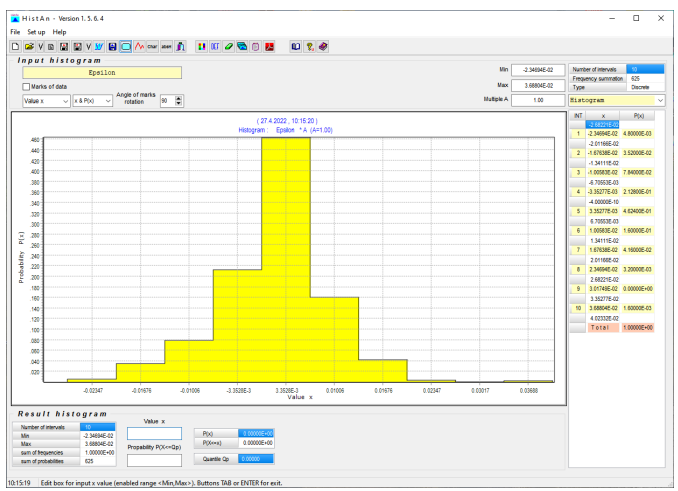
Cross-sectional variability:

$$A_{var} = A_{nom} \cdot (1 - 2 \cdot \varepsilon)$$

$$W_{var} = W_{nom} \cdot (1 - 3 \cdot \varepsilon)$$

$$I_{var} = I_{nom} \cdot (1 - 4 \cdot \varepsilon)$$

Histogram ε (Epsilon.dis) for expressing the variability of cross-sectional characteristics

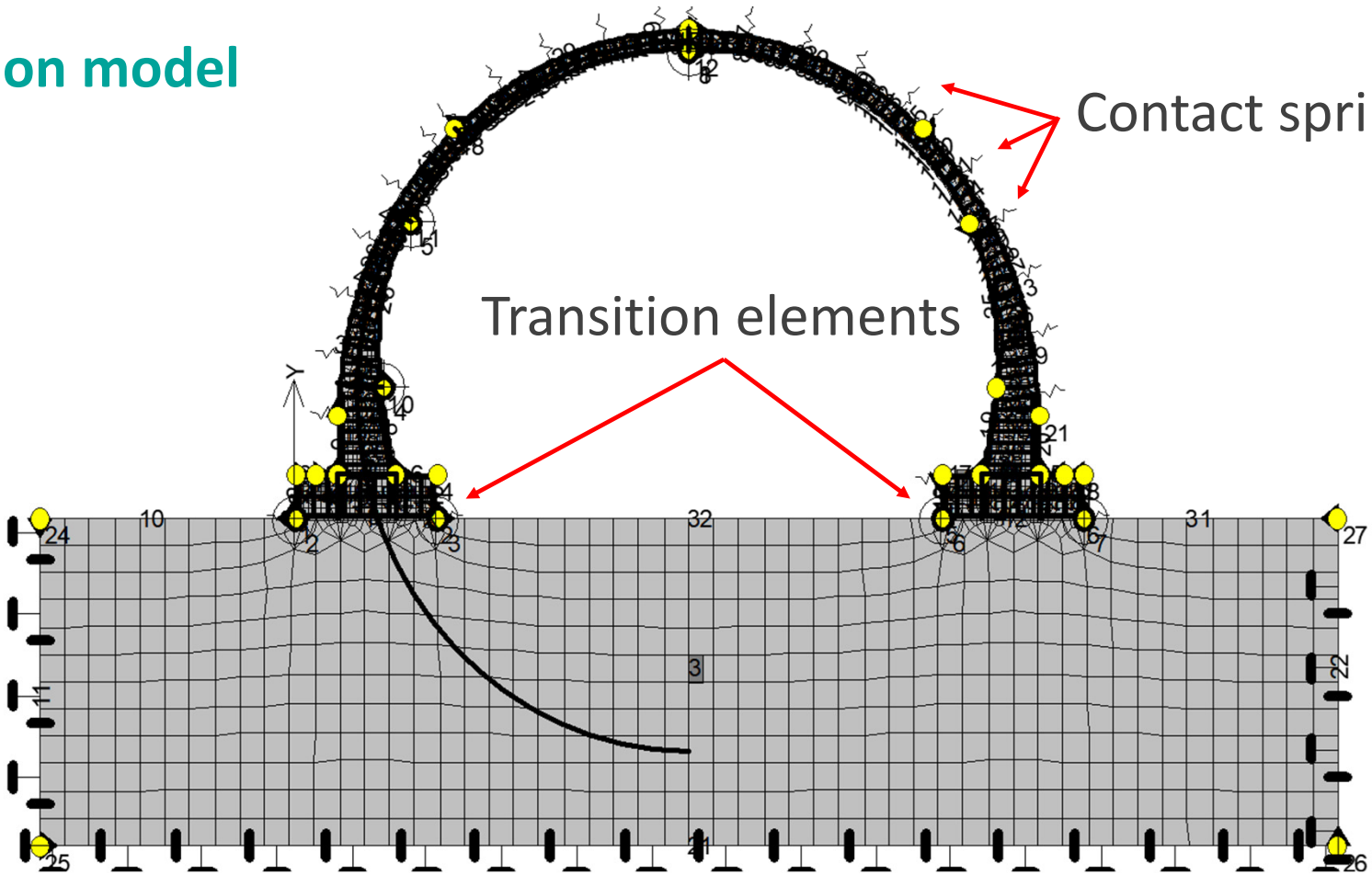


Atena-Sara-Freet: Sample Calculation

The structure under assessment is reinforced concrete road tunnel.

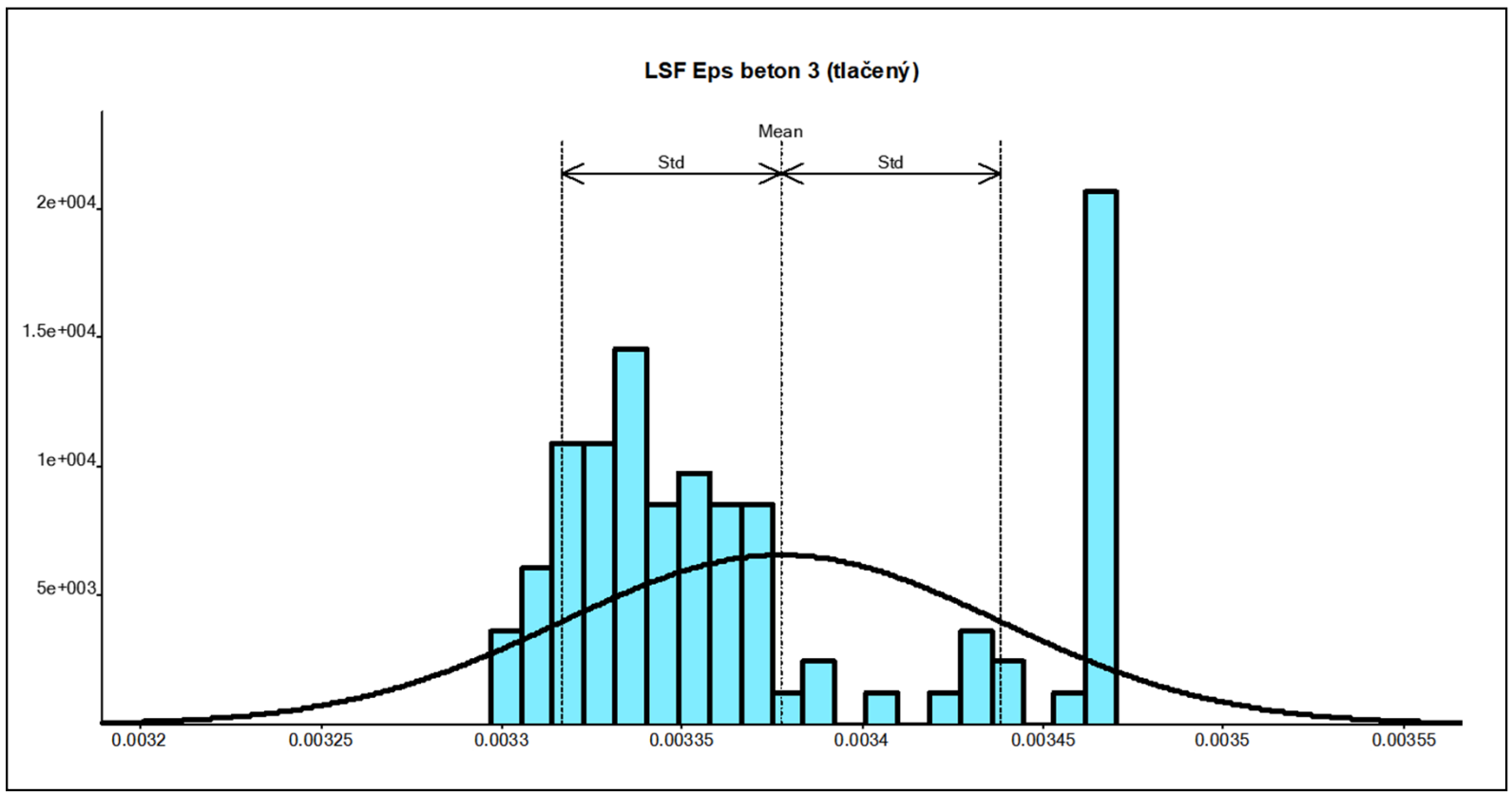
Atena-Sara-Freet: Sample Calculation

Calculation model



Atena-Sara-Freet: Sample Calculation

The results achieved, strain compression concrete



The reliability criteria:

- Deflection
- Deformation of the concrete
- Deformation of the steel reinforcement

Probability of exceeding the limit of compression strain of concrete is $\approx 10^{-42}$.