

# **Statistically Dependent Random Variables**

- Theoretical background
- Software HistAn2D and HistAn3D
- Examples

Some of input variables are **statistically dependent** however, e.g., crosssection characteristics, strength properties etc.



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#### **Statistics Analyze of the Measured Data**

**Pearson's correlation coefficient** 

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \cdot \sum_{i=1}^{n} (y_i - \bar{y})^2}}$$

where  $x_i$  and  $y_i$  are elements of the both random quantities and  $\overline{x}$  and  $\overline{y}$  are mean values of those quantities

**Spearman's coefficient of sequential correlation** 

$$\rho = 1 - \frac{6 \cdot \sum_{i=1}^{n} (p_i - q_i)^2}{n \cdot (n^2 - 1)}$$

can be determined by arranging n values  $x_i$  and  $y_i$  of the both random quantities by their size and by allocating sequence numbers  $p_i$  and  $q_i$ .

#### **Statistics Analyze of the Measured Data**



### **Example 1**

Perform a statistical analysis of the dependence of both random variables using **Pearson's** and **Spearman's correlation coefficients**.



Statistically independent random variables are entered into probabilistic calculation using **double** or **triple histograms**.



Desktop of HistAn2D: double histogram of statistically independent (left) and dependent (right) random variable

# Used double histograms for statistically dependent random cross-section properties of HE300B profile.



 $A_{var}, I_{v,var}$ 

 $I_{y,var}, W_{y,var}$ 

 $A_{var}, W_{y,var}$ 

**Theoretical Background**: In each standard histogram *A*, one axis includes the  $a_j$  class which is limited by  $a_{\min}$  and  $a_{\max}$ , while the other axis shows typically the probability,  $p_{a_j}$ , of occurrence of that class,  $a_j$ .

The sum of probabilities for each class  $a_j$  in the histogram is  $\sum p_{a_j} = 1$ .

In the double histogram of two random variables,  $Z_1$  and  $Z_2$ , the quantity  $z_1$  is limited again by  $z_{1,\min}$  and  $z_{1,\max}$ , while  $z_2$  is limited by  $z_{2,\min}$  and  $z_{2,\max}$ .

The values can be divided, using the step  $\Delta z_{1,}$  into  $N_1$  intervals for random quantities  $Z_1$ , or, using the step  $\Delta z_2$ , into  $N_2$  intervals for the random quantities  $Z_2$ . The number of intervals is as follows:

$$N_1 = \frac{z_{1,\max} - z_{1,\min}}{\Delta z_1}$$
 and  $N_2 = \frac{z_{2,\max} - z_{2,\min}}{\Delta z_2}$ .

**Theoretical Background**: If the input variable  $z_1$  is in the  $j^{\text{th}}$  class of  $z_{1,j}$  in theory,  $z_2$  could acquire following values:  $z_{2,1}, z_{2,2}, \dots, z_{2,j}, \dots, z_{2,N_2}$ . This means, it can acquire  $N_2$  values.

The double histogram of the random quantities  $z_1$  and  $z_2$  can contain  $N_1 \cdot N_2$  classes. This means, each class is determined by two values,  $z_{1,j}$  and  $z_{2,j}$ , and by the probability of occurrence of that class,  $p_{z_{1,j},z_{2,j}}$ . Again:  $\sum p_{z_{1,j},z_{2,j}} = 1$ .

The number of classes with the non-zero probability can reach the product of  $N_1 \cdot N_2$ . If the random quantities are dependent, the number of classes in the histogram with the non-zero probability can be considerably lower than the product  $N_1 \cdot N_2$ .

Special software applications HistAn2D (left) and HistAn3D (right) were developed for creation of the double and triple histograms which describe the statistical dependence between two or three random variables.

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		14	1680.28000	/8136.30839		23	1471.58320	72377.39251	0	0.000000000		
		15	1584.85500	/2444.90579		24	1471.58320	73322,47969	0	0.000000000		
		16	1589.11500	/4820.37802		25	1471.58320	74267.56686	0	0.000000000		
		17	1698.55000	//899.97664		26	1471.58320	75212.65404	0	0.000000000		
		18	1657.27000	/5650.48228		27	1471.58320	76157.74121	0	0.000000000		
		19	1553.96000	70864.80212		28	1471.58320	77102.82839	0	0.000000000		
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lax	1768.40500	58149	50.91900	81355.72067		1457.25387	4765807.36670	73038.95354	0	0.000000000
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			Raw Data			1457.25387	4765807.36670	76063.23249	0	0.000000000
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	7	1616 35500	5327557 8510	74728 57074	15	1457.25387	4876243.53010	73038.95354	0	0.000000000
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	10	1623 16500	5376478 1420	75431 43064	18	1457.25387	4876243.53010	77575.37197	0	0.000000000
	11	1649 31000	5460514 0840	76558 18721	19	1457.25387	4876243.53010	79087.51145	0	0.000000000
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	16	1589 11500	5320222 1010	74820 37802	24	1457.25387	4986679.69350	71526.81406	0	0.000000000
	17	1698.55000	5556089,8260	77899.97664	25	1457.25387	4986679.69350	73038.95354	0	0.000000000
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	20	1627 37500	5256208 4290	74611 04704	25	1457.25387	4986679.69350	79087.51145	0	0.000000000
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Desktop of HistAn2D (left) and HistAn3D (right):

raw data of rolled shape IPE 140 cross-section properties under analyses

Using the software, it is possible to view for each random variable a simple histogram with non-parametric (empirical) distribution of probability as well as a multidimensional histogram which describes the statistical dependence between the quantities.



Histograms with **non-parametric** (empirical) **distribution of probability**: Histogram of the IPE140 cross-section area A (left) and cross-section modulus  $W_y$  (right)



Double histogram for two statistically dependent random quantities – crosssection area A and cross-section modulus  $W_{v}$  Double histogram for two **statistically independent** random quantities – crosssection area *A* and cross-section modulus *W*<sub>y</sub>



**Statistically independent** random variables are entered into probabilistic calculation using ProbCalc software



bulk density vs. compressive strength the correlation 60.8% to 62.2%



cube vs. cylinder compressive strength the correlation 99.8% to 100.0%



compressive strength of concrete vs. floor in the building the correlation -21.1% to -25.8%

#### **Correlation coefficients** of a **double histogram of the statistically dependent quantities** with different numbers of intervals (Pearson's correlation coefficient for raw data is 0.9645; Spearman correlation coefficient for raw data is 0.9499)

Number of intervals in a double histogram	Pearson's correlation coefficient	Spearman's rank correlation coefficient	Number of intervals in a double histogram	Pearson's correlation coefficient	Spearman's rank correlation coefficient
<b>4</b> <sup>2</sup> = <b>16</b>	0.79985097	0.79507798	18 <sup>2</sup> = 324	0.95267109	0.94023800
6 <sup>2</sup> = 36	0.86661900	0.86360377	20 <sup>2</sup> = 400	0.96046634	0.94378886
8 <sup>2</sup> = 64	0.91530000	0.91194405	<b>22<sup>2</sup> = 484</b>	0.95940904	0.94355084
<b>10<sup>2</sup> = 100</b>	0.93984931	0.92352904	24 <sup>2</sup> = 576	0.95903334	0.94989866
12 <sup>2</sup> = 144	0.94381175	0.93613068	<b>26<sup>2</sup> = 676</b>	0.96464064	0.95260826
14 <sup>2</sup> = 196	0.95443331	0.93939308	<b>28<sup>2</sup> = 784</b>	0.96017017	0.94660574
<b>16<sup>2</sup> = 256</b>	0.94876401	0.93694950	<b>30<sup>2</sup> = 900</b>	0.95938019	0.94245225

Pearson's correlation coefficient (up) and Spearman's rank correlation coefficient (bottom) of a double histogram vs. number of intervals





	Number of zero-p	robability intervals		Number of zero-probability intervals			
Number of intervals in a double histogram	Statistically dependent quantitiesStatistically independent quantities		Number of intervals in a double histogram	Statistically dependent quantities	Statistically independent quantities		
4 <sup>2</sup> = 16	6	0	18 <sup>2</sup> = 324	288	69		
6 <sup>2</sup> = 36	24	0	20 <sup>2</sup> = 400	361	112		
8 <sup>2</sup> = 64	48	0	<b>22</b> <sup>2</sup> = 484	443	160		
10 <sup>2</sup> = 100	80	0	24 <sup>2</sup> = 576	531	216		
<b>12<sup>2</sup> = 144</b>	119	0	<b>26<sup>2</sup> = 676</b>	627	258		
14 <sup>2</sup> = 196	166	14	28 <sup>2</sup> = 784	735	322		
<b>16<sup>2</sup> = 256</b>	222	46	<b>30</b> <sup>2</sup> = 900	847	372		

The **number of classes** for double histograms **with zero probability** vs. the number of intervals chosen during creation of the histograms from the primary data

Number of intervals for the zero-probability in double histogram



**Numerical correlation index** – can characterize the dependence between random variables not only for the linear relationship between two variables, but also for nonlinear dependence, or even for more than two random

variables:

where  $T_M$  is the number of all classes in double or triple  $I_k = \frac{T_M - T_C}{T_M}$  histogram (for optimal number of intevals and raw data),  $T_c$  is the number of non-zero probability classes in double or triple histogram.

#### For statistically dependent variables:

Correction for insufficient number of data:

where  $n_1, n_2, n_3, \cdots, n_t$  are the numbers of intervals in histograms,

 $p_1, p_2, p_3, \cdots, p_t$  are the numbers of intervals without raw data.

2 dependent variables:  $T_M = (n_1 - p_1) \cdot (n_2 - p_2)$ 

*t* dependent variables:  $T_M = (n_1 - p_1) \cdot (n_2 - p_2) \cdot (n_3 - p_3) \cdot \dots \cdot (n_t - p_t)$ 



The calculation of **numerical correlation index** in HistAn2D software for variable number of intervals in double histogram

The **numerical correlation index** for two random variables - cross-sectional area A and cross-section modulus  $W_y$ 



# **Example 2, Reliability Assessment**



Scheme of the structure under assessment

#### **Reliability assessment of the column**

```
l \dots 6 m
profile HEB 300, steel S235, E \dots 2.1 \cdot 10^{11} Pa
imperfections: a \dots \pm 30 mm
```

Load	Туре	Extremal value [kN]
D	Dead	350
L	Long Lasting	75
S	Short Lasting	75
W	Wind	40
EQ	Earthquake	$\frac{1}{20} \cdot (D + L + S) = \frac{500}{20} = 25$

# **Example 2, Reliability Assessment**

#### **Ultimate limit state**

$$RF = R - E$$

$$R \dots \text{ structural resistance - yield stress } f_y$$

$$E \dots \text{ load effect - stress in outer fibres } \sigma$$

#### Serviceability limit state

 $\begin{array}{l} RF = \delta_{tol} - |\delta| \\ \delta \\ \end{array} \begin{array}{l} \delta_{tol} \\ \ldots \end{array} \text{ structural resistance - allowed deformation (35 mm)} \\ \delta \\ \ldots \end{array} \begin{array}{l} \text{load effect - maximal horizontal deformations} \end{array}$ 

#### **Random input variables:**

- 5 load components,
- cross-section variability,
- initial imperfection in column,
- yield stress  $f_y$ .



Histograms of reliability function RF, ultimate limit state



Statistically dependent cross-section parameters Failure probability  $P_f = 5.247 \cdot 10^{-5}$  (RC2/CC2) Time of calculation 9 sec.

Statistically independent cross-section parameters Failure probability  $P_f = 5.133 \cdot 10^{-5}$  (RC2/CC2) Time of calculation 3:20 min.



### Example 3

Static scheme of the elemental structure of a **parabolic arch** fixed in both ends and loaded with combination of three single loads



The reliability assessment has been made using the interaction formula:

ity  $P_f = P(RF < 0) = P\left(1 - \left[\left(\frac{N_{Ed}}{N_{Rd}}\right)^2 + \frac{M_{Ed}}{M_{Rd}}\right] < 0\right)$ 

$$\left(\frac{N_{Ed}}{N_{Rd}}\right)^2 + \frac{M_{Ed}}{M_{Rd}} \le 1$$

The failure probability  $P_f$  was determined using the reliability function RF:

#### **Example 3**



Histogram of reliability function RF, for the probabilistic calculation with **statistically independent cross-section parameters** of the cross-section area Aand cross-section modulus  $W_{\gamma}$ , failure probability  $P_f =$ 



Histogram of reliability function RF, for the probabilistic calculation with statistically dependent cross-section parameters of the cross-section area A and cross-section modulus  $W_y$ , failure probability  $1.647 \cdot 10^{-5}$ .

 $1.637 \cdot 10^{-5}$ .

### **Structure of Supercomputer Centre**

#### http://www.it4i.cz/

- 5-storey building,
- foundation slab with ribs,
- reinforced concrete structure with walls and columns.













#### Structure of Supercomputer Centre htt

#### http://www.it4i.cz/











### **Structure of Supercomputer Centre**

#### http://www.it4i.cz/

# The National Supercomputing Center **IT4 Innovations**, March 2014





# **Foundation Structure of Supercomputer Centre**

- Piles foundation under columns,
- Foundation slab with ribs upwards,
- Sliding joint at the bottom surface for volume change,
- Shrink bands elimination.





# **Foundation Structure of Supercomputer Centre**

#### Goal of the measurements:

- Development of the hydration heat during foundation slab concreting,
- Change of the stresses in the concrete,
- Comparison the tensile stresses with the numerical calculation.



# **Technology of Measurement**

- optical fiber temperature
- string gauges temperature/stress
- foil strain gauges strain (stress)
- digital thermometer temperature







# **Setting of Measuring Column**

Measuring column mounted to a reinforced concrete slab

- 7x string gauges (3 in transverse and 4 in longitudinal direction)
- 6x bundles of optical fibers





### **Ground Plan of Foundation Slab**

#### Schematic chart of the foundation slab and location of the sensors



#### **Installation of a Measuring Pillar in the Structure**



#### **Installation of a Measuring Pillar in the Structure**



### **Installation of a Measuring Pillar in the Structure**





#### **Data Transfer on Construction Site**



Time records of the temperature in the concrete foundation slab made for 9 different



Changes in time of the normal stress in the concrete slab in a longitudinal direction for



Changes in time of the normal stress in the concrete slab in a transversal direction for



Changes in time of the normal stress in the reinforcing steel in the foundation slab



# **Analyzing the Temperature in the Foundation Slab**

[mm]	325	281	237	192	148	104	59	0	-1000
325	1	0.999179	0.996923	0.995500	0.983759	0.971624	0.930670	0.785600	-0,034170
281		1	0.998172	0.997997	0.988557	0.978113	0.941768	0.793464	-0,129316
237			1	0.997222	0.988365	0.978245	0.942759	0.798301	-0.007715
192				1	0.995390	0.988138	0.957607	0.809621	0.028066
148					1	0.997432	0.978363	0.839512	0.106057
104						1	0.988934	0.859433	0.148671
59							1	0.899881	0.252198
0		sym.						1	0.298942
-1000									1

The correlation matrix which uses the **Pearson's correlation coefficients** to describe the statistics dependence of the temperature at different heights of the foundation slab and ambient temperature

[mm]	325	281	237	192	148	104	59	0	-1000
325	1	0.997100	0.994693	0.992175	0.974777	0.955080	0.907967	0.744175	-0.093153
281		1	0.998468	0.996990	0.981617	0.963234	0.919626	0.749822	-0.725701
237			1	0.998523	0.985556	0.969800	0.928818	0.762627	-0.051773
192				1	0.989439	0.975540	0.938054	0.769048	-0.024796
148					1	0.993216	0.967774	0.800925	0.068958
104						1	0.985939	0.836969	0.115915
59							1	0.881010	0.207761
0		sym.						1	0.244649
-1000									1

The correlation matrix which uses the **Spearman's correlation coefficients** to describe the statistics dependence of the temperature at different heights of the foundation slab and ambient temperature

# **Analyzing the Temperature in the Foundation Slab**



Desktop of HistAn2D: The double histogram which describes the statistic dependence of the random variable temperature at +325 mm and +281 mm using DOProC method

The **Pearson's** and **Spearman's correlation coefficients** between the temperatures measured at +325 mm and temperatures in other measuring points



### **Analyzing Changes in Normal Stress in a Slab**

	59	148	237	325
59	1	0.671255	0.548708	0.379817
148		1	0.906740	0.894987
237			1	0.906740
325	sym.			1

	59	148	237	325		
59	1	0.711119	0.535419	0.307283		
148		1	0.860507	0.784488		
237			1	0.860507		
325	sym.			1		

The correlation matrix which describes the statistic dependence of changes in the **normal stress** in a concrete slab in a **longitudinal direction** in four heights using the **Pearson's** (left) and **Spearman's** (right) **correlation coefficient** 

The Pearson's and Spearman's correlation coefficient for the randomly variable change in the **normal stress** of the foundation slab in a **longitudinal direction** measured at +59 mm combined with the values at other heights



**Analyzing Changes in Normal Stress in a Slab** 

	104	192	281
104	1	0.443257	0.092120
192		1	0.874841
281			1

	104	192	281
104	1	0.393268	0.094875
192		1	0.918306
281			1

The correlation matrix which describes the statistic dependence of changes in the **normal stress** in a concrete slab in a **transversal direction** in three heights using the **Pearson's** (left) and **Spearman's** (right) **correlation coefficient** 

The **Pearson's** and **Spearman's correlation coefficient** for the randomly variable change in the **normal stress** of the concrete foundation slab in a **transversal direction** measured at +104 mm combined with the values measured at other heights



#### **Analyzing Changes in Normal Stress in Reinforcement**

sensor	1	3	4	5	6	sensor	1		3	4	5	6
1	1	-0.326076	0.020921	0.438671	-0.013040	1	1	-	-0.340521	-0.244565	0.300965	-0.067029
3		1	0.659559	-0.267758	0.357575	3			1	0.369197	-0.254437	-0.172053
4			1	-0.426226	0.861452	4				1	-0.541736	0.632246
5				1	-0.401796	5					1	-0.290020
6					1	6						1

The correlation matrix which describes the statistic dependence of changes in the **normal stress** in **reinforcing steel** in the concrete slab using five sensors and the **Pearson's correlation coefficient** (left) and the **Spearman's correlation coefficient** (right)

The **Pearson's** and **Spearman's correlation coefficient** for the randomly variable change in the **normal stress** in **reinforcing steel** in the foundation slab measured by the sensor #1 combined with the values measured by the sensors #3 through #6



Analyzing the statistic dependence of the **dynamic modulus of elasticity of the concrete** and **compressive cube strength of the concrete**:

- **Dynamic modulus of elasticity** of the concrete and **compressive cube strength** of the concrete analyze using the **non-destructive tests**.
- The statistic dependence between the two randomly variable quantities can be described again using a pair of correlation coefficients (the Pearson's and Spearman's correlation coefficient equal to 0.541351 and 0.524191, respective) or using a double histogram.

Desktop of HistAn2D: A **double histogram** which describes the statistic dependence of the **dynamic modulus of elasticity** of the concrete and **compressive cube strength** of the concrete



Statistical analysis of concrete in the frame structure:

- Destructive testing detected physical and mechanical parameters of concrete in supporting structure and the floors during the construction and technical survey of the building of the Faculty of Mechanical Engineering in Brno.
- The aim of this survey was the need to assess the **quality of concrete** in selected parts of the **horizontal and vertical load-bearing structures**.

View on the structure with old cladding during inspection



Statistical analysis of concrete in the frame structure

Detail of the concrete frame under cladding before reconstruction



Statistical analysis of concrete in the frame structure:

- 32 cores 45 mm in diameter on the peripheral columns over the entire height of the 17 storied building was done.
- In the internal load-bearing columns between 1<sup>st</sup> and 10<sup>th</sup> over ground floors of building were performed 25 cores.
- In the horizontal supporting structures were done 6 cores in total on floors 2, 4 and 6.
- From all of the cores were created 166 test specimens to determine strength parameters of concrete.



Statistical analysis of concrete in the frame structure:

Using the statistical and sensitivity analysis was subsequently found a statistical dependence between the **bulk density of concrete** and **concrete compressive strength** with the correlation in range 60.8% to 62.2% and between the **cube** and **cylinder compressive strength of concrete** with the correlation of 99.8% to

100.0%.



bulk density vs. compressive strength the correlation 60.8% to 62.2%



cube vs. cylinder compressive strength the correlation 99.8% to 100.0%

Statistical analysis of concrete in the frame structure:

- A study based on strength properties of concrete depending on the floor of sample was also performed.
- The study pointed to a slight statistical correlation between the compressive strength of concrete and the ground floor, where the drill core sample was executed.
- The floor decreases with increasing compressive strength of concrete.

compressive strength of concrete vs. floor in the building the correlation -21.1% to -25.8%.

