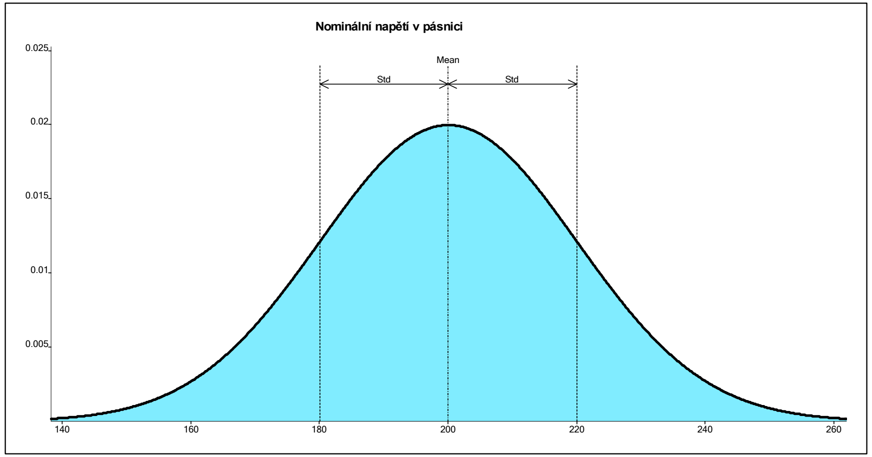
Parametric probability distributions

- Parametric probability distributions of a continuous random variable
- An overview of important continuous probability distributions
- Histogram creation with parametric probability distribution

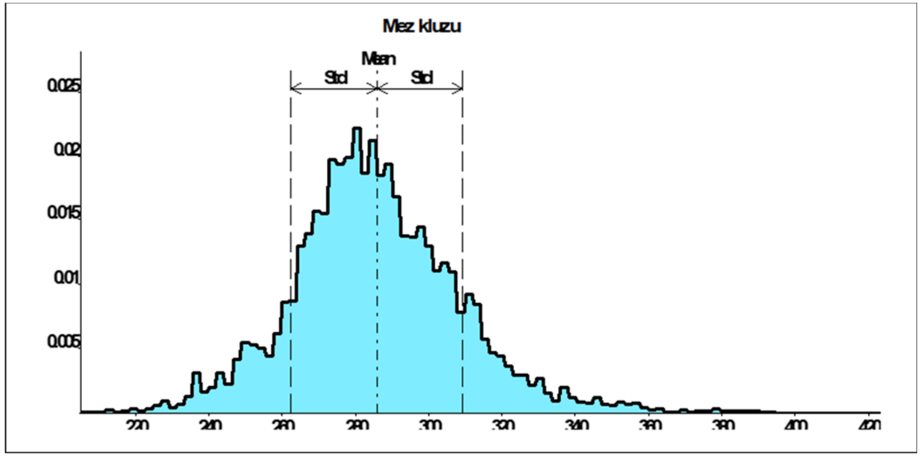
Probability distribution

Parametric probability distribution - probabilities defined by analytical function – e.g., common expression of normal (Gaussian) probability distribution:

$$f(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Parameters - characteristics of random variable probability distribution (e.g., μ mean value and σ standard deviation)

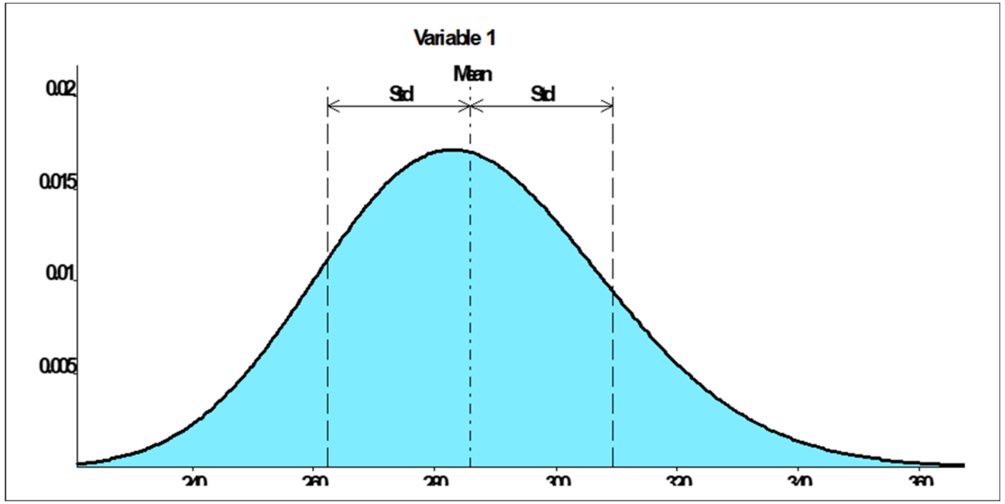


Non-parametric (empirical) **probability distribution** - definition based on measurements (often long-term)

Parametric probability distributions

Important parametric probability distributions for continuous random variables:

- Uniform distribution
- Normal (Gaussian) distribution
- Exponential distribution
- Laplace distribution
- Logistic distribution
- Maxwell distribution
- Student's t-distribution
- Fisher's z-distribution
- Chi-square distribution

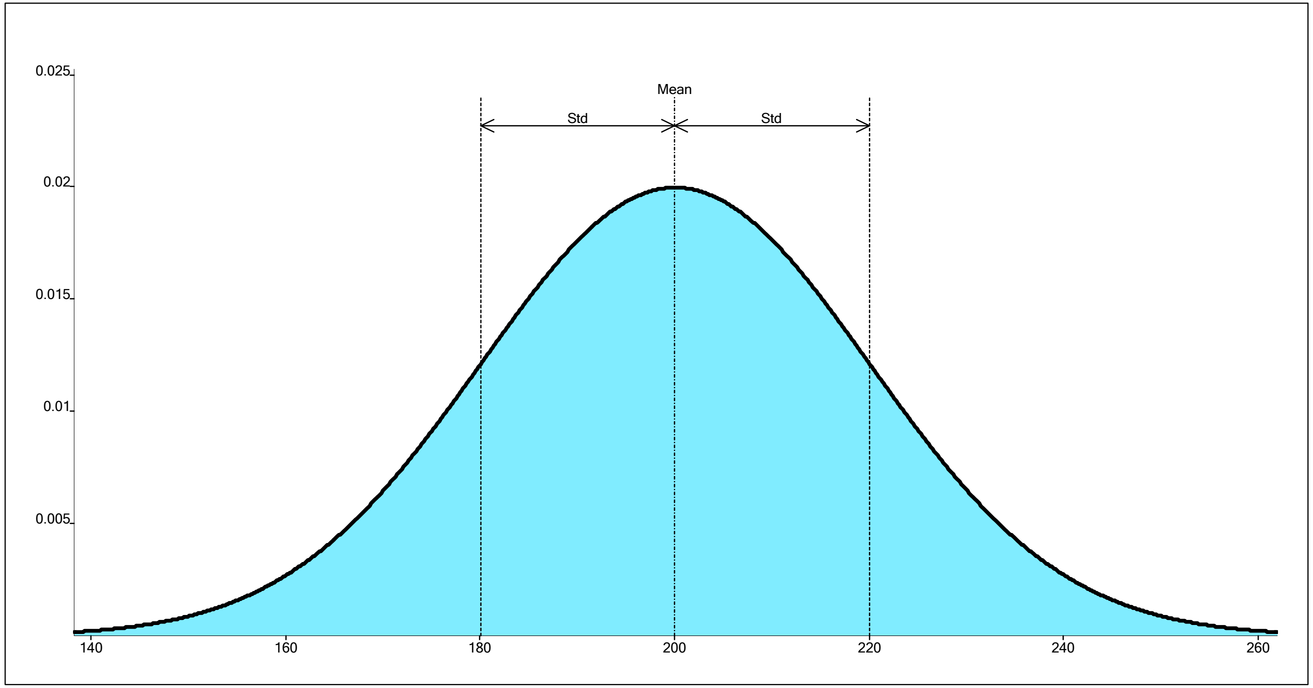


Parameters - characteristics of random variable probability distribution
(e.g., μ mean value and σ standard deviation)

Normal (Gaussian) probability distribution

Common expression of **normal** (**Gaussian**) **probability distribution**:

$$f(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



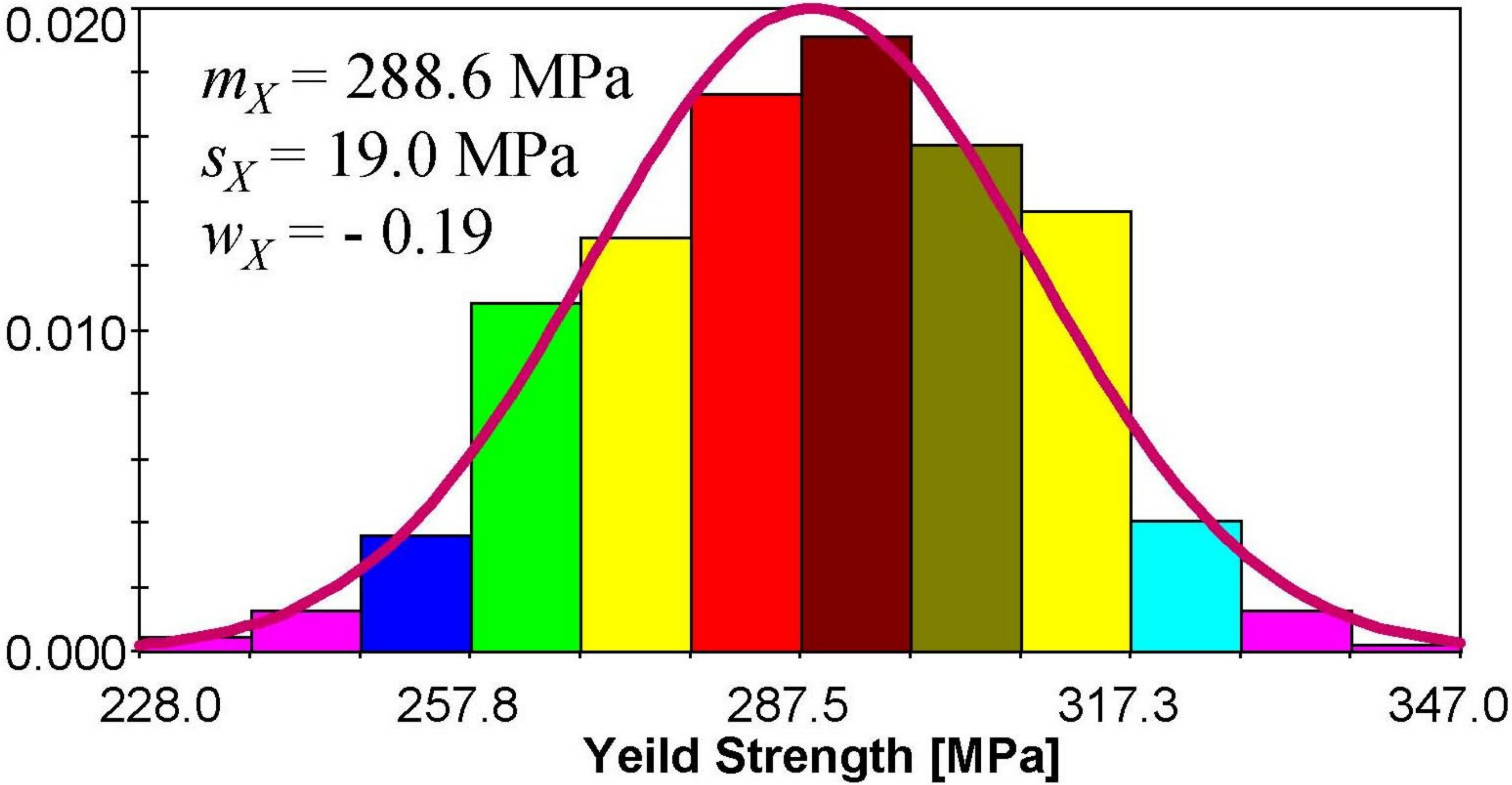
μ ... **mean value**

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

σ ... **standard deviation**

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2}$$

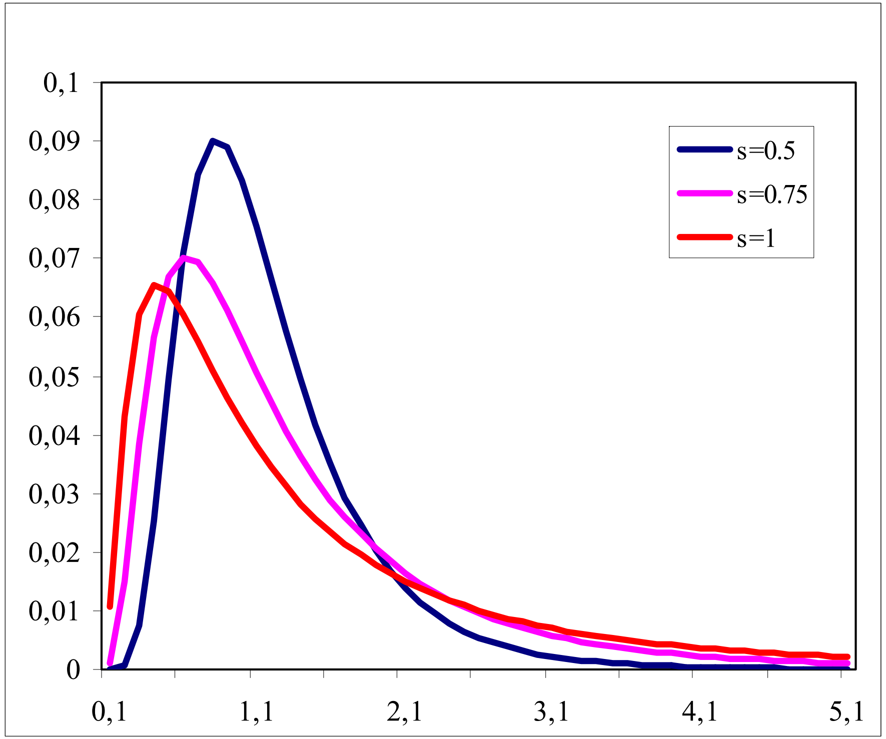
Yield stress of the S235 steel



Log-normal probability distribution

Common expression of **log-normal probability distribution**:

$$f(x|\mu,\sigma) = \frac{1}{x\sqrt{2\pi}\sigma} e^{\frac{-(\ln x - \mu)^2}{2\sigma^2}}$$



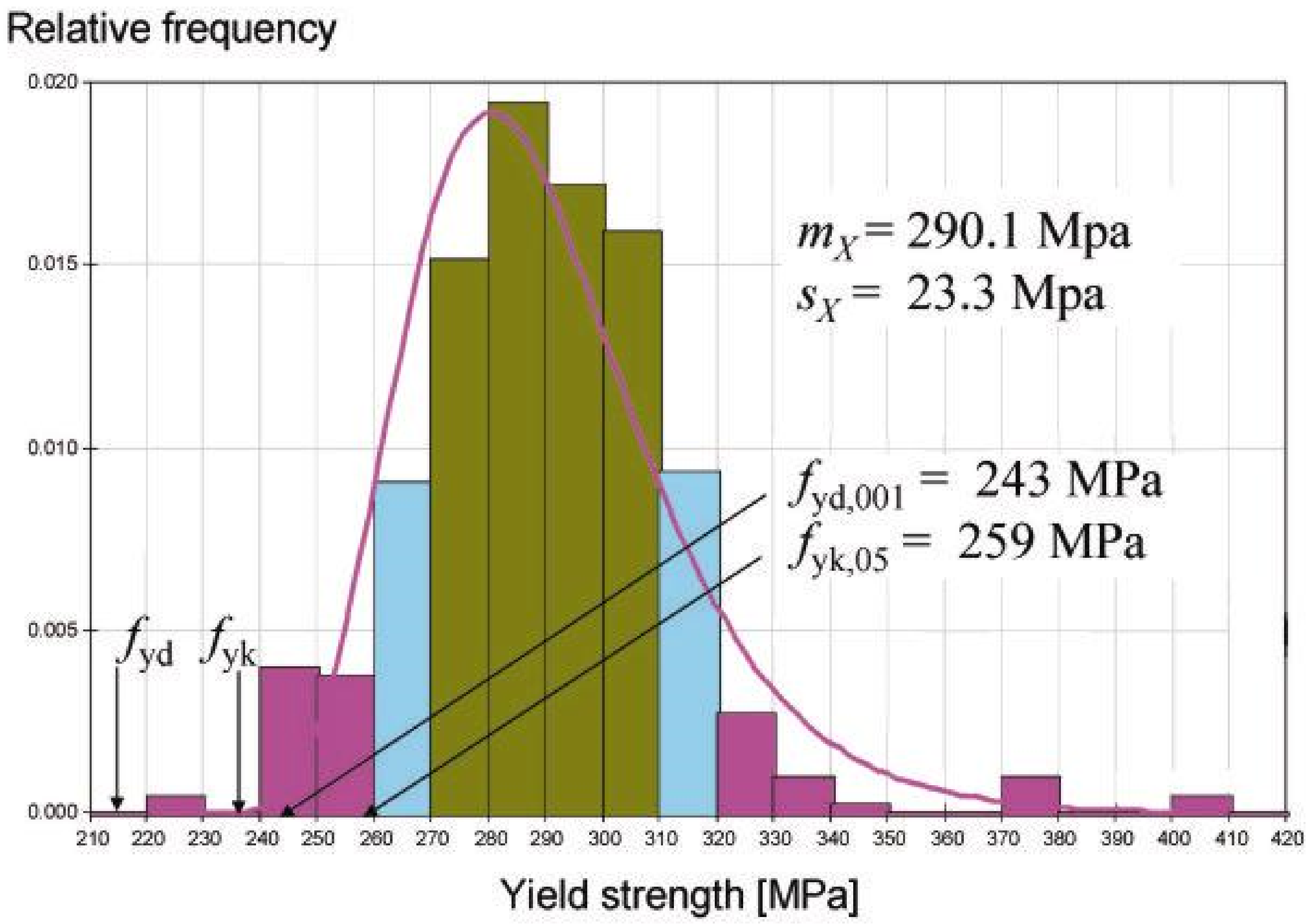
μ ... **mean value**

$$\mu = \frac{1}{n}\sum_{i=1}^{n} \ln(x_i)$$

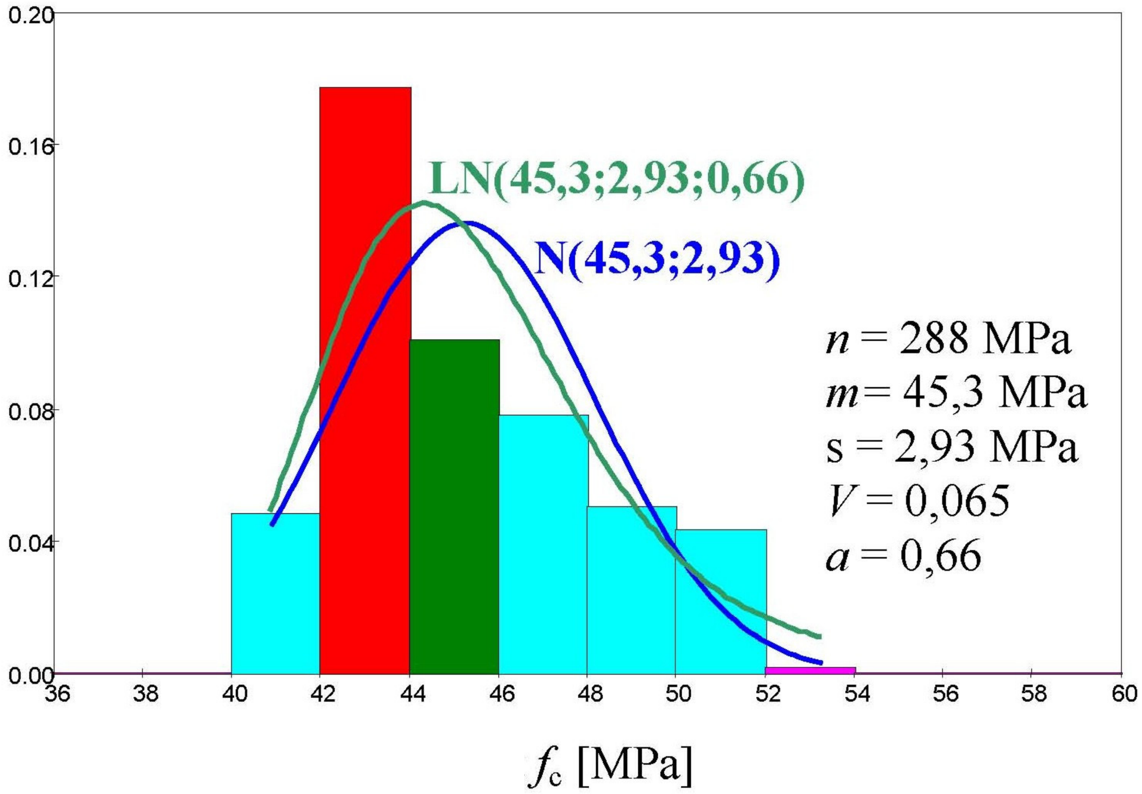
σ ... **standard deviation**

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (\ln(x_i) - \mu)^2}$$

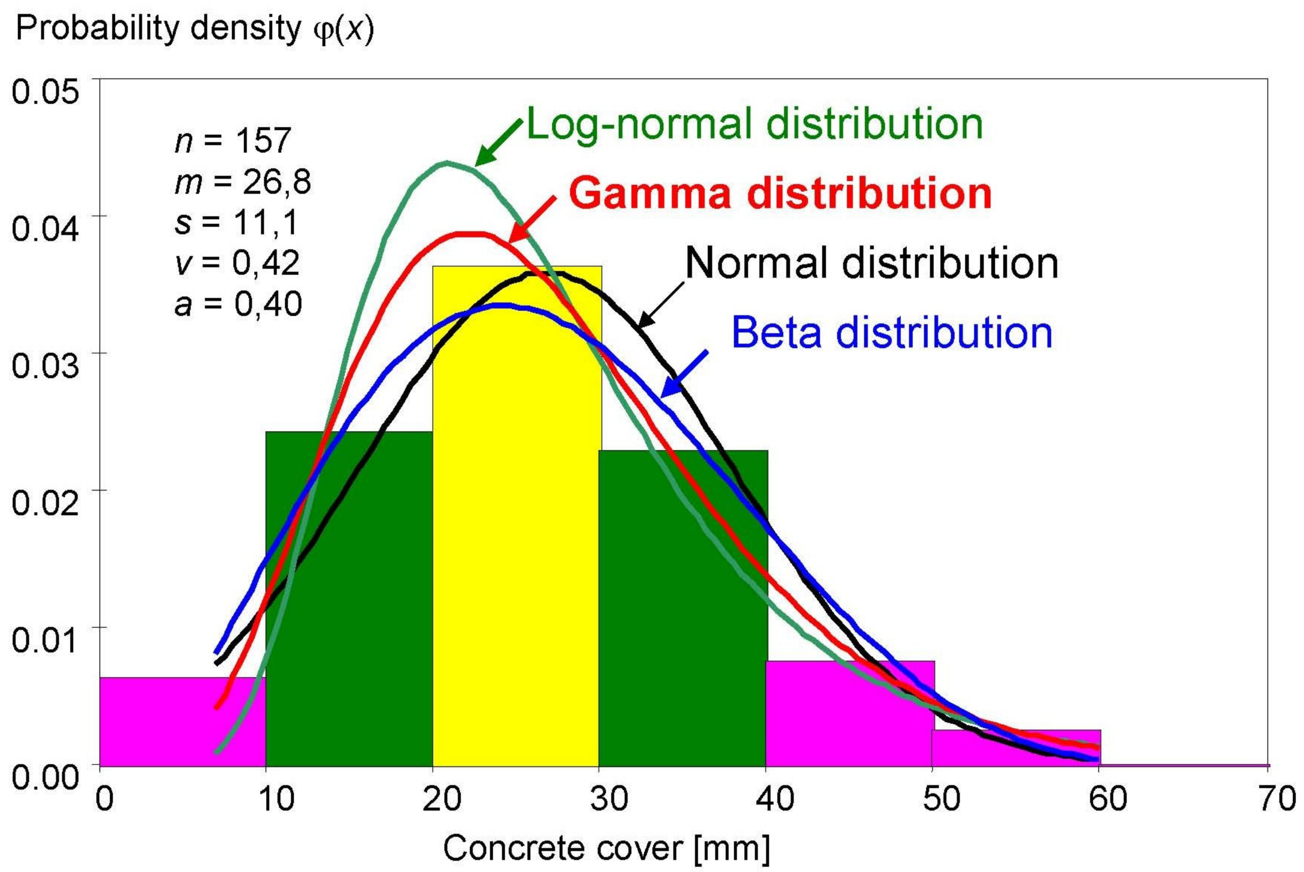
Yield stress of the S235 steel



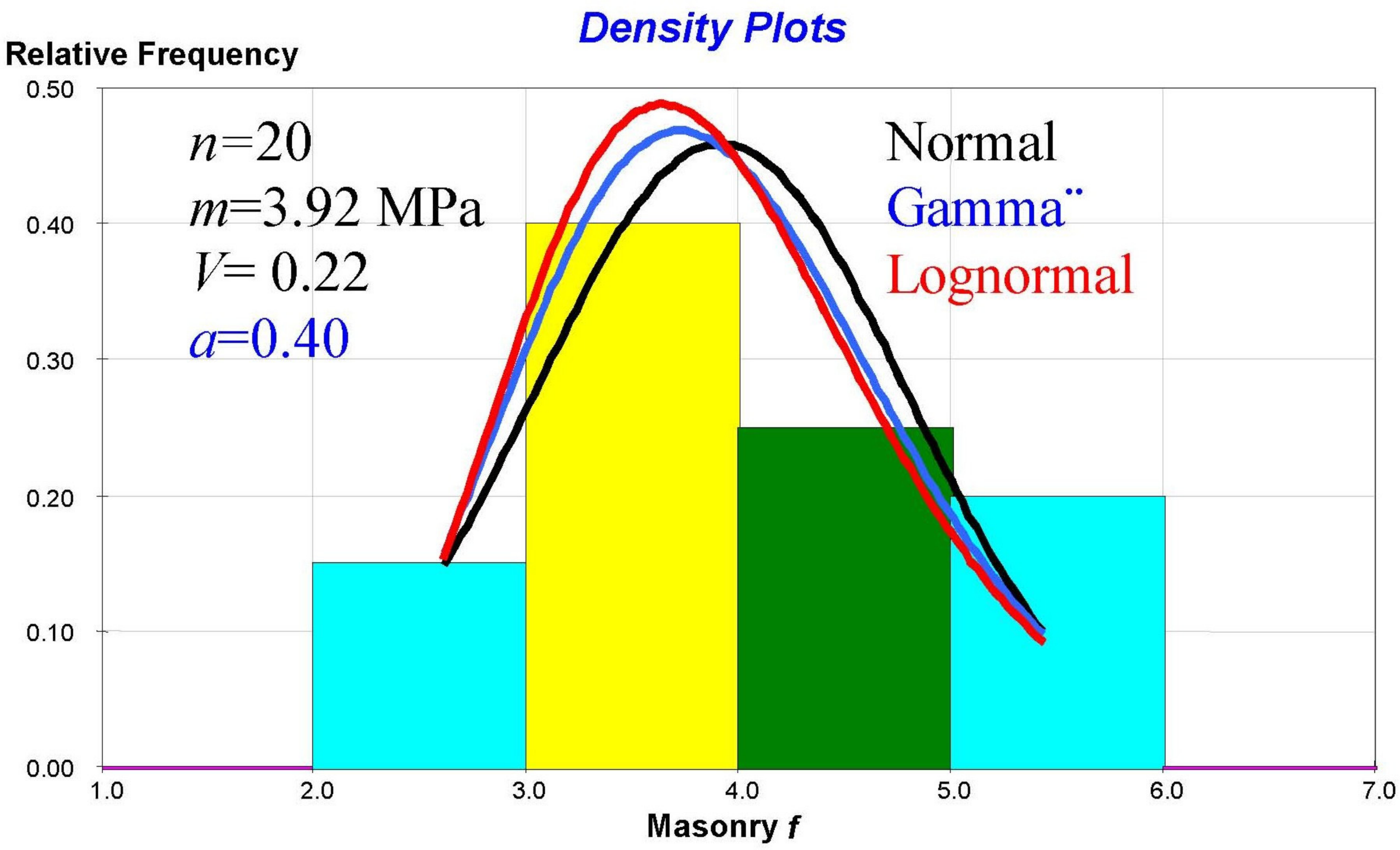
Compressive strength of concrete



Concrete cover layer thickness



Compressive strength of masonry



Parametric probability distributions

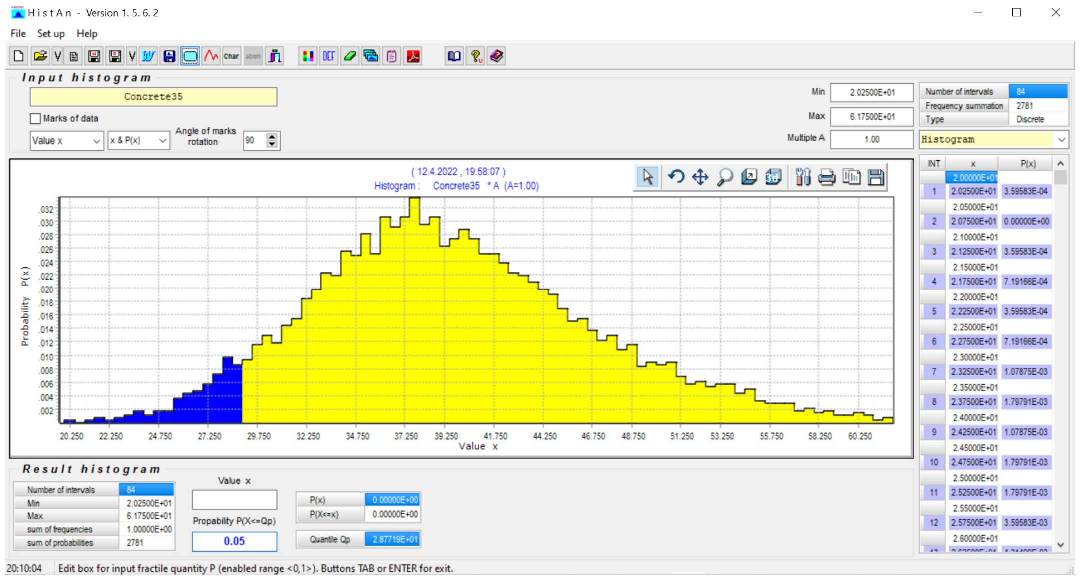
Distribution	Abbreviation	PDF	Parameters
Kappa	KAP	$F(x)=\left\{1-\gamma_2\left[1-\gamma_1(x-\alpha)/\beta\right]^{1/\gamma_1}\right\}^{1/\gamma_2}$ $f(x)=\beta^{-1}\left[1-\gamma_1(x-\alpha)/\beta\right]^{(1/\gamma_1)-1}\times[F(x)]^{1-\gamma_2},\ \beta>0$	4
Generalized extreme value type III	GEV	$f(x)=\frac{1}{\beta}\left(1+\gamma\frac{x-\alpha}{\beta}\right)^{-1/\gamma-1}\exp\left[-\left(1+\gamma\frac{x-\alpha}{\beta}\right)^{-1/\gamma}\right]$	3
Generalized Logistic	GLO	$f(x)=\frac{\gamma\exp\left(-\frac{x-\alpha}{\beta}\right)}{\beta\left(1+\exp\left(-\frac{x-\alpha}{\beta}\right)\right)^{\gamma+1}}$	3
Generalized Pareto	GPA	$f(x)=\frac{1}{\beta}\left(1+\frac{\gamma(x-\alpha)}{\beta}\right)^{-1/\gamma-1}$	3
Lognormal	LN3	$f(x)=\frac{1}{(x-\gamma)\sqrt{2\pi}\beta}\exp\left[-\frac{1}{2}\left(\frac{\ln(x-\gamma)-\alpha}{\beta}\right)^2\right]$	3
Pearson Type III	P3	$f(x)=\frac{1}{\beta^\gamma\Gamma(\gamma)}(x-\alpha)^{\gamma-1}\exp\left(-\frac{x-\alpha}{\beta}\right)$	3
Exponential	E	$f(x)=\begin{cases}\lambda\exp(-\lambda x), & x\geq 0\\ 0, & x<0\end{cases}$	2
Gumbel	G	$f(x)=\frac{1}{\beta}\exp\left[\frac{x-\alpha}{\beta}-\exp\left(\frac{x-\alpha}{\beta}\right)\right]$	2
Normal	N	$f(x)=\frac{1}{\sqrt{2\pi\beta^2}}\exp\left(-\frac{(x-\alpha)^2}{2\beta^2}\right)$	2
Logistic	L	$f(x)=\frac{\exp\left(-\frac{x-\alpha}{\beta}\right)}{\beta\left(1+\exp\left(-\frac{x-\alpha}{\beta}\right)\right)^2}$	2
Uniform	U	$f(x)=\begin{cases}\frac{1}{b-a}, & a<x<b\\ 0, & x<a \text{ or } x>b\end{cases}$	1

Distribution	Abbreviation	PDF	Parameters
Gamma	G2	$f(x)=\frac{x^{\gamma-1}\exp\left(-\frac{x}{\beta}\right)}{\Gamma(\gamma)\beta^\gamma}$	2
Generalized Pareto	GP2	$f(x)=\frac{1}{\beta}\left(1+\frac{\gamma x}{\beta}\right)^{(-1/\gamma-1)}$	2
Lognormal	LN2	$f(x)=\frac{1}{x\beta\sqrt{2\pi}}\exp\left(-\frac{(\ln x-\alpha)^2}{2\beta^2}\right)$	2
Weibull	W2	$f(x)=\frac{\gamma}{\beta}\left(\frac{x}{\beta}\right)^{\gamma-1}\exp\left[-\left(\frac{x}{\beta}\right)^\gamma\right],\ x\geq 0$	2

HistAn software tool

Program for more detailed **analysis of input histograms**:

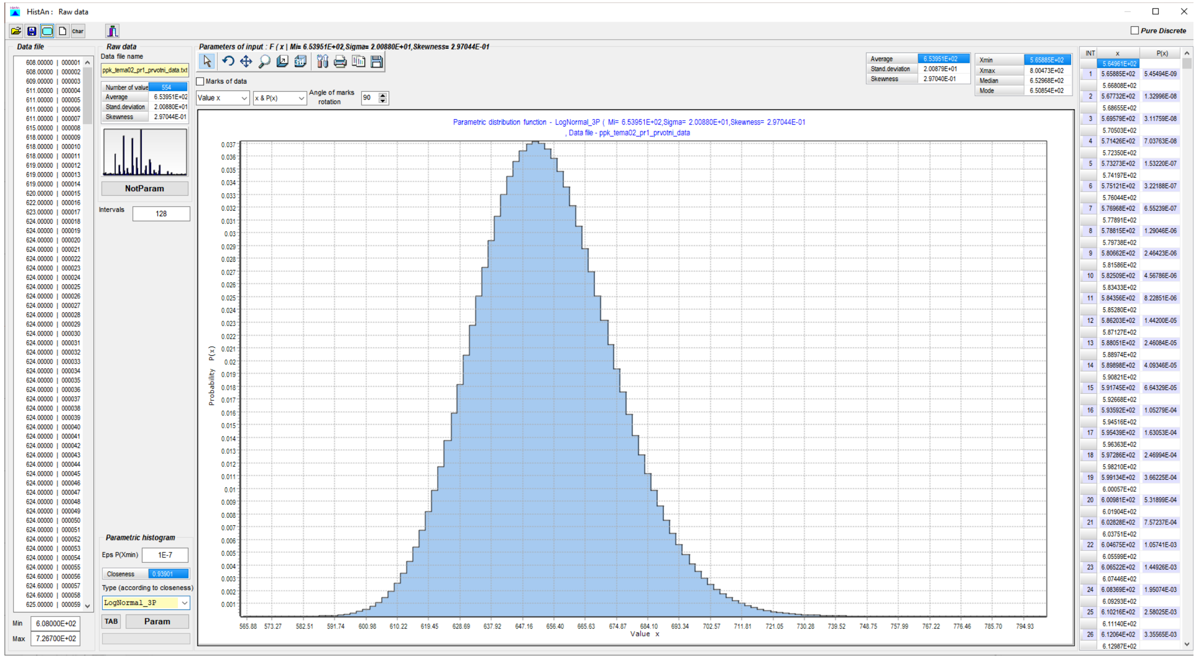
- **Minimum** and **maximum values** of a random variable
- **Number of histogram classes** (intervals) and frequencies defined in them
- **Simple probabilistic calculations** with histograms (determination of p-quantile and probability of exceedingly the determined value of a random variable)
- Determining the **combination of several input histograms**
- Creation of **histograms with parametric distribution**
- Processing of **measured raw data**



Desktop of the **HistAn software tool**

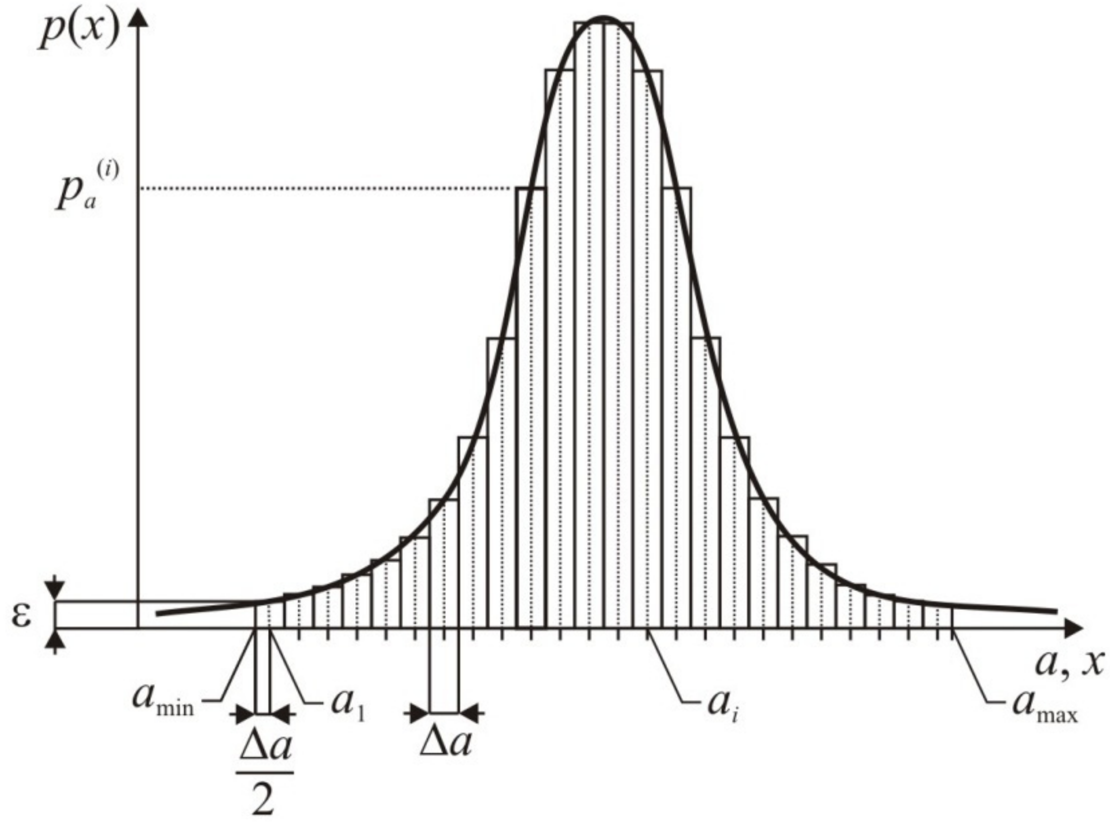
Parametric distributions in HistAn software tool

- Implementation of a module for entering set of measured data and for their evaluation.
- Creation of histograms with non-parametric and parametric distribution (23 types) with the possibility of choosing the number of intervals.
- Choice of appropriate probability distribution according to **coefficient of determination**.



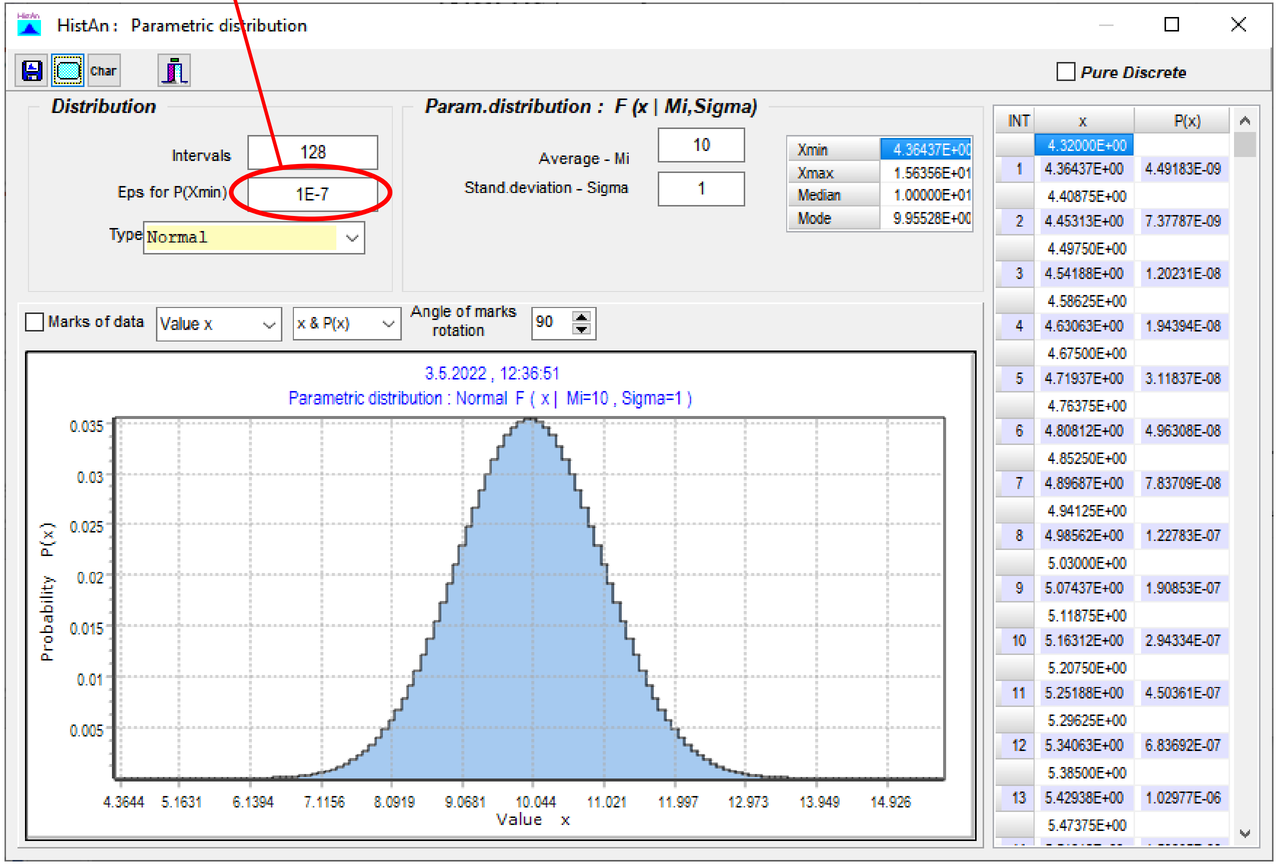
Histogram of random variable

Histogram of discretized continuous random variable with parametric probability distribution



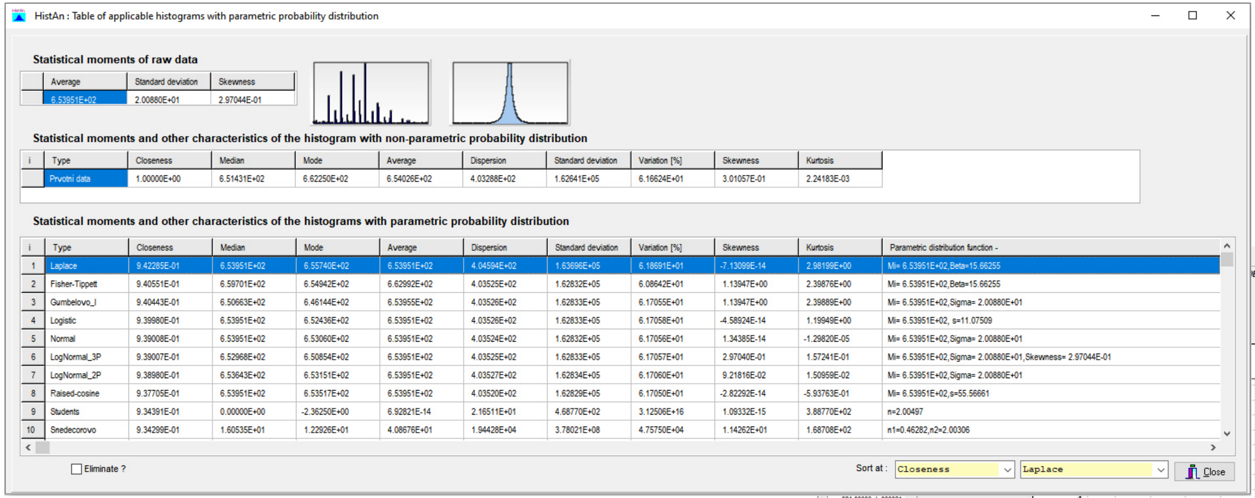
Parametric distributions in HistAn

Probability for cutting of the probability distribution

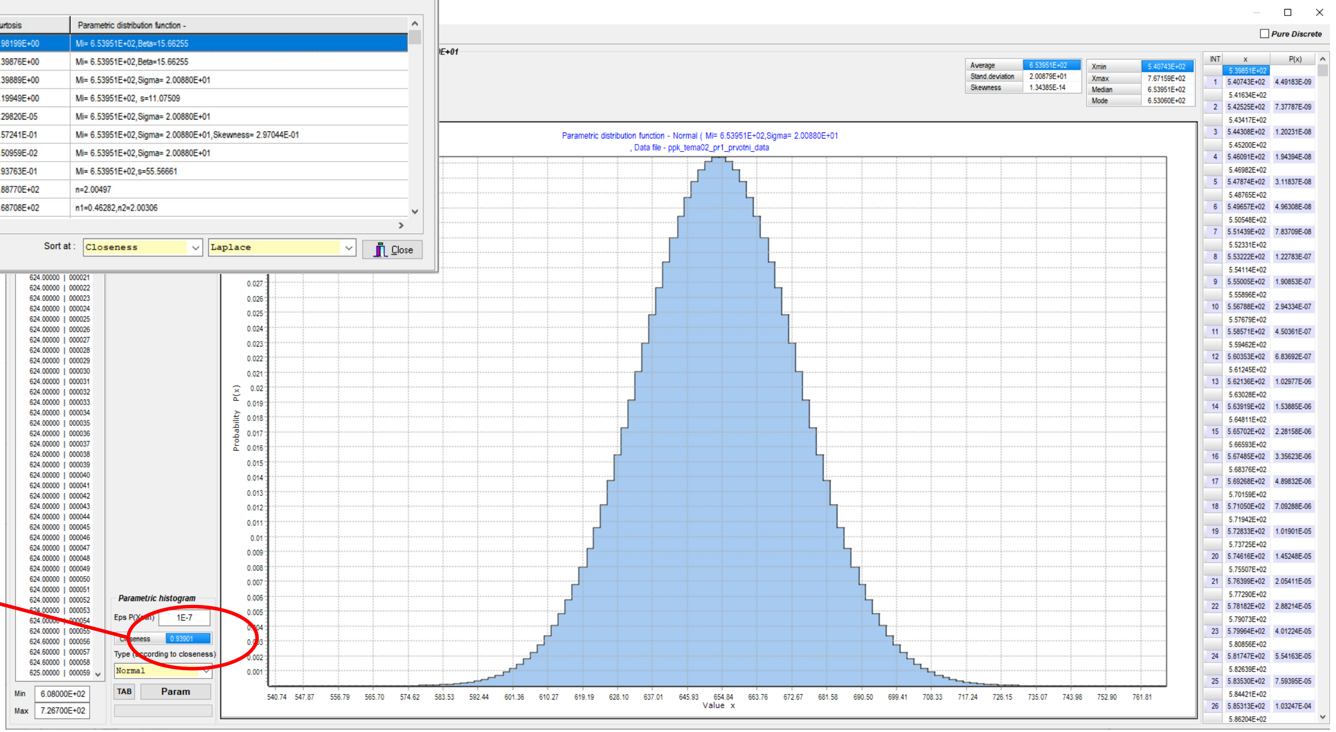


- Normal
- Log-Normal
- Gumbel I and II
- Raised-Cosine
- Cauchy
- Fischer-Tippett
- Laplace
- Logistic
- Weibull
- Rayleigh
- Lévy
- Student
- Beta
- Gama
- Snedecor's F distribution
- Pareto
- Uniform
- Triangular
- Exponential
- Chi-square
- Half-Logistic

Parametric distributions in HistAn



Characteristics of derived parametric data



Selection of a suitable distribution according to the **coefficient of determination**

Coefficient of determination

$$\frac{s_Y^2}{s_y^2} = 1 - \frac{s_{y,x}^2 + \frac{2}{n}.\sum_i (y_i - Y_i).(Y_i - \bar{y})}{s_y^2}$$

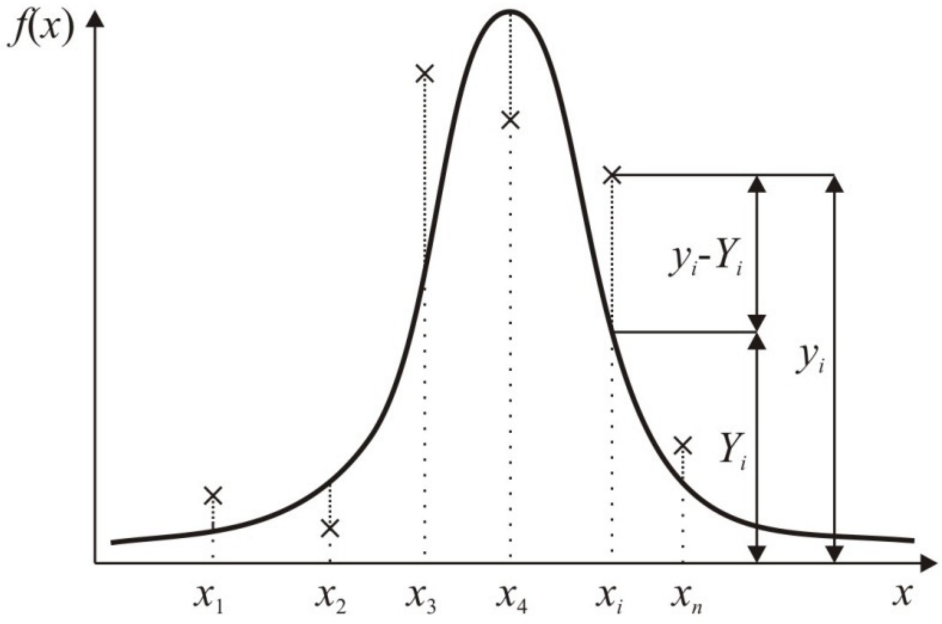
$$\frac{s_Y^2}{s_y^2} \in \langle 0,1 \rangle$$

variances for n intervals

$$s_y^2 = \frac{1}{n}.\sum_i (y_i - \bar{y})^2$$

$$s_Y^2 = \frac{1}{n}.\sum_i (Y_i - \bar{y})^2$$

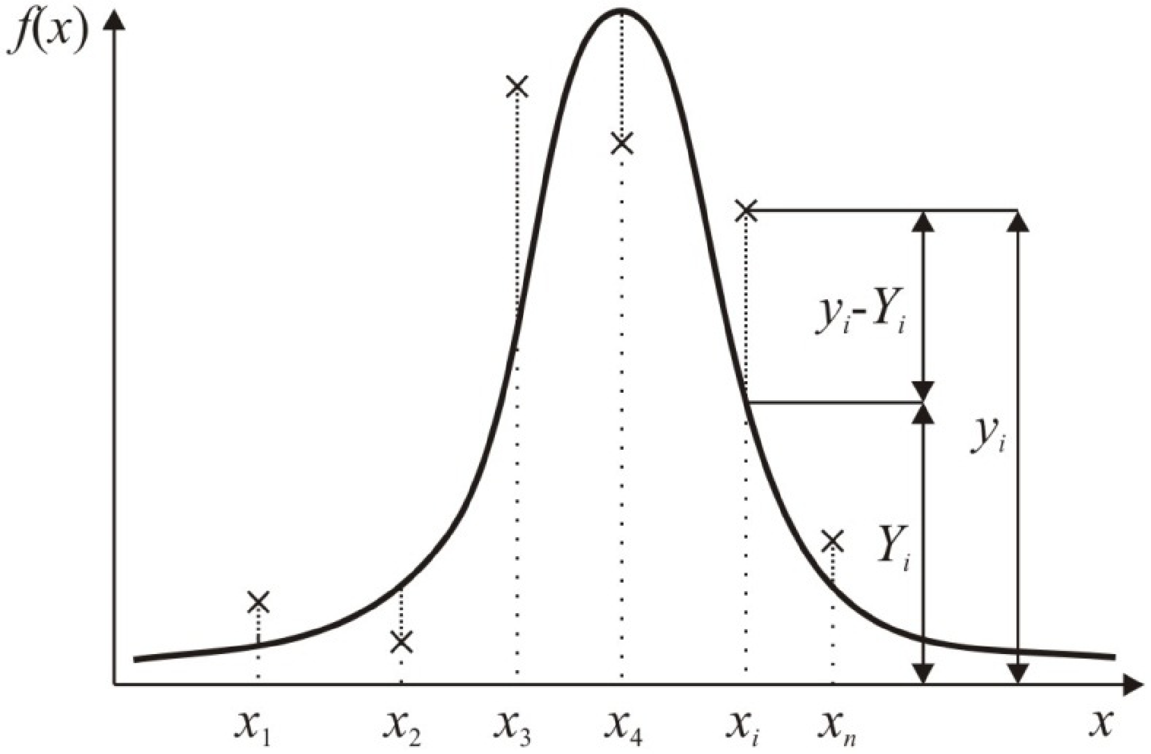
$$s_{y,x}^2 = \frac{1}{n}.\sum_i (y_i - Y_i)^2$$



Y_i ... the value of the probability density function of the parametric distribution at the respective value x_i

$\bar{y}$ ... mean value from all y_i

Residual sum of squares



Scatter $$s_{y,x}^2 = \frac{1}{n} \cdot \sum_i (y_i - Y_i)^2$$

Desired minimum value

Y_i ... the value of the probability density function of the parametric distribution at the respective value x_i

Use of parametric distributions in HistAn