VSB TECHNICAL UNIVERSITY OF OSTRAVA | FACULTY OF CIVIL ENGINEERING | DEPARTMENT OF STRUCTURAL MECHANICS

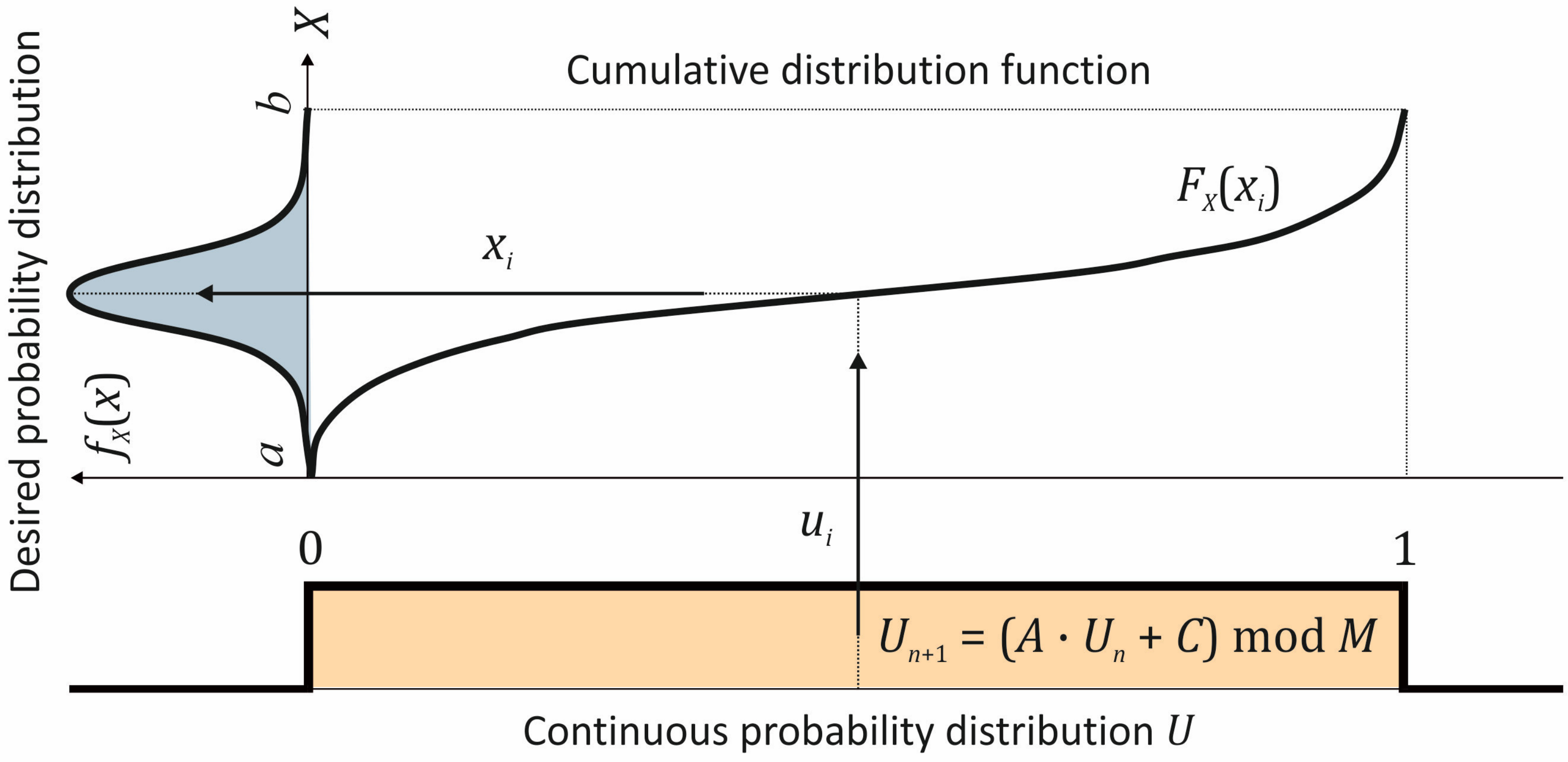
Topic 4:

Simulation Based Reliability Assessment (SBRA method)

- Introduction to SBRA method
- AntHill software
- Examples

The Principle of SBRA Method

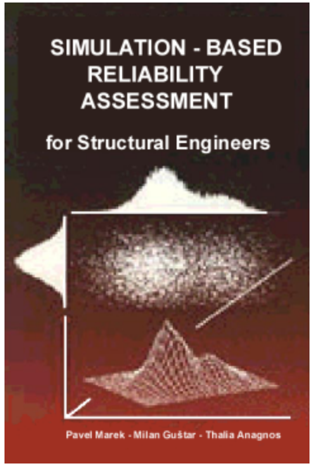
Generating limited distribution and transformation of the desired distribution



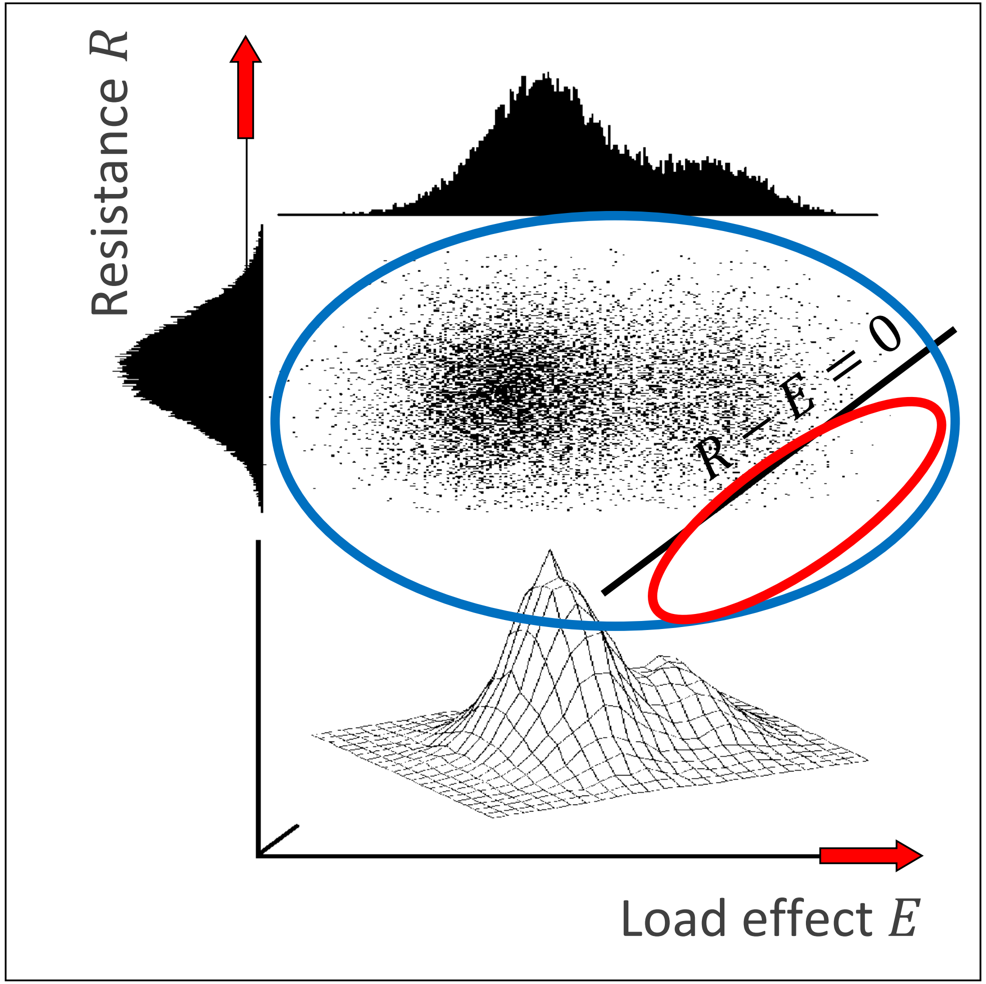
Reliability Assessment Using SBRA

- Input variables characterized using **bounded histograms** with nonparametric probability distribution.
- Analysis of the reliability function using **Monte Carlo method**.
- **Reliability** is expressed as $P_f < P_d$, where P_f is **probability of failure**, and P_d is **designed value** of failure probability:

$$P_f = \frac{\Sigma}{\Sigma} < P_d$$



Marek et al., CRC Press, 1995

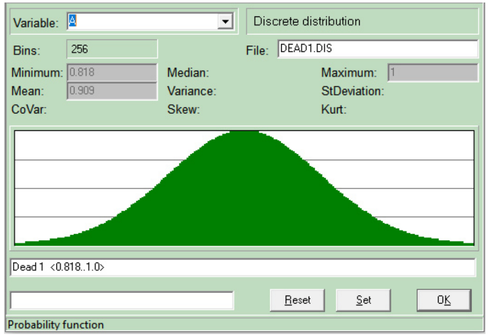


The Principle of SBRA Method

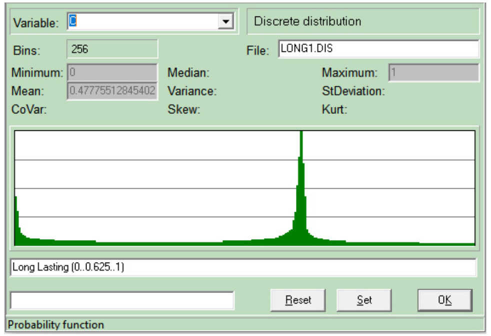
Variable **load** values, variability of cross-section and **strength** characteristics

Representation of random variables with **non-parametric histogram** - (empirical) probability distribution

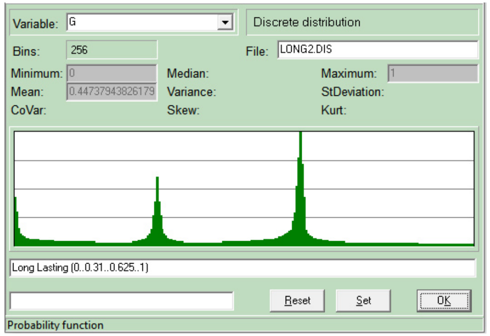
Dead load



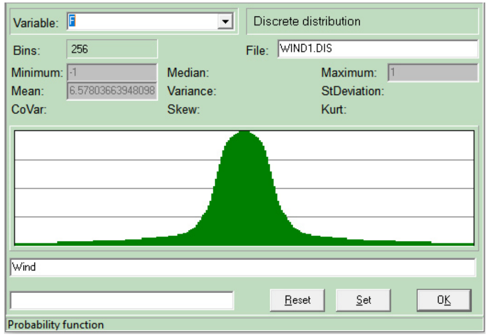
Long lasting load



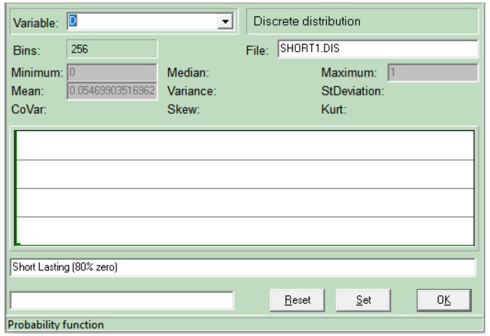
Long lasting load



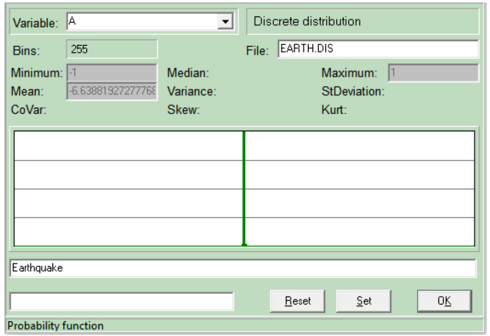
Wind load



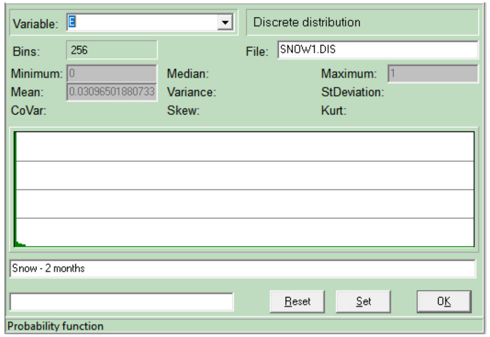
Short lasting load



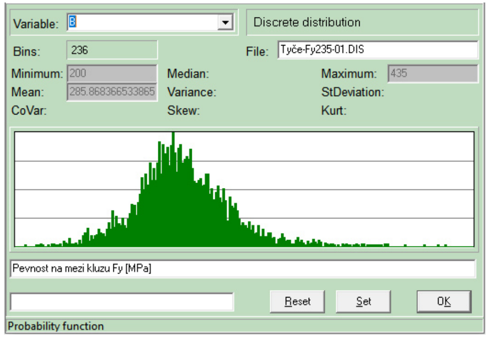
Earthquake load



Snow load

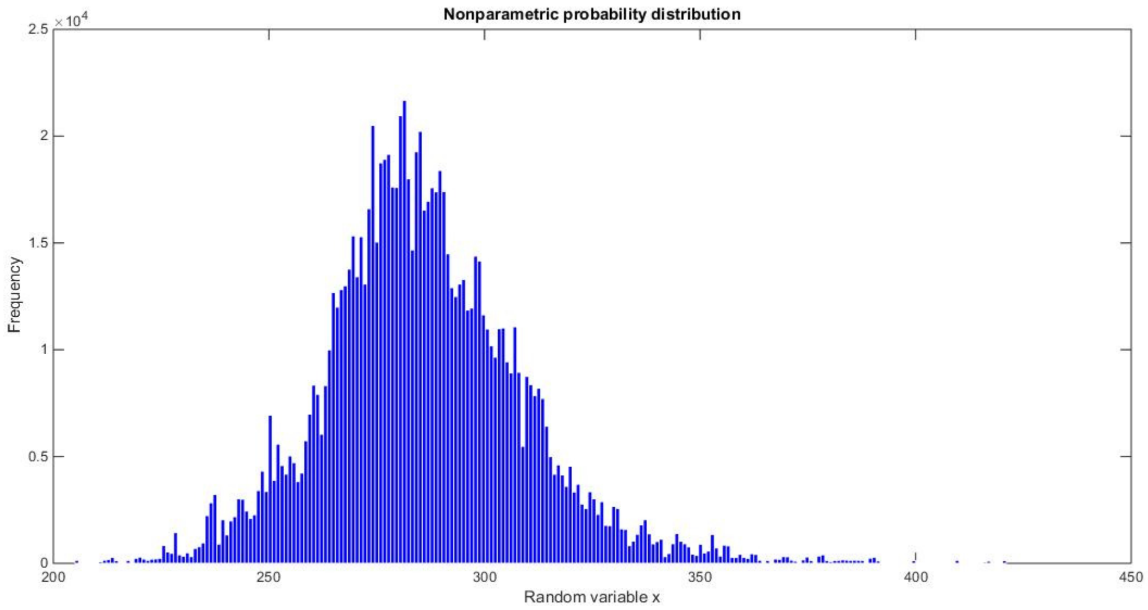


Strength of steel



The Principle of SBRA Method

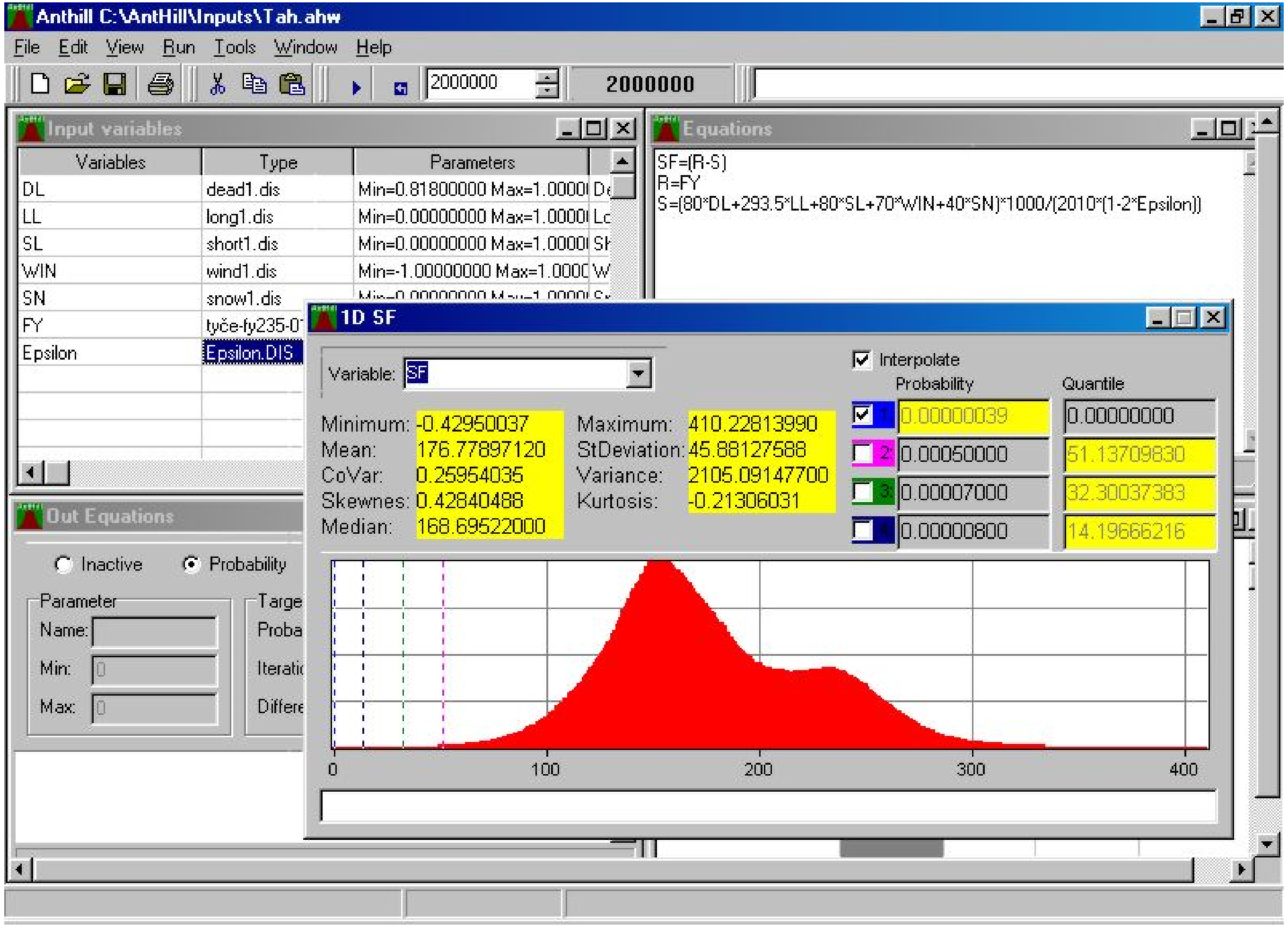
Strength of steel – **yield stress**, histogram with **non-parametric** (empirical) probability distribution




```
Inputs:
-------------------------------------------
Number of simulations N   =       1.0e+06
File name *.dis           =  T235FY01.DIS

Statistical parameters
-------------------------------------------
Minimal value             =       204.982
Maximal value             =       421.052
Range                     =       216.070
Mean value                =       286.026
Standard deviation        =        23.434
Scatter                   =       549.139
Coefficient of variation  =         8.19%
Skewness                  =         0.571
Kurtosis                  =         4.658
Median                    =       284.300
Quantile 5%               =       249.748
Percentile 5%             =       249.748
Probability(X<235)        =         0.94%
```

Calculation using SBRA, program AntHill

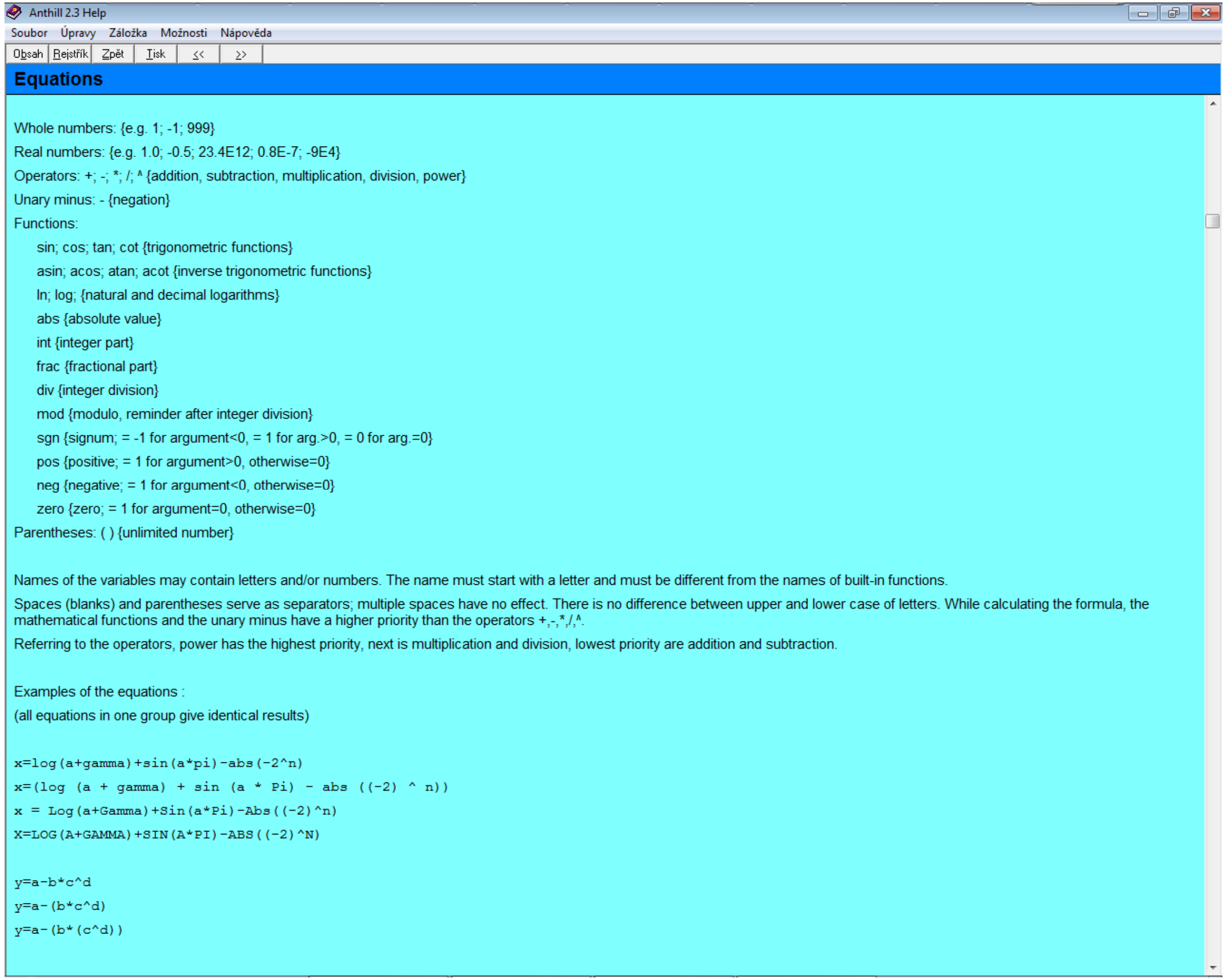


Desktop of the **AntHill** program

(Resulting histogram of reliability function with calculated probability of failure)

http://www.noise.cz/SBRA/software.html

Calculation using SBRA, program AntHill


Anthill 2.3 Help
Soubor Úpravy Záložka Možnosti Nápověda
Obsah Rejstřík Zpět Tisk << >>

Equations

Whole numbers: {e.g. 1; -1; 999}
Real numbers: {e.g. 1.0; -0.5; 23.4E12; 0.8E-7; -9E4}
Operators: +; -; *; /; ^ {addition, subtraction, multiplication, division, power}
Unary minus: - {negation}
Functions:
- sin; cos; tan; cot {trigonometric functions}
- asin; acos; atan; acot {inverse trigonometric functions}
- ln; log; {natural and decimal logarithms}
- abs {absolute value}
- int {integer part}
- frac {fractional part}
- div {integer division}
- mod {modulo, reminder after integer division}
- sgn {signum; = -1 for argument<0, = 1 for arg.>0, = 0 for arg.=0}
- pos {positive; = 1 for argument>0, otherwise=0}
- neg {negative; = 1 for argument<0, otherwise=0}
- zero {zero; = 1 for argument=0, otherwise=0}

Parentheses: ( ) {unlimited number}

Names of the variables may contain letters and/or numbers. The name must start with a letter and must be different from the names of built-in functions.

Spaces (blanks) and parentheses serve as separators; multiple spaces have no effect. There is no difference between upper and lower case of letters. While calculating the formula, the mathematical functions and the unary minus have a higher priority than the operators +,-,*,/,^.

Referring to the operators, power has the highest priority, next is multiplication and division, lowest priority are addition and subtraction.

Examples of the equations :
(all equations in one group give identical results)

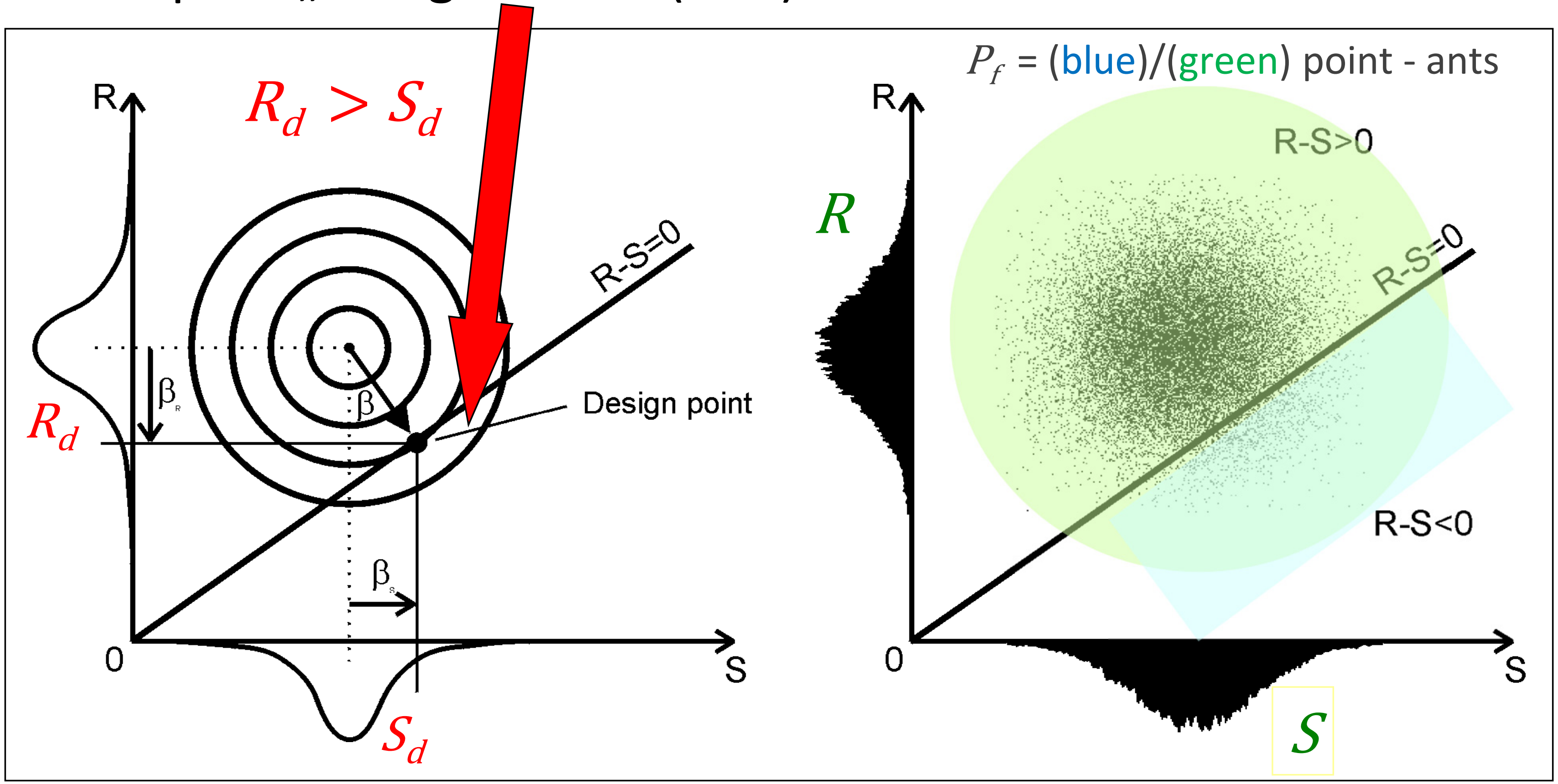
```
x=log(a+gamma)+sin(a*pi)-abs(-2^n)
x=(log (a + gamma) + sin (a * Pi) - abs ((-2) ^ n))
x = Log(a+Gamma)+Sin(a*Pi)-Abs((-2)^n)
X=LOG(A+GAMMA)+SIN(A*PI)-ABS((-2)^N)

y=a-b*c^d
y=a-(b*c^d)
y=a-(b*(c^d))
```

Desktop of the **AntHill** program
- help window
(Creating a mathematical model using arithmetic expressions and functions)

Concepts of Reliability Assessment

Concept of „Design Point“ (PFD) Probabilistic alternative

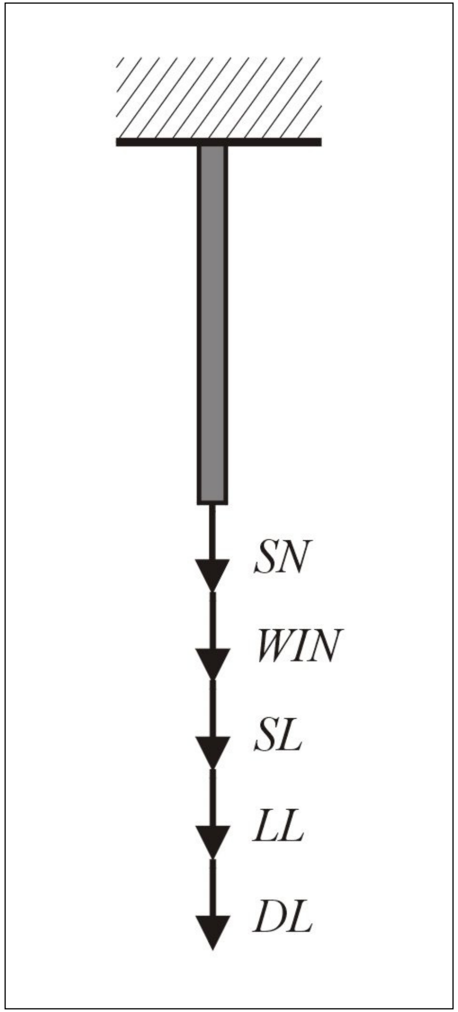


The Principle of SBRA Method

- Input random variables are expressed using **bounded histograms** with non-parametric (empirical) probability distribution,
- The failure probability P_f is obtained by analysis of the reliability function RF using **Monte Carlo simulation techniques**,
- The reliability is assessed on the basis of inequality $P_f < P_d$, where P_d is the **design failure probability** given by design standards such as EN 1990,
- The result is always different, important to select a **sufficient number of simulation steps**,
- **Universal method**, for more advanced calculations **is not effective**.

Example 1, Reliability Assessment

Expression and idealization of the structure under actual static or dynamic loads in space and time using mathematics-physical relationships determining the stress, strain, acceleration etc. from a time dependent load variable.



E.g.:

Reliability function RF:

$$RF = R - \text{abs}(E)$$

Structural resistance R (axial load capacity N_{Rd}) :

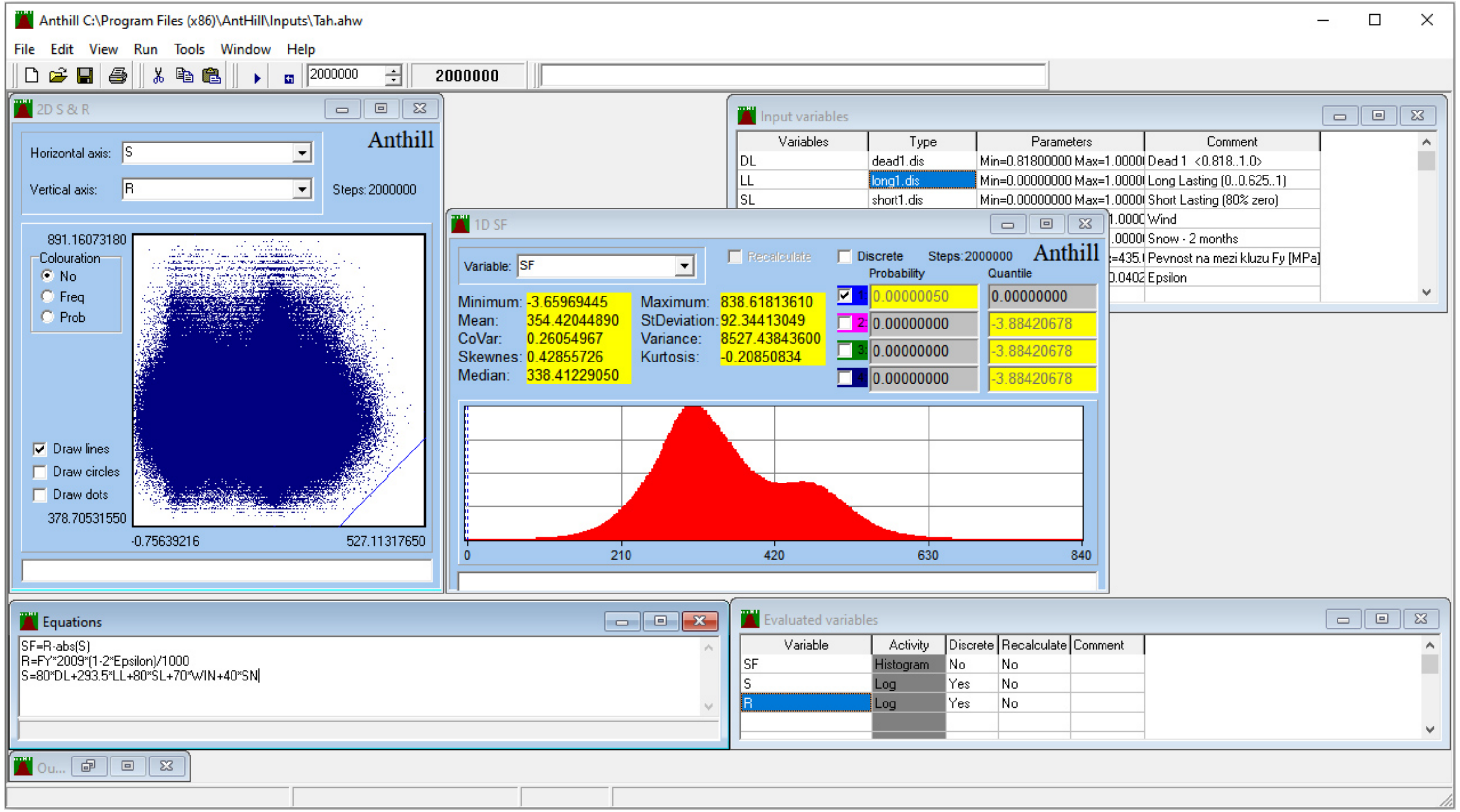
$$R = N_{Rd} = A_{var} \cdot f_y$$

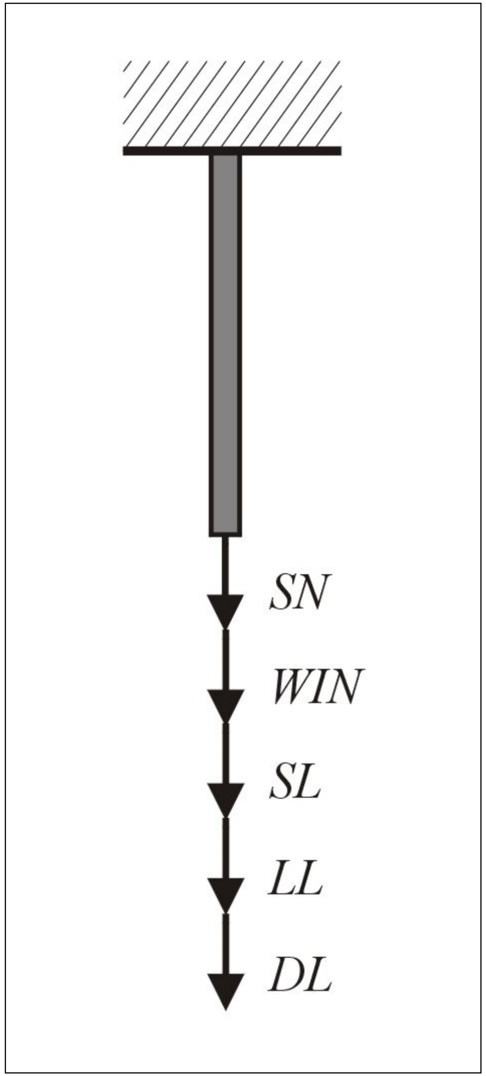
Load effect E (axial force N_{Ed}):

$$E = N_{Ed} = 80 \cdot DL + 293.5 \cdot LL + 80 \cdot SL + 70 \cdot WIN + 40 \cdot SN$$

Example 1, Computational Model

Desktop of the **AntHill** program

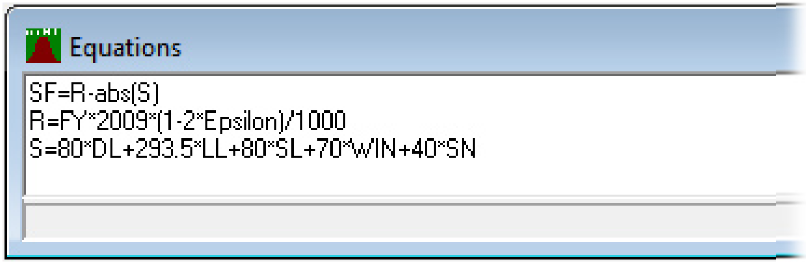




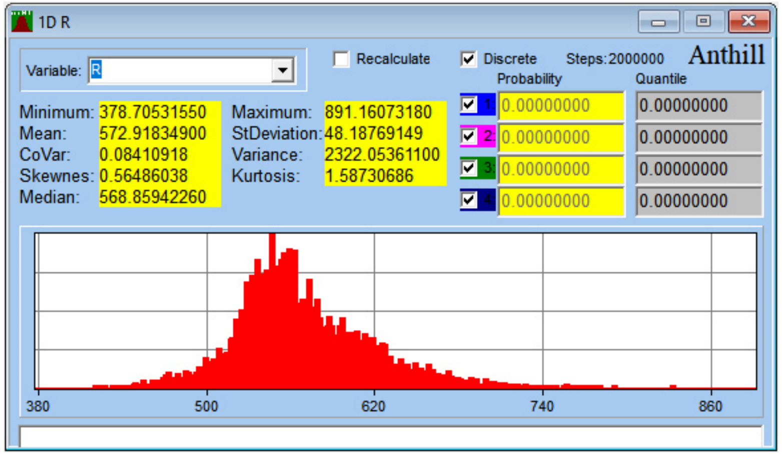
Example 1, Results

Computational model, definition

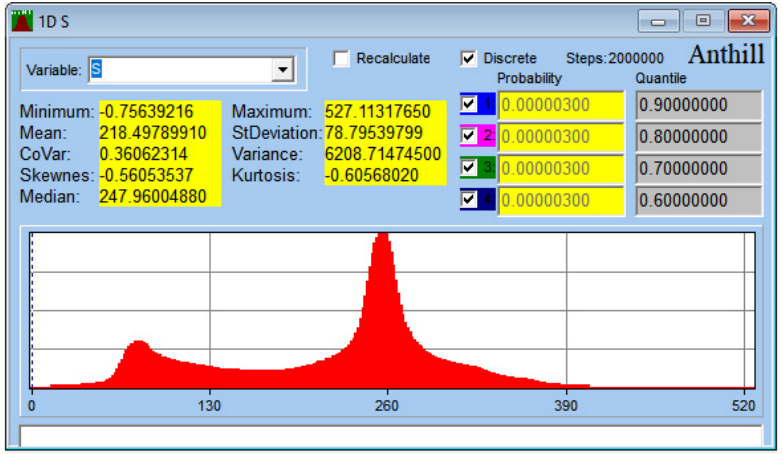
```
SF=R-abs(E)
R=FY*2009*(1-2*Epsilon)/1000
E=80*DL+293.5*LL+80*SL+70*WIN+40*SN
```



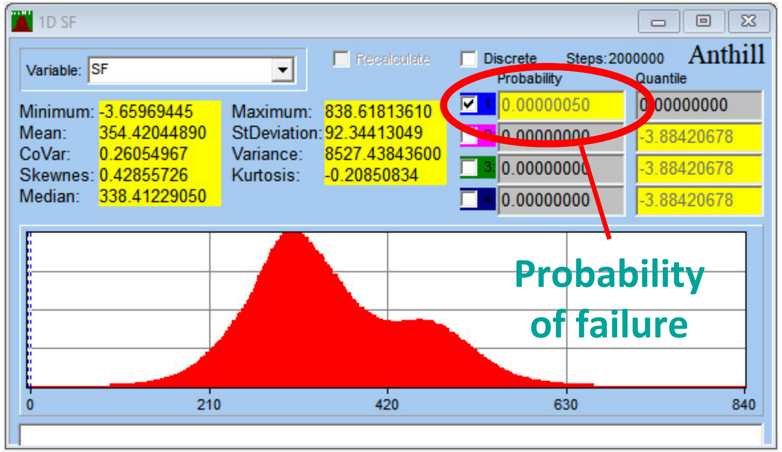
Resulting **histogram of structural resistance**

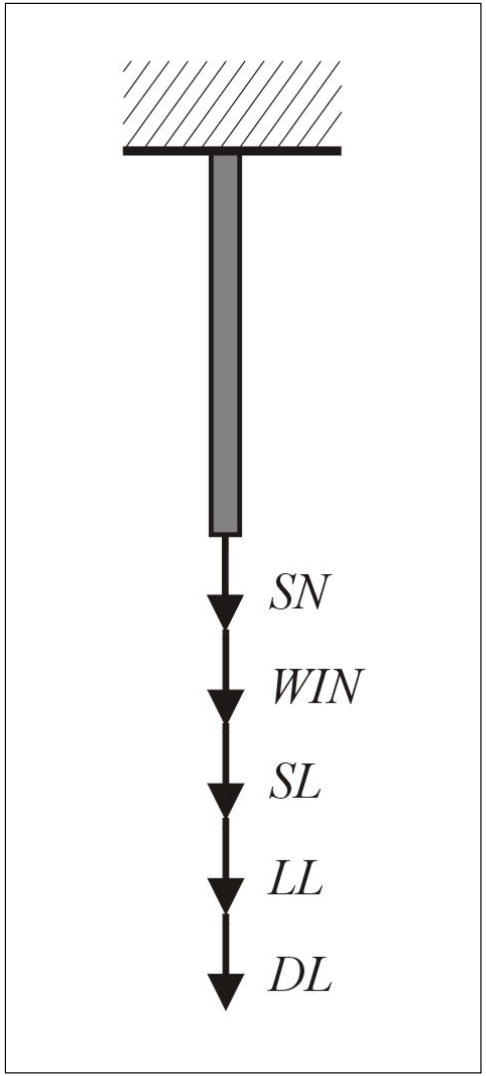


Resulting **histogram of load effect**



Resulting **histogram of reliability function**



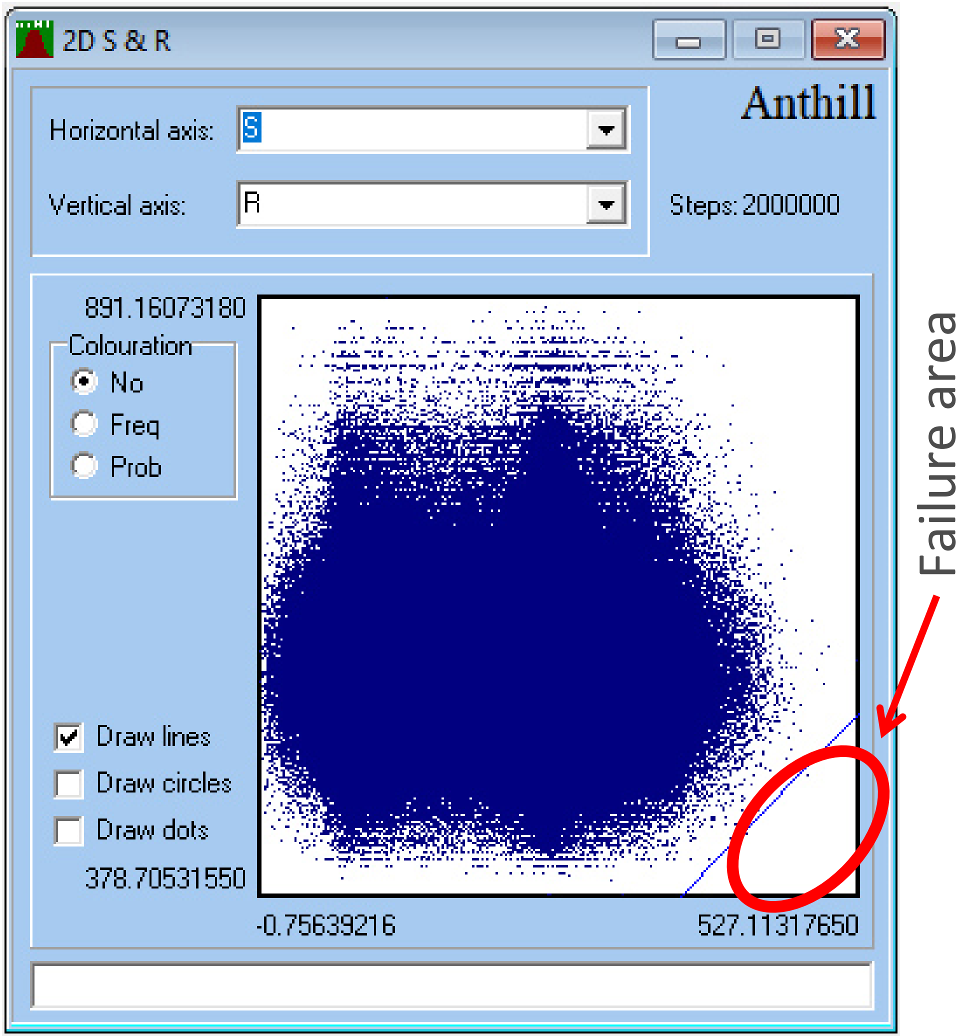


Example 1, Results

Summary:

- Input values are defined using **non-parametric bounded histograms**,
- Analysis of reliability function using **Monte Carlo simulation**,
- Reliability is expressed:

$$p_f = \frac{N_f}{N} \leq p_d$$

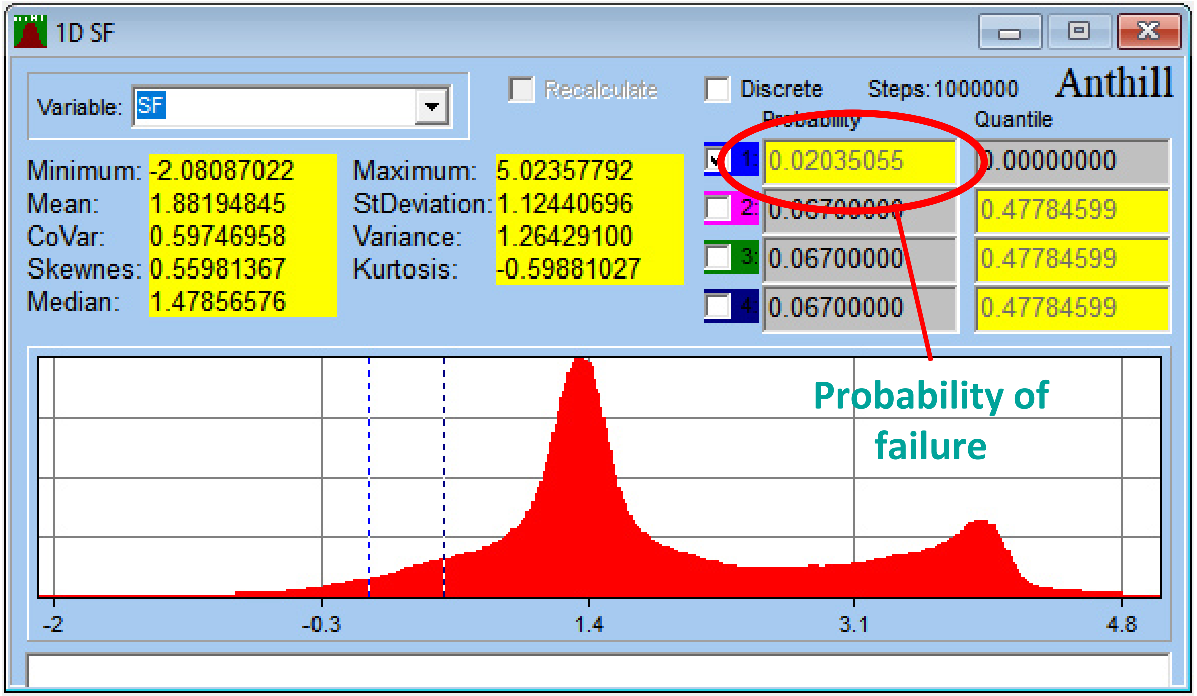


Example 1, Serviceability Limit States

Computational model, definition ($\delta_{lim} = 5$ mm)

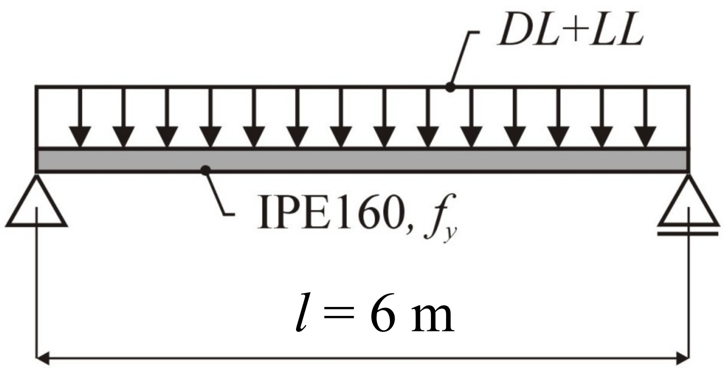
```
SF=5-DeltaL
DeltaL=F*1000*6/(210*Avar)
Avar=2009*(1-2*Epsilon)
F=(80*DL+293.5*LL+80*SL+70*WIN+40*SN)
```

Resulting **histogram of reliability function**



Example 2, Reliability Assessment

Mathematical model of probabilistic calculation:



Reliability function: $RF = R - E$

Structural resistance (ultimate bending moment):

$$R = M_{Rd} = W_{y,var} \cdot f_y$$

Load effect (maximal bending moment):

$$E = M_{Ed} = \frac{1}{8} \cdot (2.1 \cdot DL + 3.5 \cdot LL) \cdot l^2$$

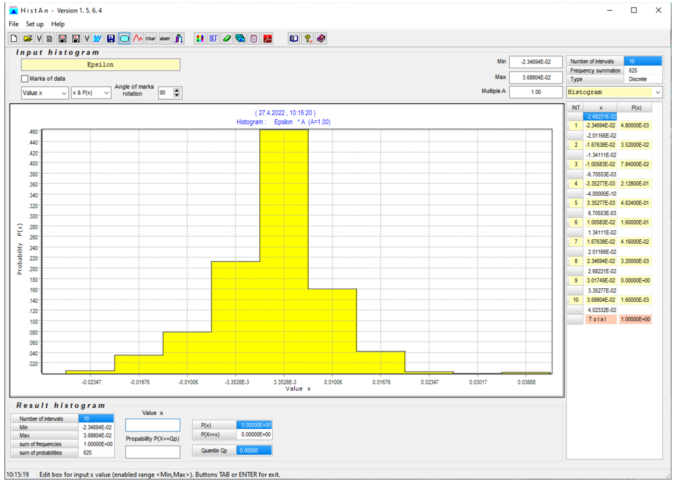
Cross-sectional variability:

$$A_{var} = A_{nom} \cdot (1 - 2 \cdot \varepsilon)$$

$$W_{var} = W_{nom} \cdot (1 - 3 \cdot \varepsilon)$$

$$I_{var} = I_{nom} \cdot (1 - 4 \cdot \varepsilon)$$

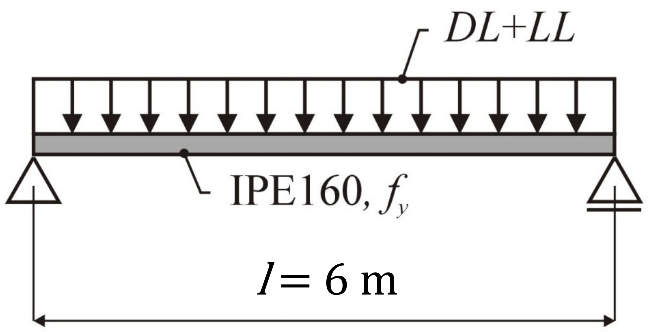
Histogram ε (Epsilon.dis) for expressing the variability of cross-sectional characteristics



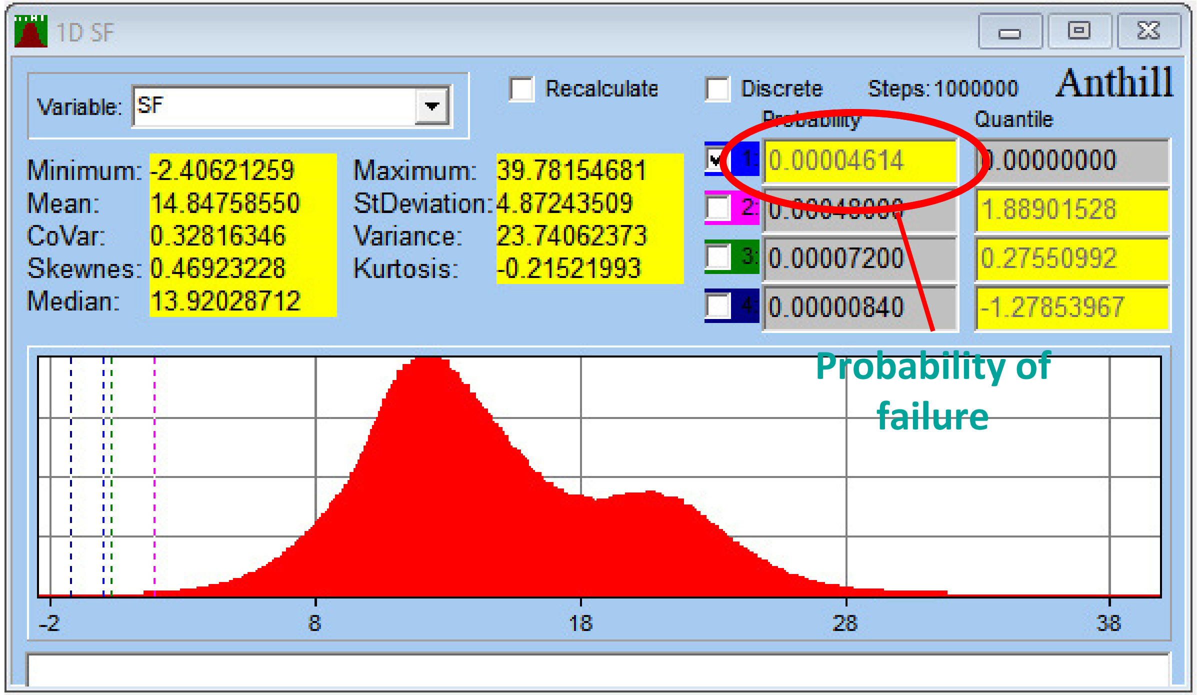
Example 2, Results

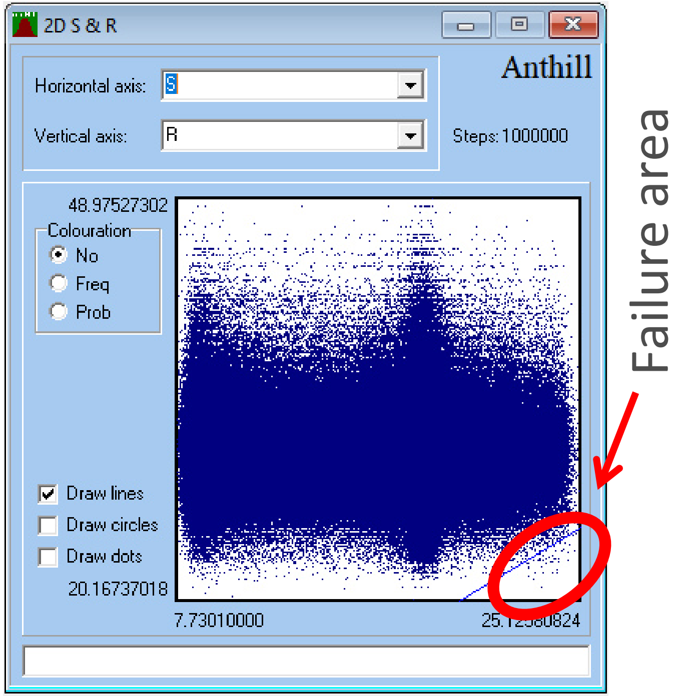
Computational model, definition

```
SF=R-E
R=Fy*108700*1e-6*(1-3*Eps)
E=1/8*(2.1*DL+3.5*LL)*Span^2
```

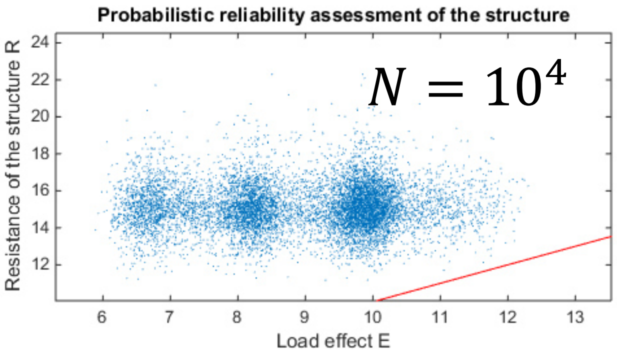


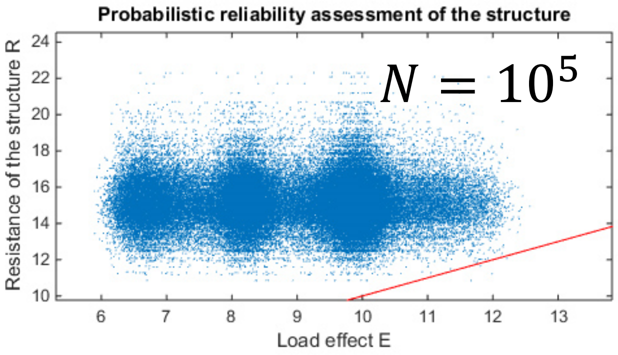
Resulting **histogram of reliability function**

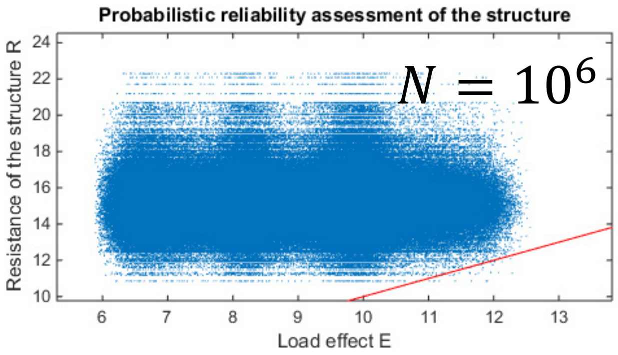




Example 2, Reliability Assessment of IPE160



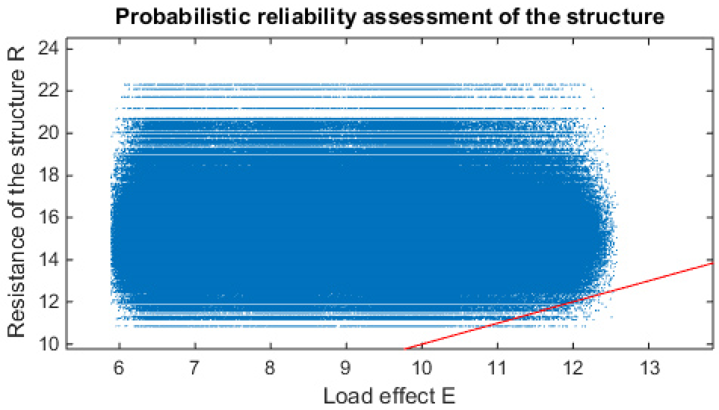




$$P_f = \frac{N_f}{N} = \frac{0}{10^4} = 0$$

$$P_f = \frac{N_f}{N} = \frac{4}{10^5} = 4 \cdot 10^{-5}$$

$$P_f = \frac{N_f}{N} = \frac{57}{10^6} = 5.7 \cdot 10^{-5}$$

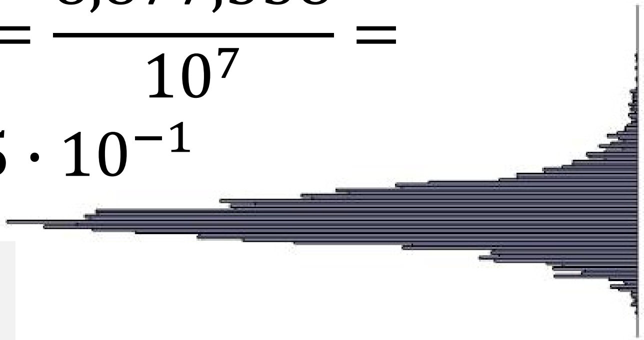


$$N = 10^7$$

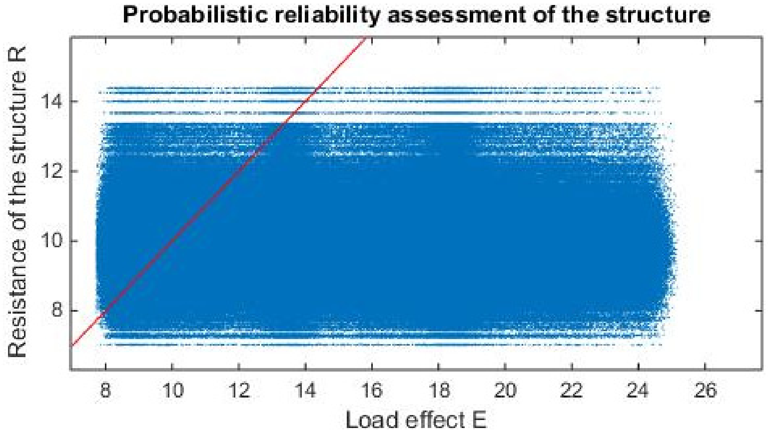
$$P_f = \frac{N_f}{N} = \frac{470}{10^7} = 4.7 \cdot 10^{-5}$$

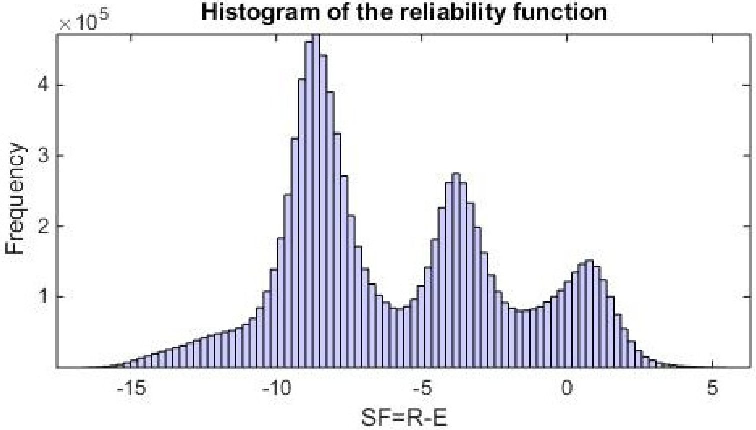
Example 2, Reliability Assessment of IPE100

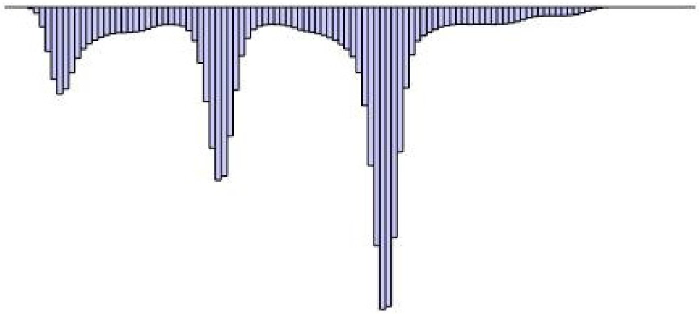
$$P_f = \frac{N_f}{N} = \frac{8{,}877{,}538}{10^7} = $$
$$= 8.8775 \cdot 10^{-1}$$



$$N = 10^7$$



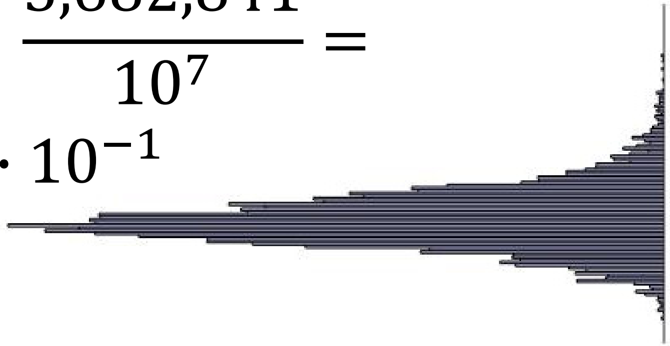


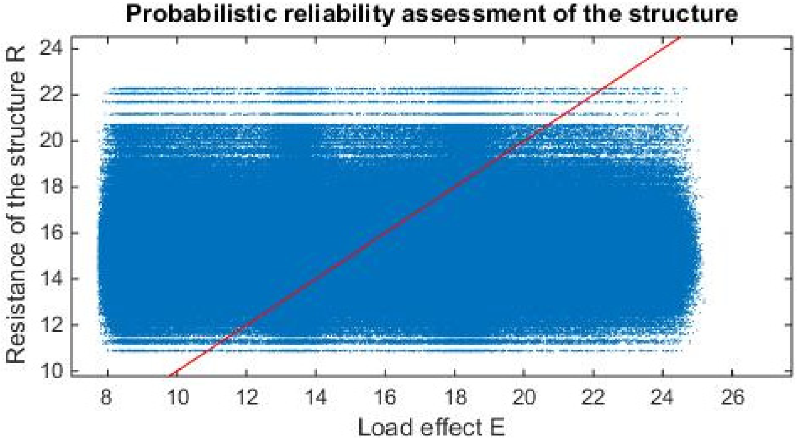


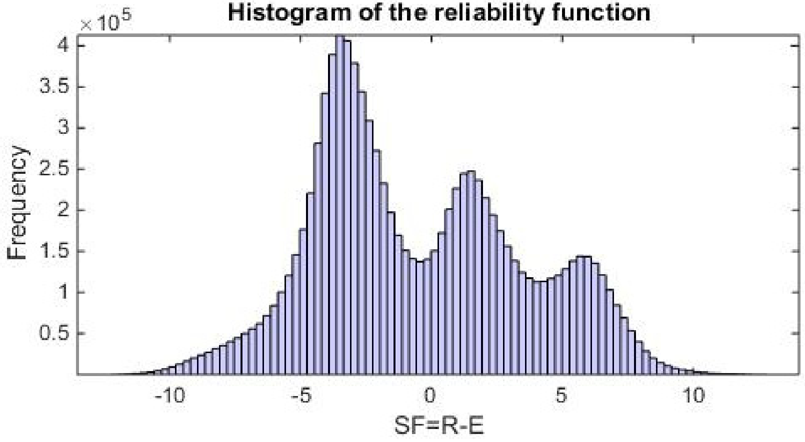
Example 2, Reliability Assessment of IPE120

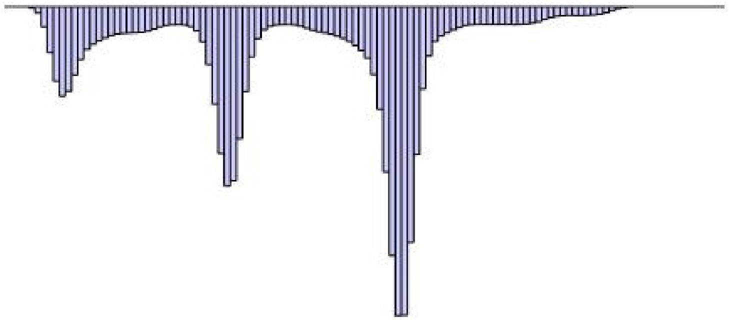
$$P_f = \frac{N_f}{N} = \frac{5{,}682{,}841}{10^7} = $$
$$= 5.6828 \cdot 10^{-1}$$

$$N = 10^7$$





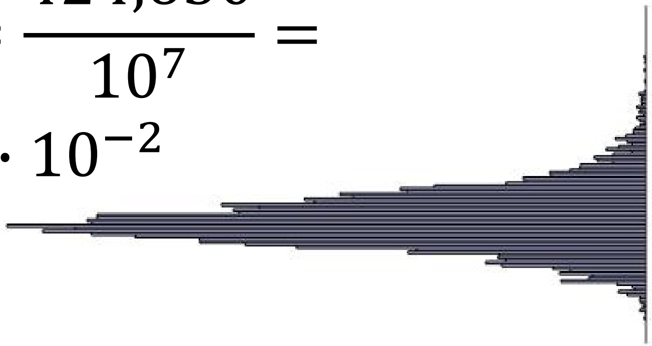


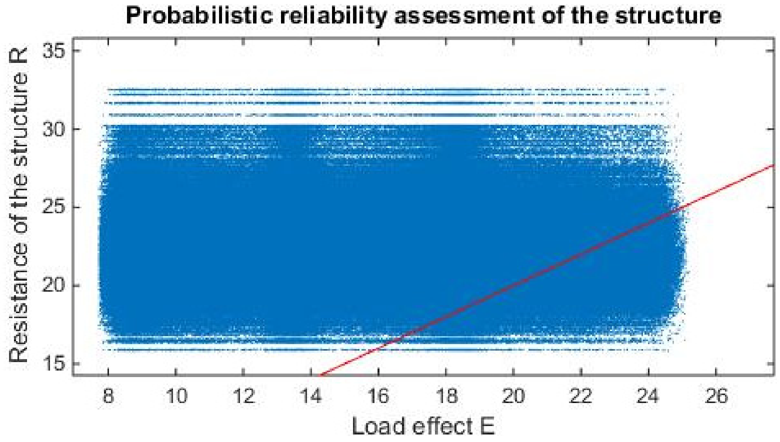


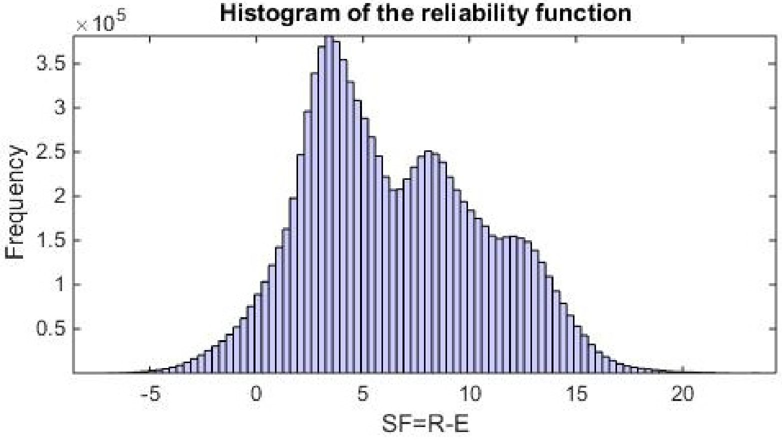
Example 2, Reliability Assessment of IPE140

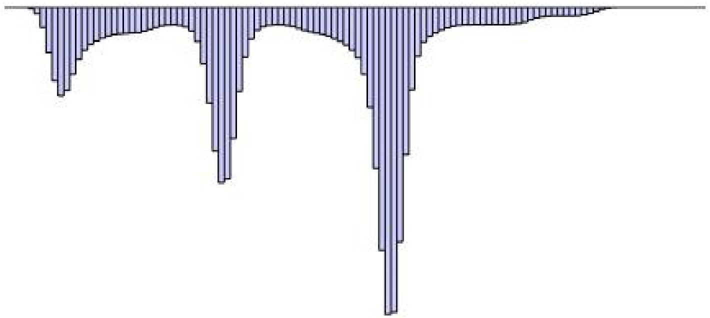
$$P_f = \frac{N_f}{N} = \frac{424{,}630}{10^7} = $$
$$= 4.2463 \cdot 10^{-2}$$

$$N = 10^7$$





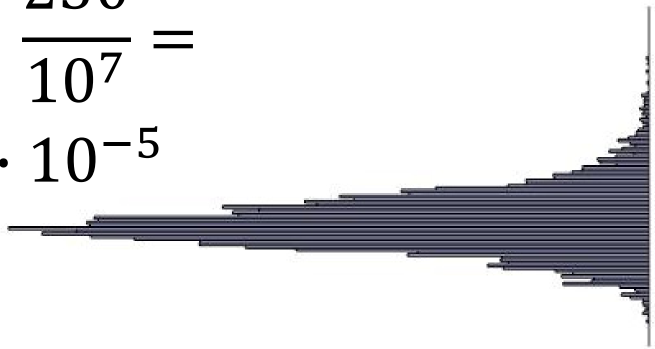


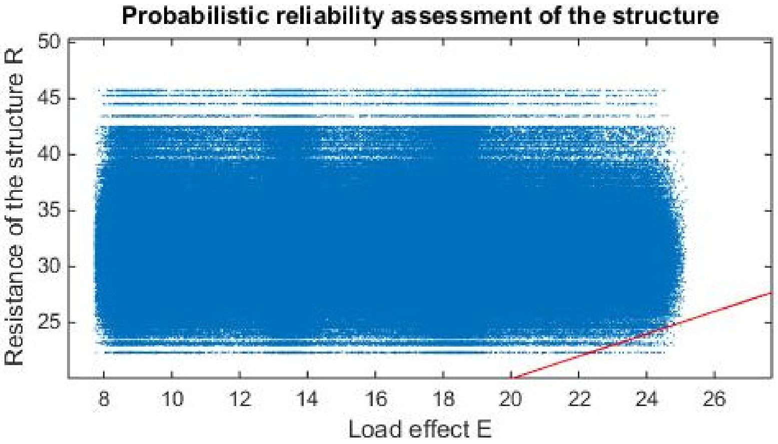


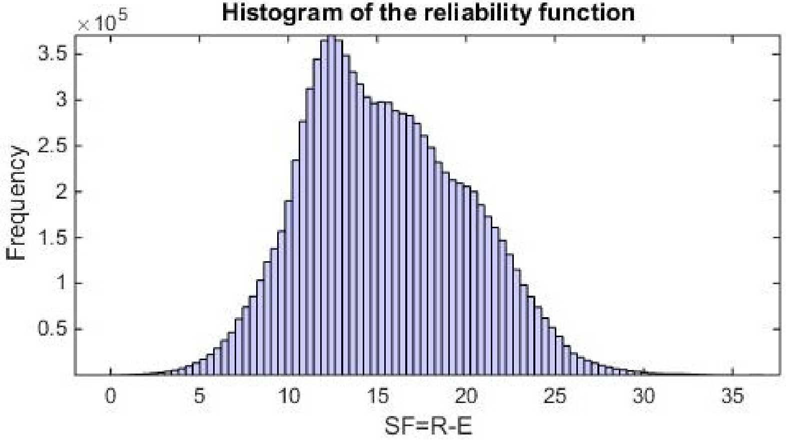
Example 2, Reliability Assessment of IPE160

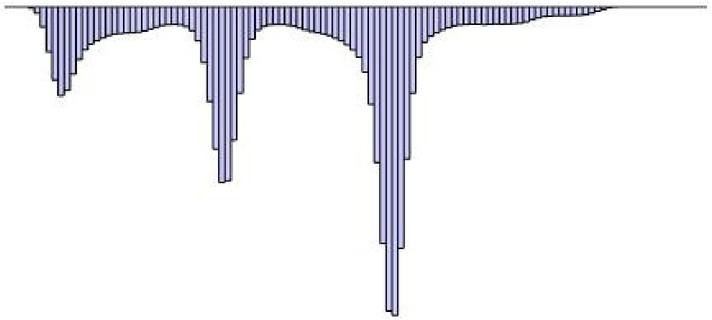
$$P_f = \frac{N_f}{N} = \frac{230}{10^7} = $$
$$= 2.3000 \cdot 10^{-5}$$

$N = 10^7$









Example 3, Reliability Assessment

Mathematical model of probabilistic calculation:

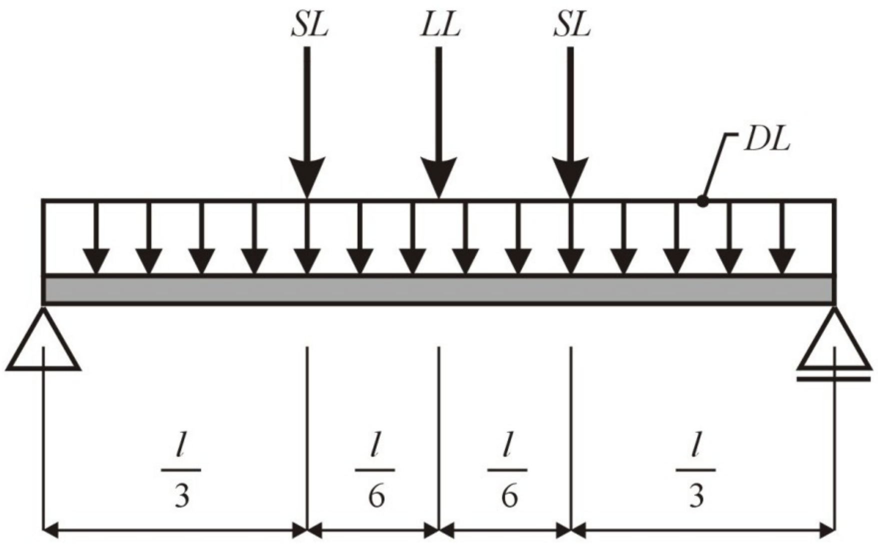
Reliability function: $RF = R - E$

Structural resistance (ultimate bending moment) :

$$R = M_{Rd} = W_{y,var} \cdot f_y$$

Load effect (maximal bending moment):

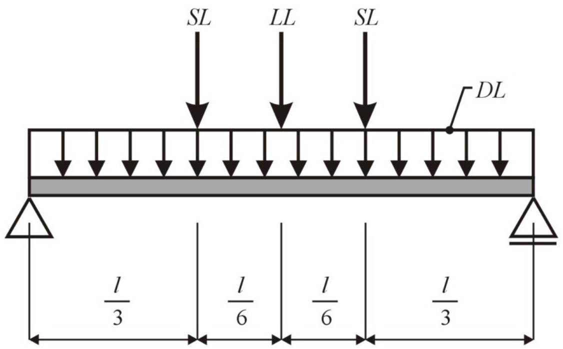
$$E = M_{Ed} =$$
$$= \frac{1}{8} \cdot 5 \cdot DL \cdot l^2 + \frac{1}{4} \cdot 75 \cdot LL \cdot l +$$
$$+ \frac{1}{3} \cdot 45 \cdot SL \cdot l$$



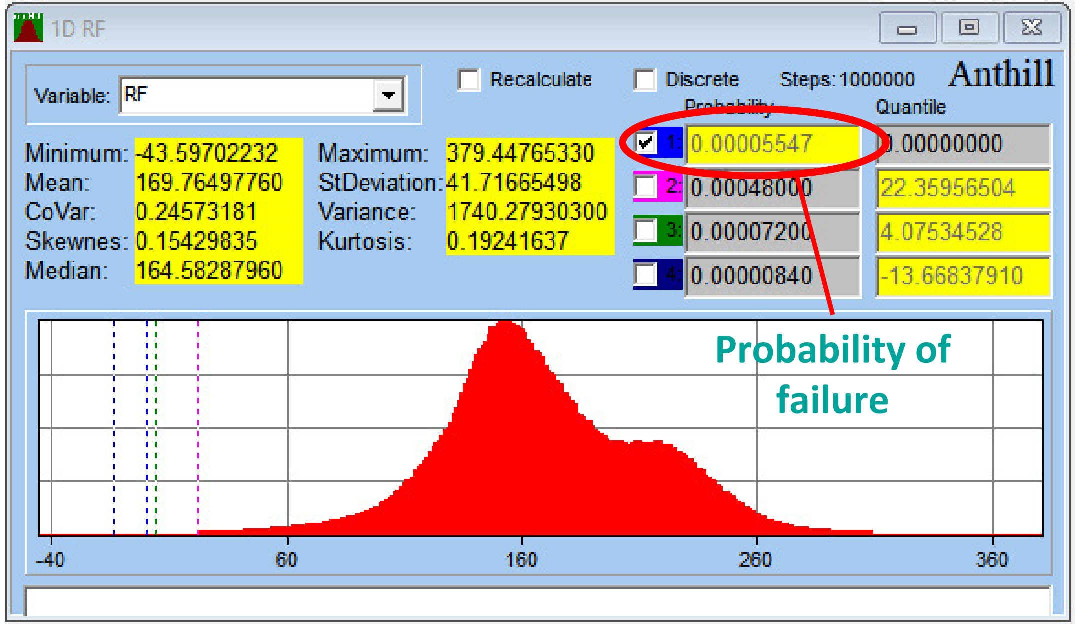
Example 3, Results

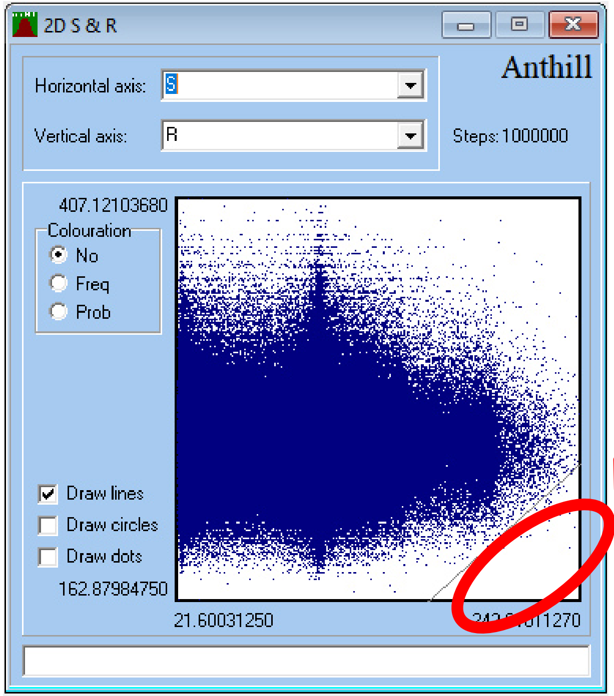
Computational model, definition

```
RF=R-E
R=Fy*Wyvar*0.1
E=6.5*POM
Wyvar=9.036*(1-3*Epsilon)
POM=5*DL*6.5/8+75*LL/4+45*SL/3
```



Resulting **histogram of reliability function**





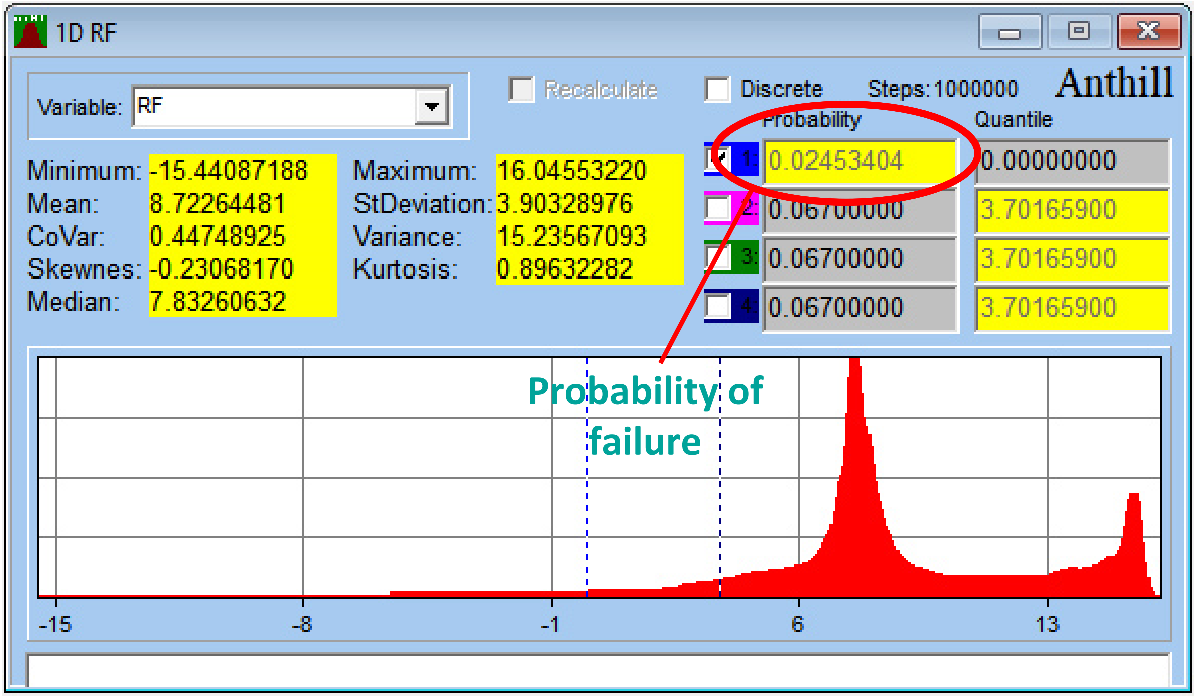
Failure area

Example 3, Serviceability Limit States

Computational model, definition $\left(\delta_{lim} = \frac{l}{350}\right)$

```
RF=6500/350-E
E=POM*6.5^3/(21*Iy)
POM=5*5*DL*6.5/384+75*LL/48+23*45*SL/648
Iy=1.627*(1-4*Epsilon)
```

Resulting **histogram of reliability function**

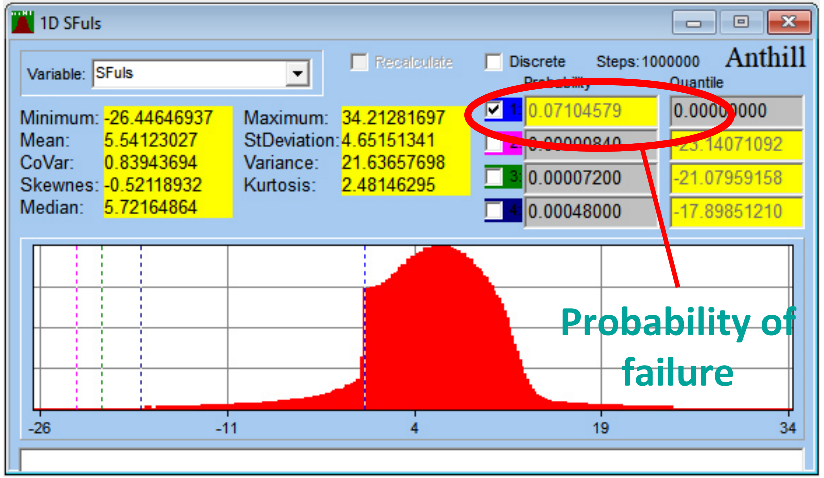


Example 4, Ultimate & Serviceability Limit States

Computational model

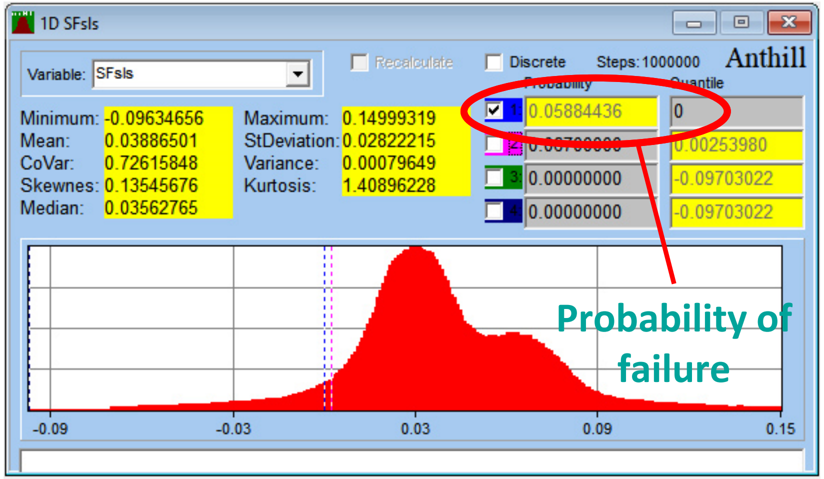
```
; Serviceability limits state
SFsls=Llim-abs(Ldelta)
Llim=0.150                     ; m
Ldelta=(5*q*(L^4))/(384*E*I)*1000 ; m
E=210e9                        ; Pa
I=1/12*b*(h^3)                 ; m4
; Ultimate limit state
SFuls=R-S
R=neg(S-Mrdel)*Mrdel+zero(S-Mrdel)*Mrdel+pos(S-Mrdel)*Mrdpl ; kNm
Mrdel=Wyel*fy*1000             ; kNm
Mrdpl=Wypl*fy*1000             ; kNm
Wypl=1/4*b*(h^2)*(1-3*EPS)     ; m3
Wyel=1/6*b*(h^2)*(1-3*EPS)     ; m3
S=abs(1/8*q*L*L)               ; kNm
fy=fyvar                       ; MPa
; Input parameters
q=4.5*DL+3.5*LL+5.0*WIN        ; kN/m
b=0.05                         ; m
h=0.1                          ; m
L=6                            ; m
```

Example 4, Results

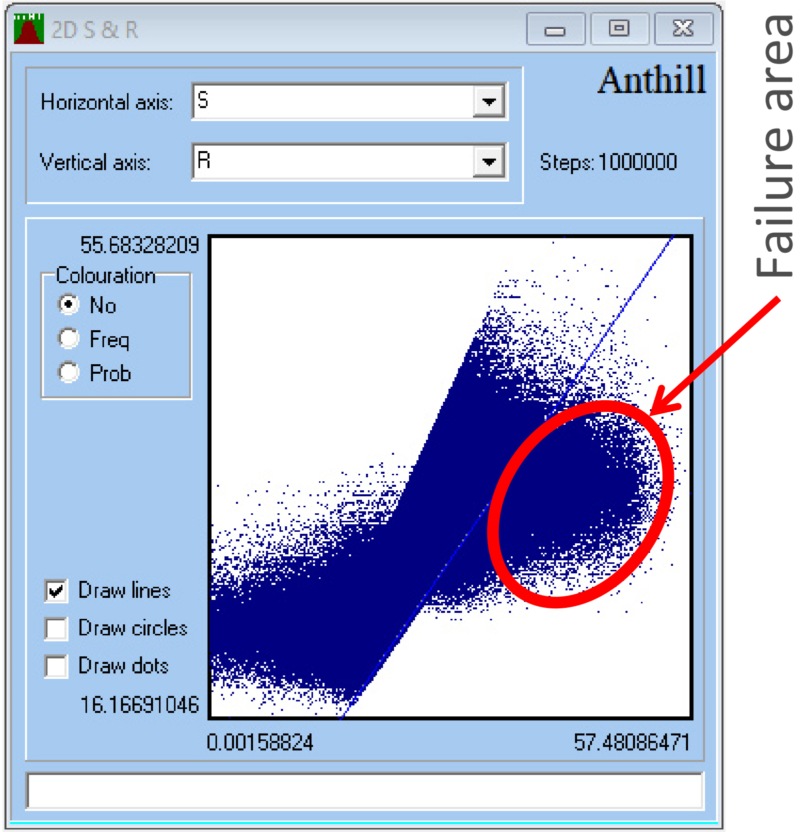


Resulting **histograms of reliability function**

Ultimate limit state



Serviceability limit state



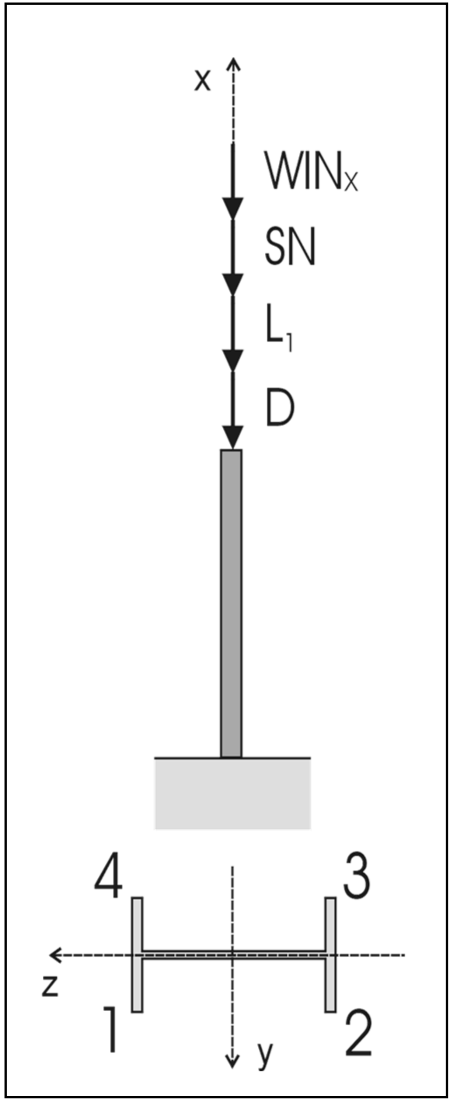
Ultimate limit state

Reliability Assessment

Column stressed by **one-component load effect** (axial force N)

$$\sigma = \pm\frac{N}{A}$$

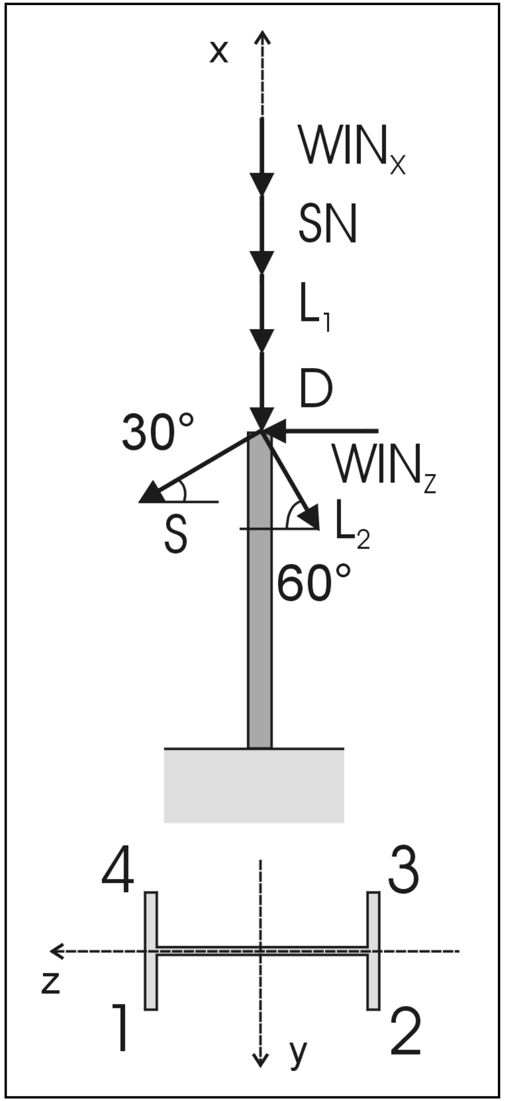
$$N_{Ed} \leq N_{Rd} = f_y \cdot A$$



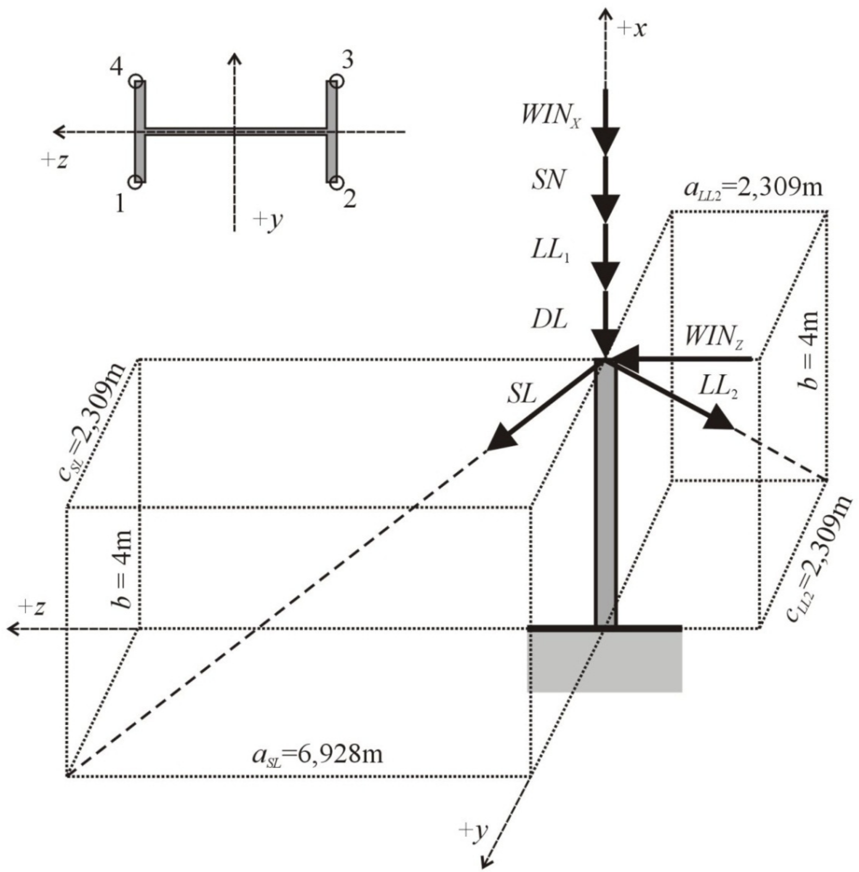
Reliability Assessment

Column stressed by **two-component load effect** (axial force N and bending moment M)

$$\sigma = \pm\frac{N}{A} \pm \frac{M_y}{W_y} \pm \left(\frac{M_z}{W_z}\right)$$

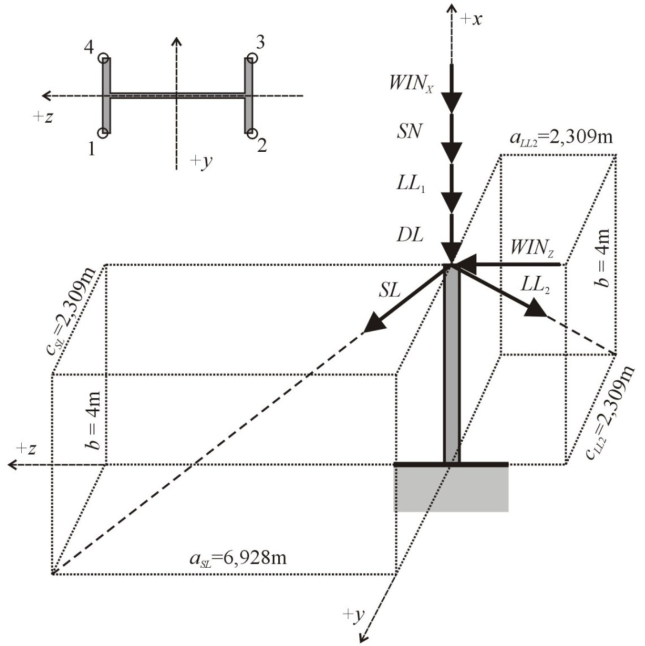


Example 5, Reliability Assessment



Column stressed by **three-component load effect**
(axial force N and bending moments M_y and M_z)

Example 5, Reliability Assessment



Mathematical model of probabilistic calculation:

Reliability function: $RF = R - E$

Structural resistance (strength of material) :

$$R = f_y$$

Load effect (maximal normal stress):

$$E = \frac{N_{Ed}}{A_{var}} \pm \frac{M_{y,Ed}}{W_{y,var}} \pm \frac{M_{z,Ed}}{W_{z,var}}$$

$$N_{Ed} = -DL - LL_1 - SN - WIN_x - SL_x - LL_{2,x}$$

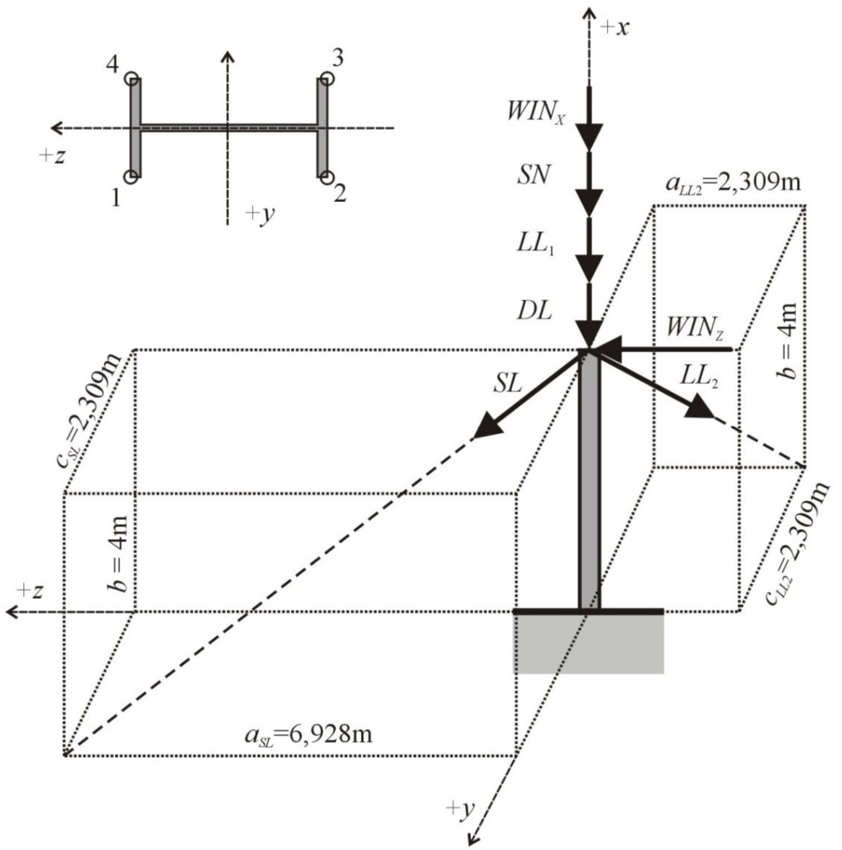
$$M_{y,Ed} = b \cdot \left(SL_z - LL_{2,z} + WIN_z\right)$$

$$M_{z,Ed} = b \cdot \left(SL_y - LL_{2,y}\right)$$

Example 5, Reliability Assessment

Computational model - Fiber 1

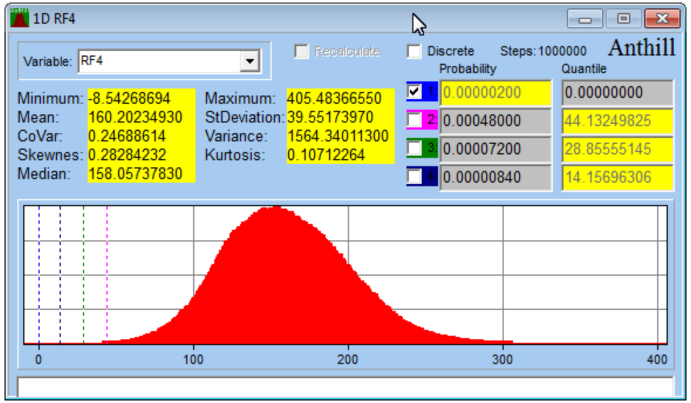
```
RF1=Fy-Sigma
Sigma=abs(Q/A-My/Wy-Mz/Wz)/1000
Q=-(Zat+80*WINvar+S*sin(pi/6)+LS*sin(pi/3))
Mz=6*(S*sin(pi/6)/tan(pi/3)-LS*sin(pi/3)/tan(pi/3))
My=6*(S*cos(pi/6)-LS*cos(pi/3)+3*WINvar)
Wz=0.00009852*(1-3*Eps)
Wy=0.0007131*(1-3*Eps)
A=0.006261*(1-2*Eps)
Eps=Epsilon
Zat=400*DL+300*LL1+100*SN
S=6*SL
LS=4*LL2
WINvar=WIN
```



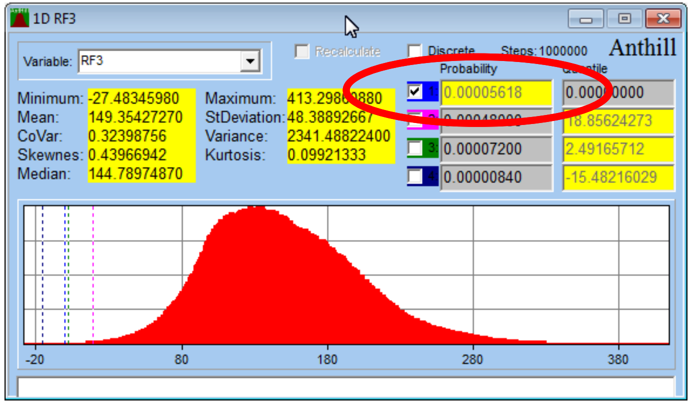
Example 5, Reliability Assessment

Reliability assessment in fibers of critical cross section 1..4:

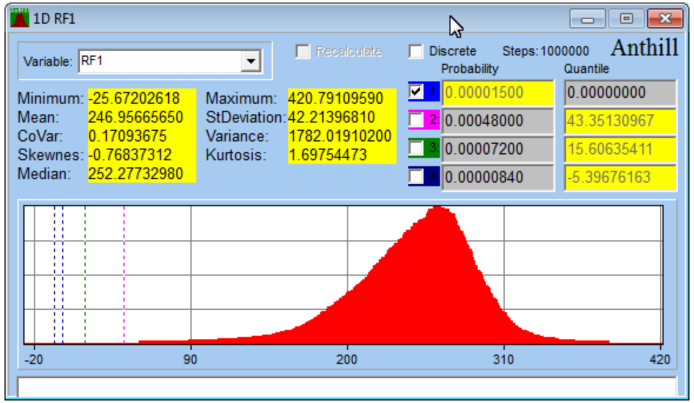
4)



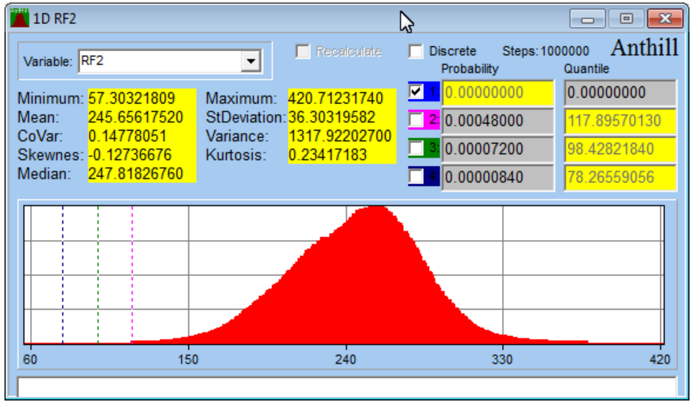
3)



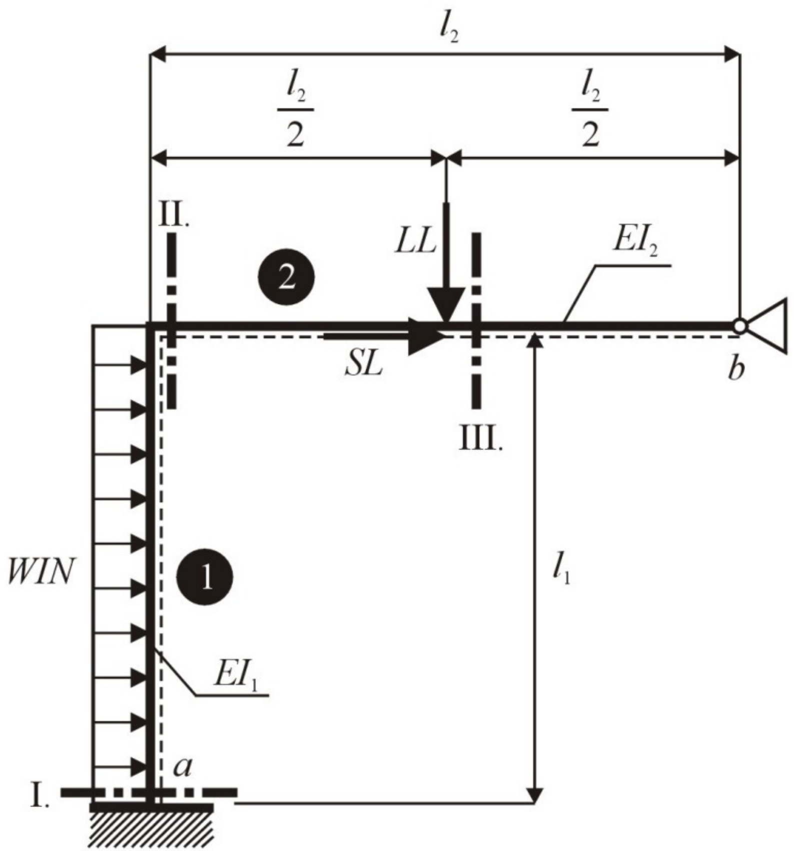
1)



2)



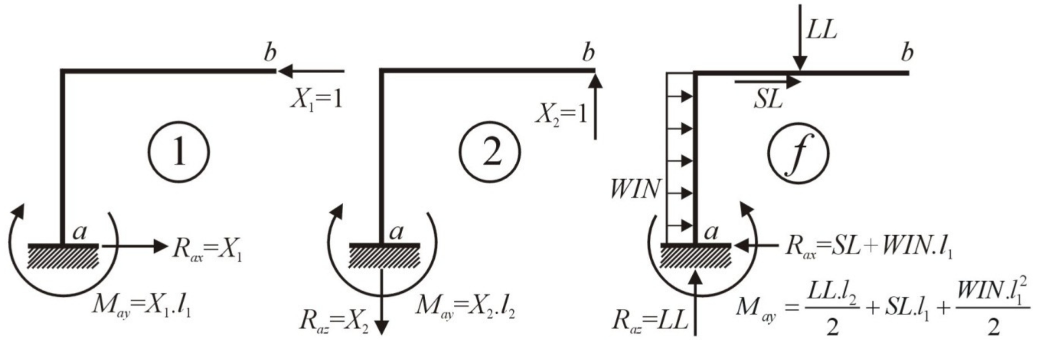
Example 6, Reliability Assessment



Statically indeterminate planar frame

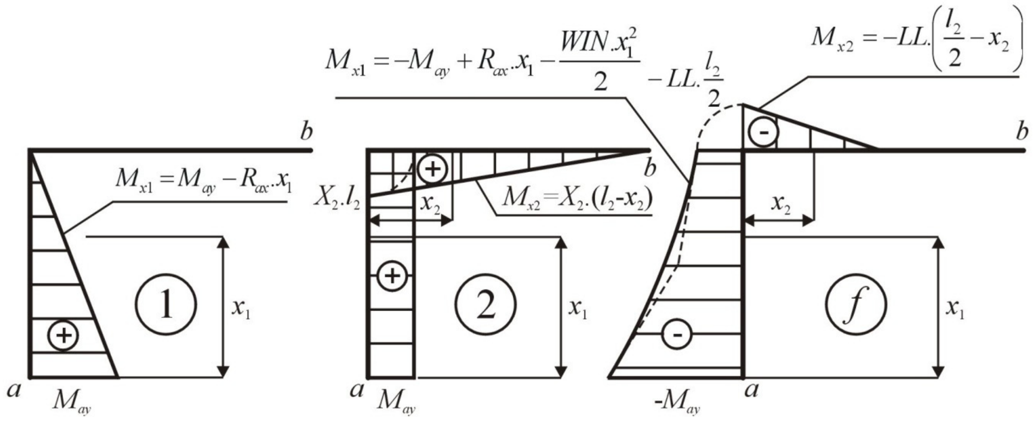
?

Example 6, Reliability Assessment



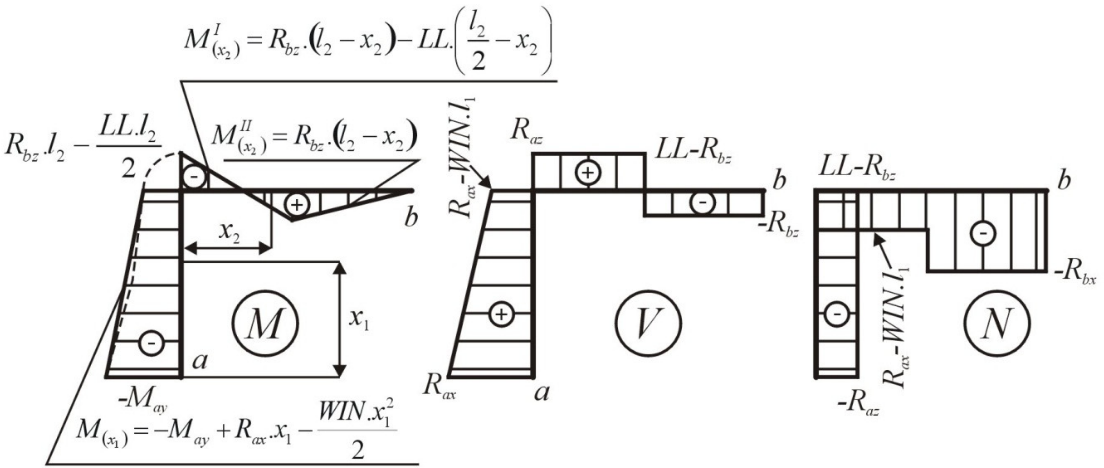
Force method:

Load schemes on a statically determined basic system



Load-induced bending moments on the basic system

Example 6, Reliability Assessment



Force method:

Real courses of internal forces

Derived analytical relations for **statically indeterminate quantities**:

$$X_1 = R_{bx} = \frac{I_{y,1} \cdot l_2 \cdot [9 \cdot LL \cdot l_2 + 4 \cdot l_1 \cdot (3 \cdot l_1 \cdot WIN + 8 \cdot SL)] + 12 \cdot I_{y,2} \cdot l_1^2 \cdot (l_1 \cdot WIN + 2 \cdot SL)}{8 \cdot l_1 \cdot \left(4 \cdot I_{y,1} \cdot l_2 + 3 \cdot I_{y,2} \cdot l_1\right)}$$

$$X_2 = R_{bz} = \frac{5 \cdot I_{y,1} \cdot LL \cdot l_2^2 + I_{y,2} \cdot l_1 \cdot (6 \cdot LL \cdot l_2 - l_1^2 \cdot WIN)}{4 \cdot l_2 \cdot \left(4 \cdot I_{y,1} \cdot l_2 + 3 \cdot I_{y,2} \cdot l_1\right)}$$

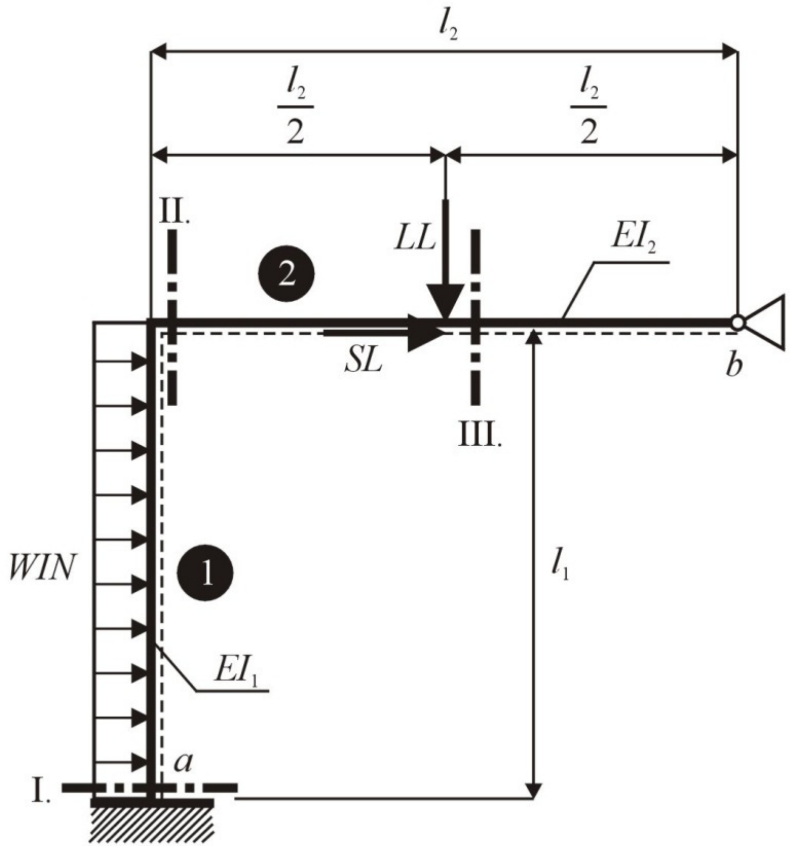
Example 6, Reliability Assessment

Mathematical model of probabilistic calculation:

Reliability function:

$$RF = R - E$$

In Sections I, II and III &
Fibers 1 and 2 (bottom, top)



Structural resistance (strength of material) :

$$R = f_y$$

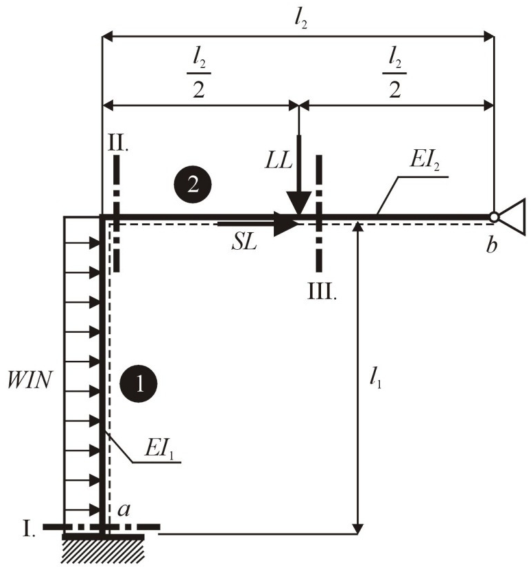
Load effect (maximal normal stress):

$$E = \left|\frac{N_{Ed}}{A_{var}}\right| + \left|\frac{M_{y,Ed}}{I_{y,var}} \cdot \frac{h}{2}\right|$$

Example 6, Reliability Assessment

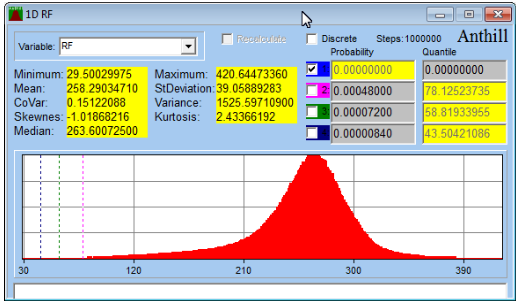
Computational model - Section I, Fiber 1

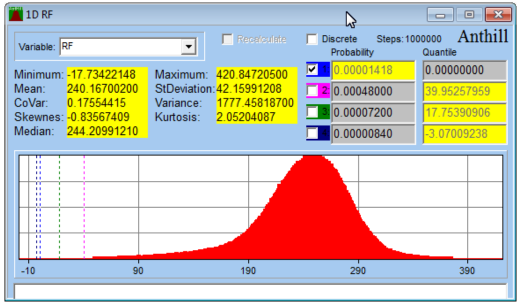
```
RF=Fy-abs(Sigma)
Sigma=(N/(0.004590*(1-2*Eps1))+M*0.135/I1)/1000
M=X1*3.5+X2*2.5-P1*2.5/2-P2*3.5-Q*(3.5)^2/2
N=-P1+X2
X1=(I1*2.5*(9*P1*2.5+4*3.5*(3*3.5*Q+8*P2))+12*I2*(3.5)^2*(3.5*Q+2*P2))/P
P=8*3.5*(4*I1*2.5+3*I2*3.5)
X2=(5*I1*P1*(2.5)^2+I2*3.5*(6*P1*2.5-
(3.5)^2*Q))/(4*2.5*(4*I1*2.5+3*I2*3.5))
I2=0.00008356*(1-4*Eps2)
I1=0.0000579*(1-4*Eps1)
Eps2=Epsilon2
Eps1=Epsilon1
P1=200*LL
Q=60*WIN
P2=260*SL
```



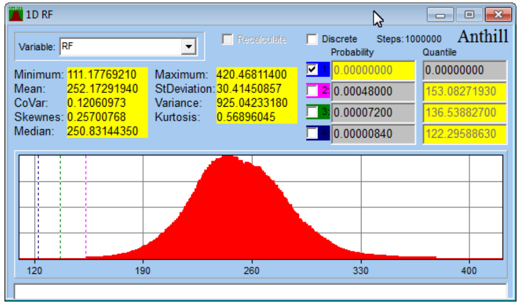
Example 6, Reliability Assessment

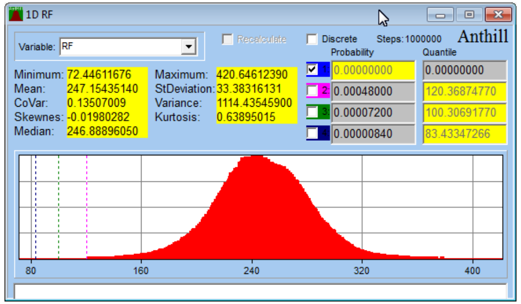
Section I, Fibers 1 and 2



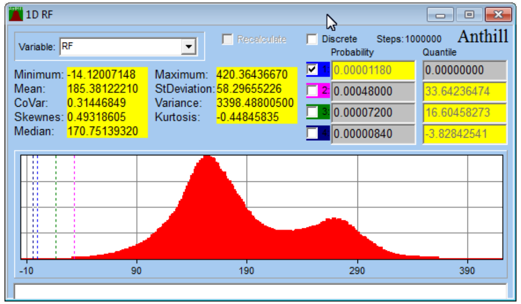


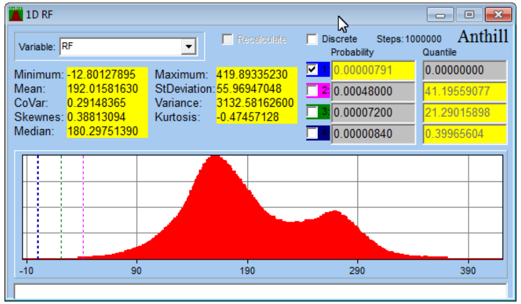
Section II, Fibers 1 and 2



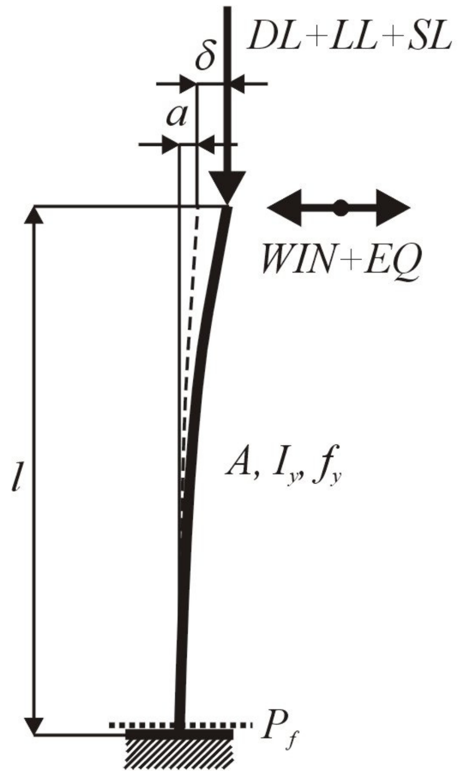


Section III, Fibers 1 and 2





Example 7, Reliability Assessment



Scheme of the structure under assessment

Reliability assessment of the column

l ... 6 m
profile HEB 300, **steel S235**, E ... $2.1 \cdot 10^{11}$ Pa
imperfections: a ... ± 30 mm

Load	Type	Extremal value [kN]
D	Dead	350
L	Long Lasting	75
S	Short Lasting	75
W	Wind	40
EQ	Earthquake	$\frac{1}{20} \cdot (D + L + S) = \frac{500}{20} = 25$

Example 7, Probabilistic Calculation Model

Calculation of maximal horizontal deformation δ using theory of II[nd] order considering the effect of initial imperfections:

$$\delta = \frac{W + EQ + \frac{a}{l} \cdot F}{\frac{F}{l \cdot K}}$$

where

$$K = \frac{\tan\left(l \cdot \sqrt{\frac{F}{EI}}\right)}{l \cdot \sqrt{\frac{F}{EI}}} - 1$$

Bending moment in critical cross-section:

$$M = \frac{\delta \cdot (1 + K)}{K} \cdot F$$

Stress in outer fibres:

$$S = \sigma = \left|\frac{M}{W}\right| + \left|\frac{F}{A}\right| = F \cdot \left(\frac{|\delta| \cdot (1 + K)}{K \cdot W} + \frac{1}{A}\right)$$

Example 7, Reliability Assessment

Ultimate limit state

$$RF = R - E$$

R ... structural resistance – yield stress f_y

E ... load effect – stress in outer fibres σ

Serviceability limit state

$$RF = \delta_{tol} - |\delta|$$

δ_{tol} ... structural resistance – allowed deformation (35 mm)

δ ... load effect – maximal horizontal deformations

Random input variables:

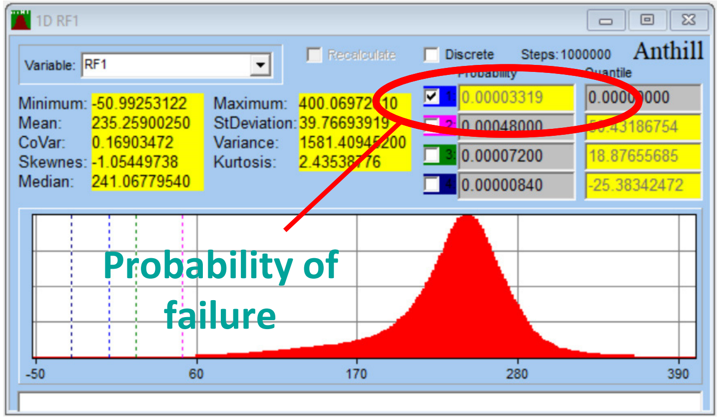
- 5 load components,
- cross-section variability,
- initial imperfection in column,
- yield stress f_y.

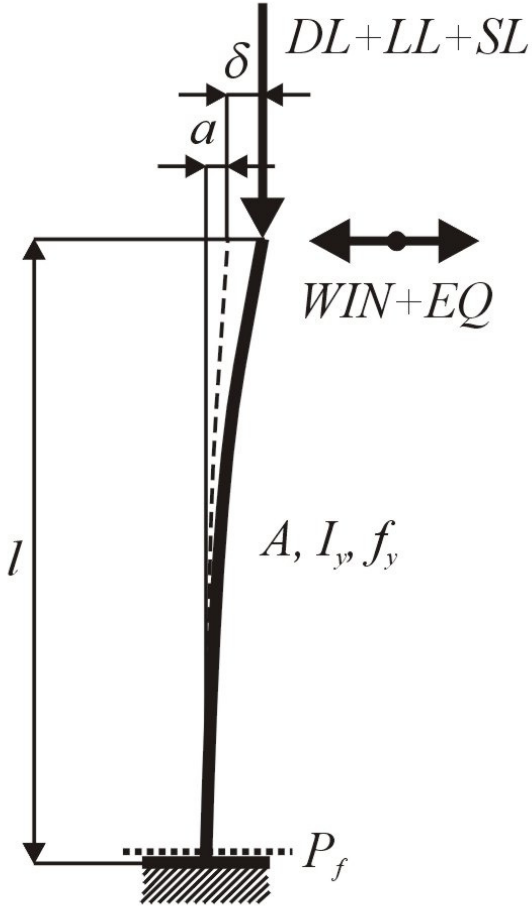
8 random input variables in total

Example 7, Reliability Assessment

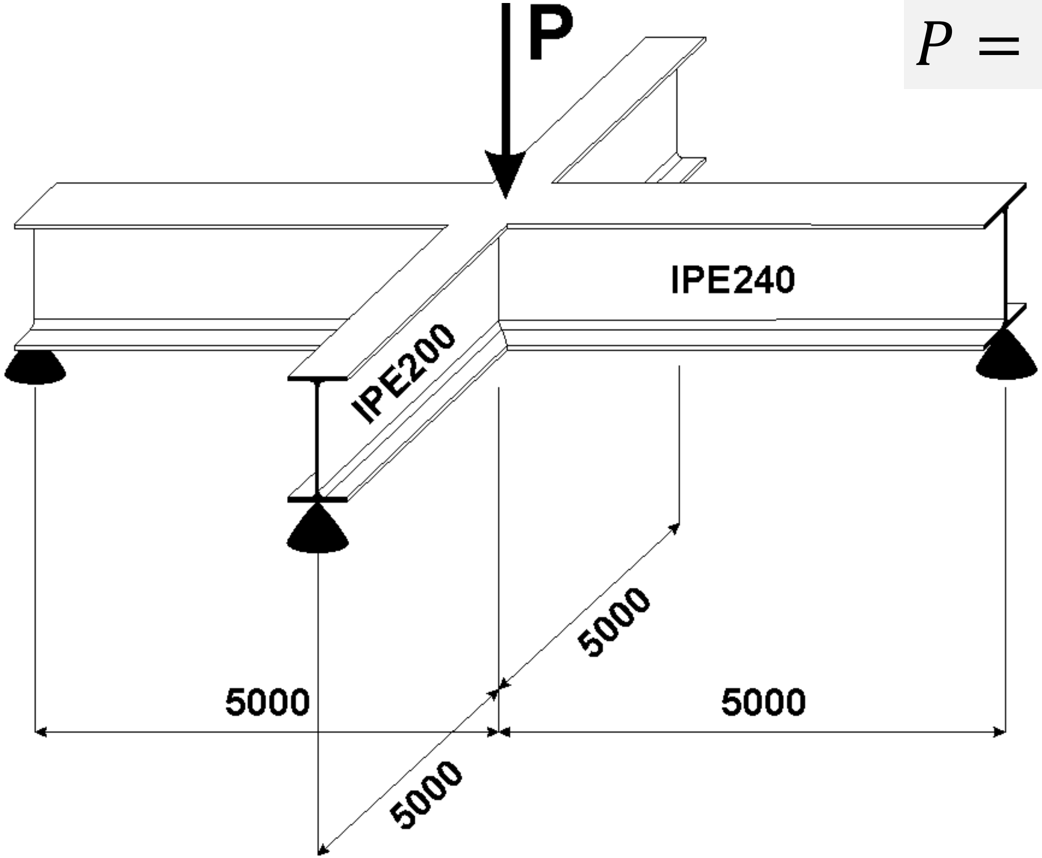
Computational model

```
RF1=R-Sigma
R=Fy
Sigma=F*(abs(DELTA)*(1+K)/(K*W)+1/A)/10^6
DELTA=(H+F*0.03*Imp/6)/(F/(6*K))
K=tan(POM)/POM-1
POM=6*(F/((2.1*10^11)*I))^0.5
H=45000*WIN+30000*EQ
F=320000*DL+120000*LL+75000*SL
I=0.0002517*(1-4*Eps)
W=0.001678*(1-3*Eps)
A=0.01491*(1-2*Eps)
Eps=Epsilon
```





Example 8, Reliability Assessment



$$P = DL \cdot DL_{var} + LL \cdot LL_{var} + SL \cdot SL_{var}$$

Steel statically indeterminate grate stressed by a combination of several mutually independent loads

Example 8, Reliability Assessment

Reliability criterion: Reaching stress at the yield point

Calculation of **elastic deformation** and maximum bending moment

$$P = P_1 + P_2$$

$$w_{1,\max} = w_{2,\max} = \frac{P_1 \cdot L_1^3}{48 \cdot E \cdot I_1} = \frac{P_2 \cdot L_2^3}{48 \cdot E \cdot I_2}$$

$$M_1 = \frac{P.L_1}{4} \cdot \left(\frac{L_2^3 \cdot I_1}{L_1^3.I_2 + L_2^3.I_1} \right)$$

$$M_2 = \frac{P \cdot L_2}{4} \cdot \left(\frac{L_1^3 \cdot I_2}{L_1^3 \cdot I_2 + L_2^3 \cdot I_1} \right)$$

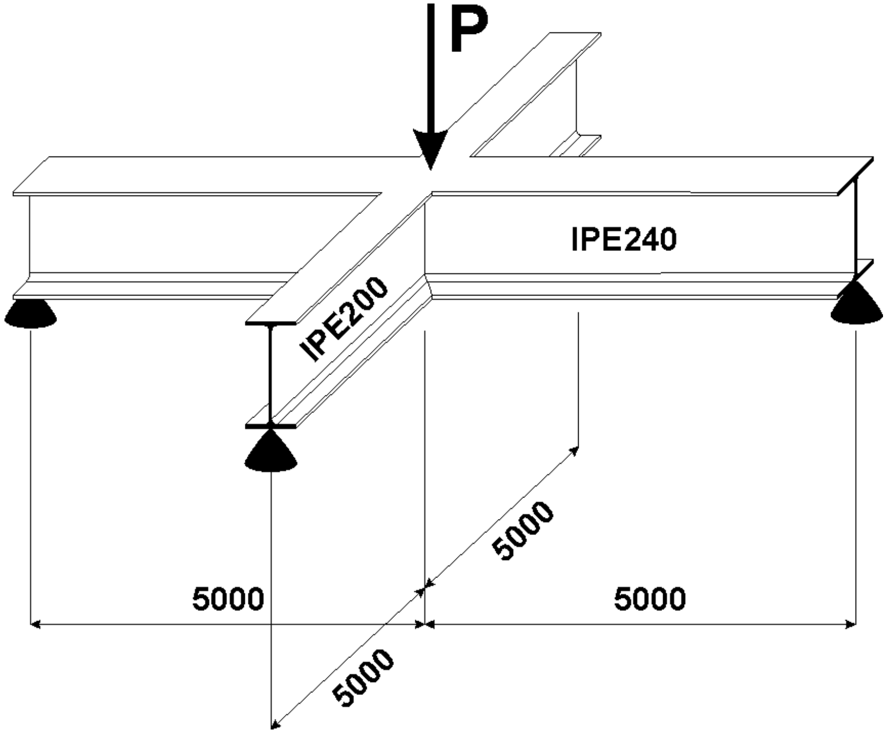
$$w_{\max} = P \cdot \frac{L_1^3 \cdot L_2^3}{48 \cdot E \cdot (L_1^3 \cdot I_2 + L_2^3 \cdot I_1)}$$

Reliability function: $RF = \mathrm{pos}(R_1 - E_1) \cdot \mathrm{pos}(R_2 - E_2)$

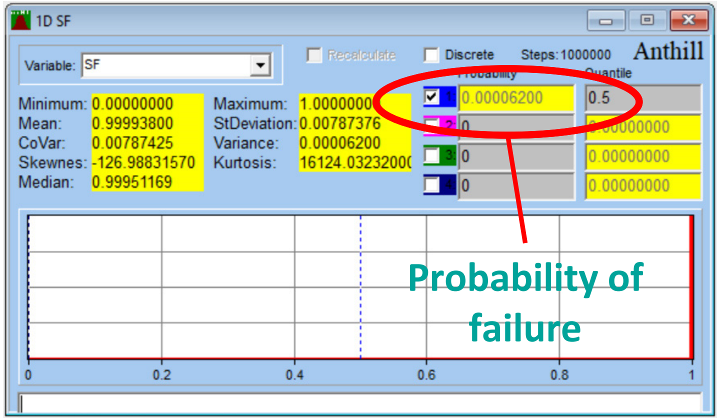
Example 8, Reliability Assess

Computational model

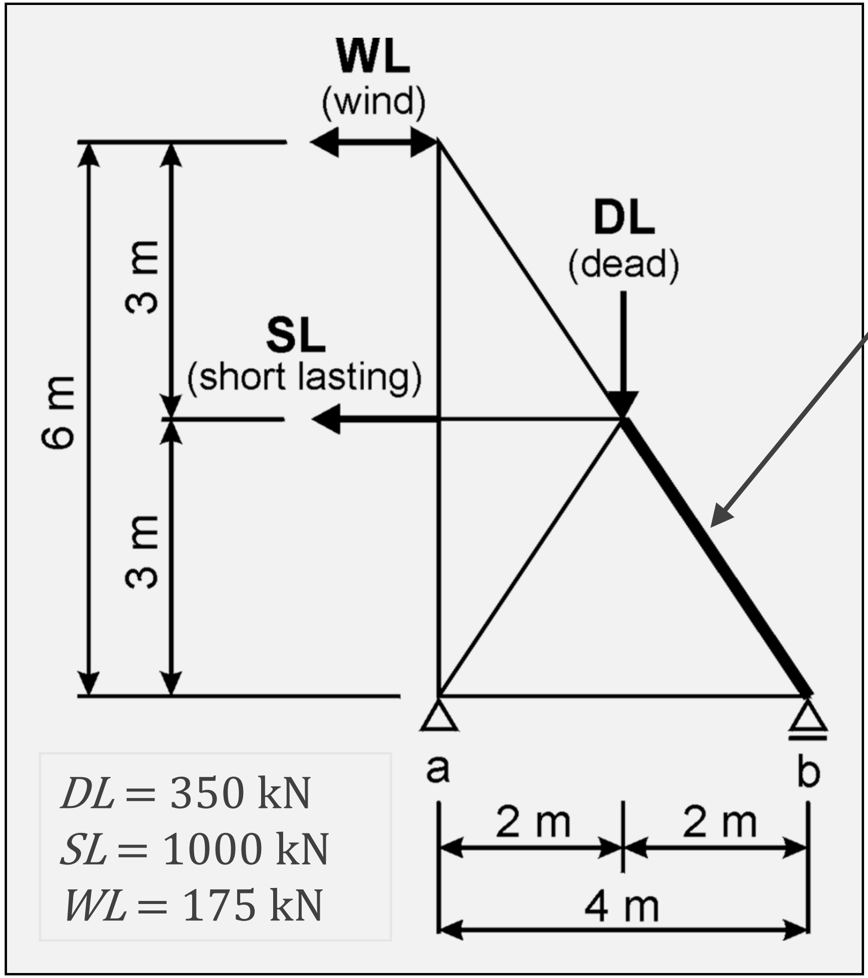
```
SF = pos(R1-E1)*pos(R2-E2)
R1 = 0.9*Fy1
R2 = 0.9*Fy2
E1 = (P*L1/4*(L2^3*I1)/(L1^3*I2+L2^3*I1))/(W1*Wvar*1000)
E2 = (P*L2/4*(L1^3*I2)/(L1^3*I2+L2^3*I1))/(W2*Wvar*1000)
P  = (80*DLvar+240*LLvar+120*SLvar+25*WINvar+40*SNvar)
W1 = 0.001928
W2 = 0.0007152
I1 = 0.0004820*Ivar
I2 = 0.0001180*Ivar
L1 = 6.0
L2 = 5.0
```



Resulting **histogram of reliability function**



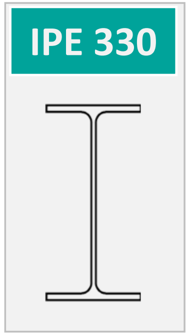
Example 9, Reliability Assessment



Steel geometry and loading of a planar truss

Axial force in the investigated bar:

Investigated bar

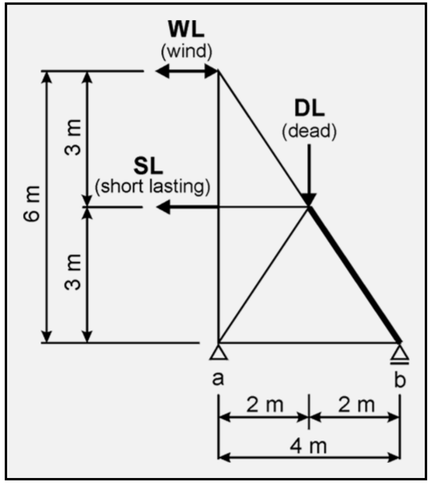


$$N_{DL} = -\frac{DL.\sqrt{13}}{6}$$

$$N_{SL} = \frac{SL.\sqrt{13}}{4}$$

$$N_{WL} = \pm\frac{WL.\sqrt{13}}{2}$$

Example 9, Reliability Assessment



Resistance in compression (buckling) $N_{R,c}$:

$$N_{R,c} = \frac{R_1 - \sqrt{{R_1}^2 - \frac{4 \cdot \left(\frac{L}{r}\right)^2}{\pi^2 \cdot E \cdot A} \cdot f_y \cdot A}}{\left(\frac{2.\,(L/r)^2}{\pi^2 \cdot E \cdot A}\right)}$$

$$R_1 = 1 + \frac{f_y \cdot \left(\frac{L}{r}\right)^2}{\pi^2 \cdot E} + \frac{L \cdot E_0 \cdot (1 + 0.1 \cdot Res_{\mathrm{var}}) \cdot c}{r^2}$$

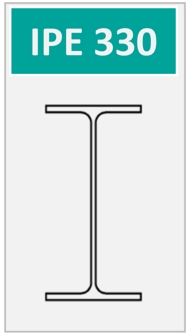
Resistance in tension $N_{R,t}$:

$$N_{R,t} = f_y \cdot A$$

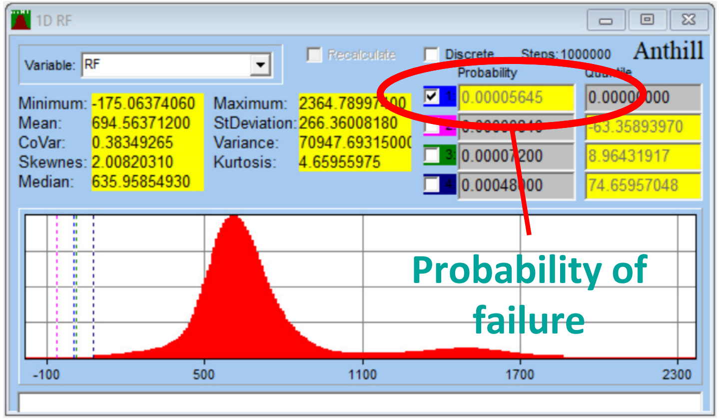
f_y ... yield stress
A ... cross-sectional area
E ... elastic modulus
c ... distance of outside fibres from centroidal axis
L ... length of the bar
r ... radius of gyration
E_0 ... initial eccentricity
Res_{var} ... effect of residual stresses

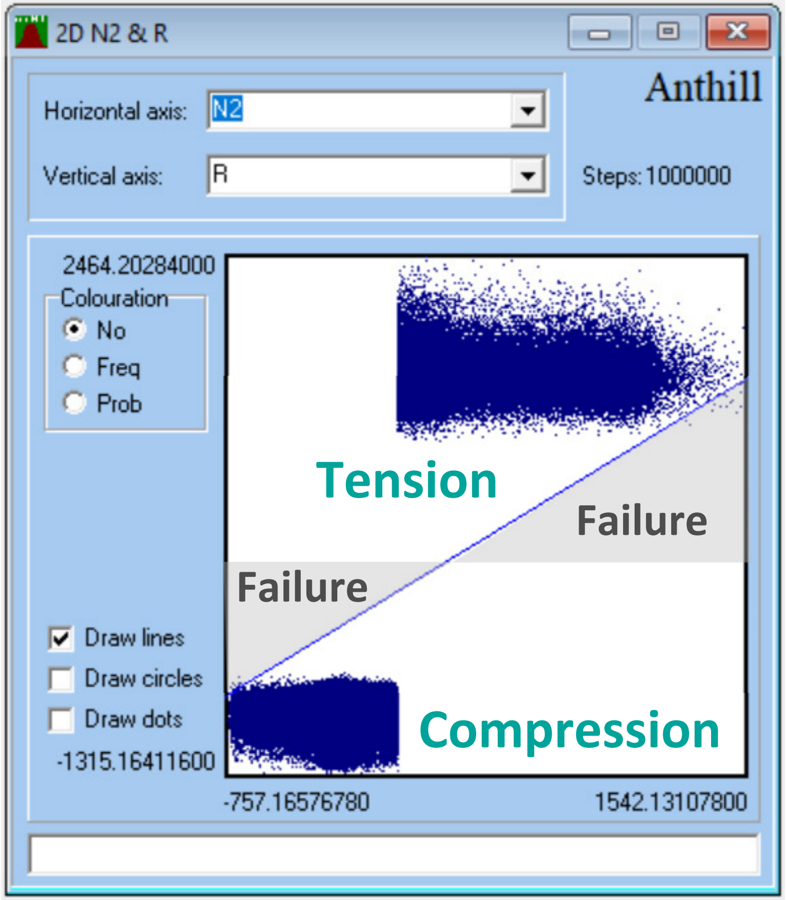
Example 9, Results

Computational model, definition

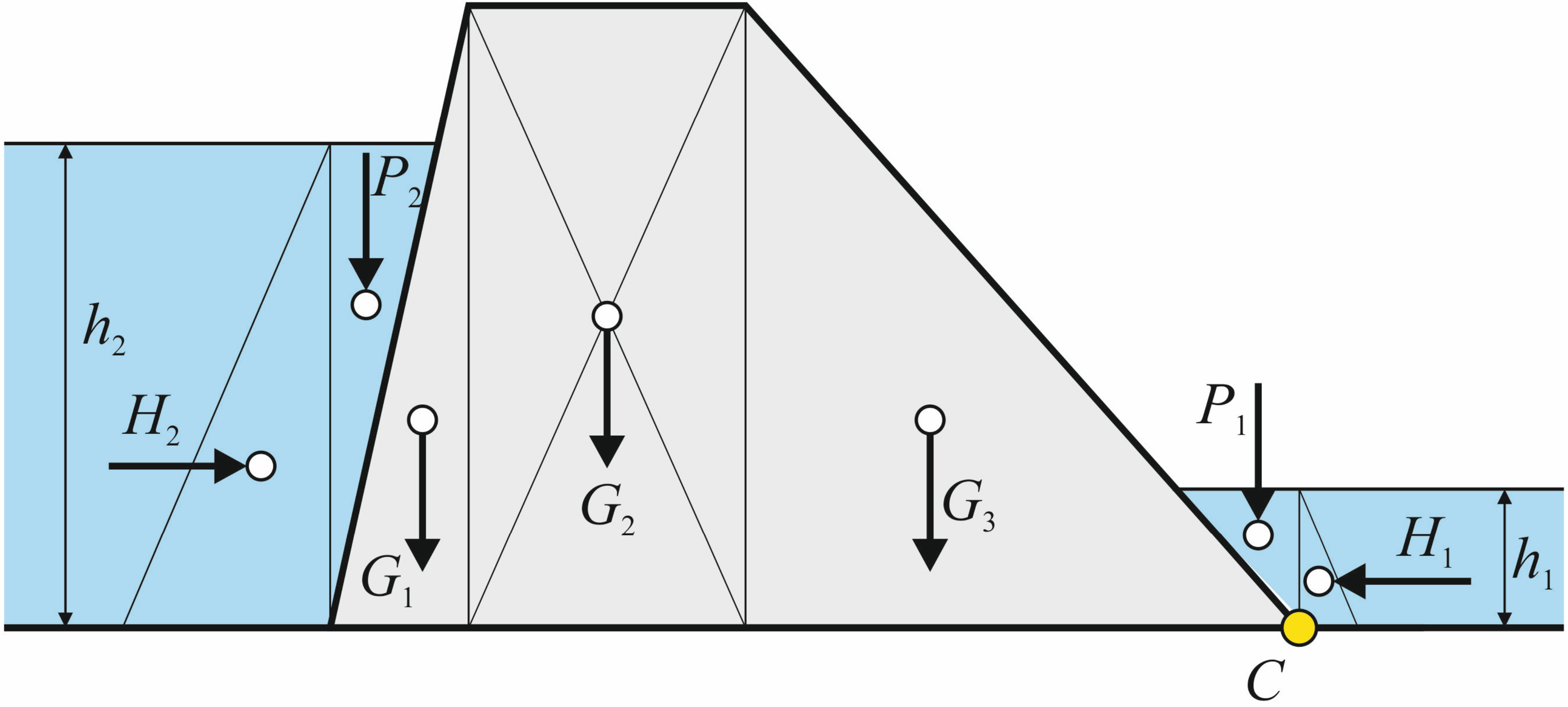



```
RF=abs(R)-abs(N2)
R=-pos(-N2)*RM+pos(N2)*RP
RP=0.9*Fy*Avar*1000
RM=(R1-(R1^2-
4*Lr2Pi2EA*(Fy*1E6*0.9)*Avar)^0.5)/(2*Lr2Pi2EA)/1000
R1=1+FyLr2Pi2E+EOcr2
EOCr2=L*EO*(1+0.1*Resvar)*ex/iz^2
FyLr2Pi2E=((Fy*1E6*0.9)*(L/iz)^2)/(Pi^2*Ecko)
Lr2Pi2EA=(L/iz)^2/(Pi^2*Ecko*Avar)
iz=0.03547937
ex=0.165
Avar=0.006261*(1-2*Eps)
Ecko=2.1E11
N2=-Vb/SinA
SinA=3/L
L=(2^2+3^2)^0.5
Vb=(F1*6-F2*3+F3*2)/4
F1=1.5*175*WL
F2=1.5*1000*SL
F3=1.35*350*DL
```





Example 10, Reliability Assessment of Gravity Dam

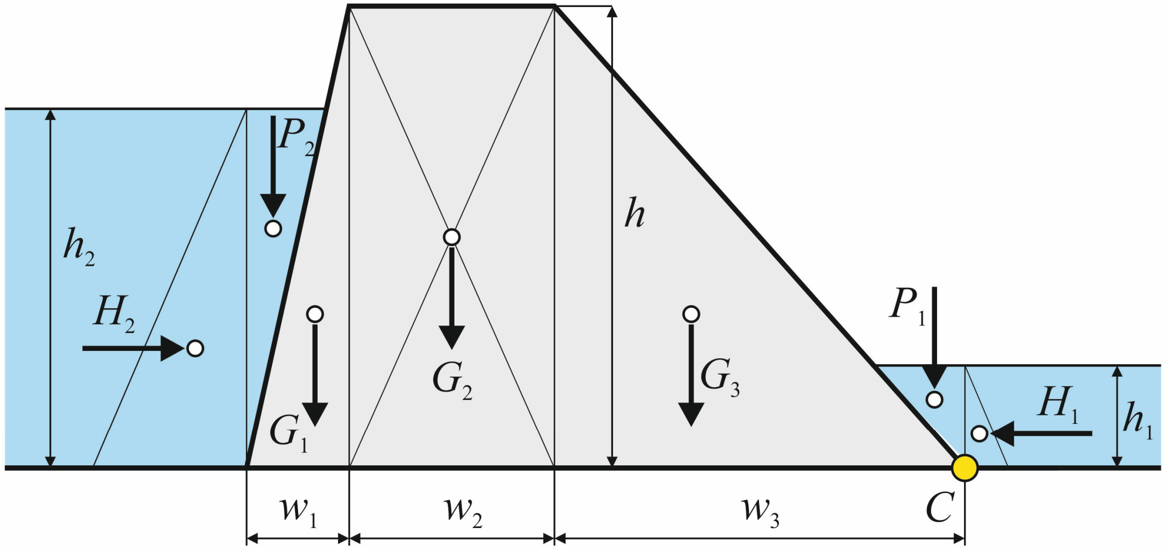


Example 10: Flipping around point C

Reliability function: $RF = \frac{R}{\beta} - E$,

where β is **coefficient of safety**

R and E are **statical moments**



E load effect (**unstable**): $E = H_2 \cdot \frac{1}{3} \cdot h_2$

R structural resistance (**stable**):

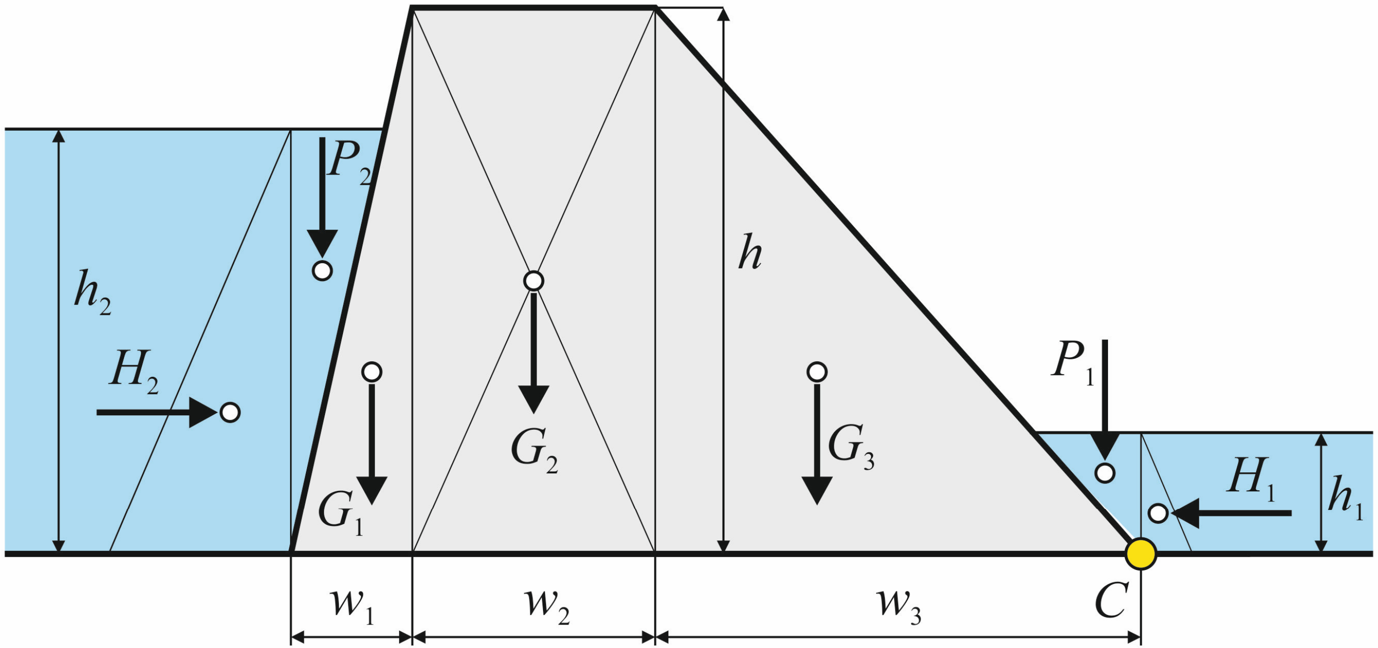
$$R = P_2 \cdot \left(w_1 + w_2 + w_3 - \frac{1}{3} \cdot \left(w_1 \cdot \frac{h_2}{h} \right) \right) + G_1 \cdot \left(\frac{w_1}{3} + w_2 + w_3 \right) + G_2 \cdot \left(\frac{w_2}{2} + w_3 \right) + G_3 \cdot \frac{2}{3} \cdot w_3 + P_1 \cdot \frac{1}{3} \cdot \left(w_3 \cdot \frac{h_1}{h} \right) + H_1 \cdot \frac{1}{3} \cdot h_1$$

Example 10: Slipping in the contact surface

Reliability function:

$$RF = \frac{R}{\beta} - E$$

where β is **coefficient of safety** ($\beta = 2$), R and E are **shear forces in the contact surface** with the **coefficient of friction** $\text{COF} = 0.79 \div 0.81$.



E load effect (**action**): $E = H_2$

R structural resistance (**against slipping**):

$$R = \text{COF} \cdot \sum F_{\text{vert}} + H_1 = \text{COF} \cdot (P_2 + G_1 + G_2 + G_3 + P_1) + H_1$$

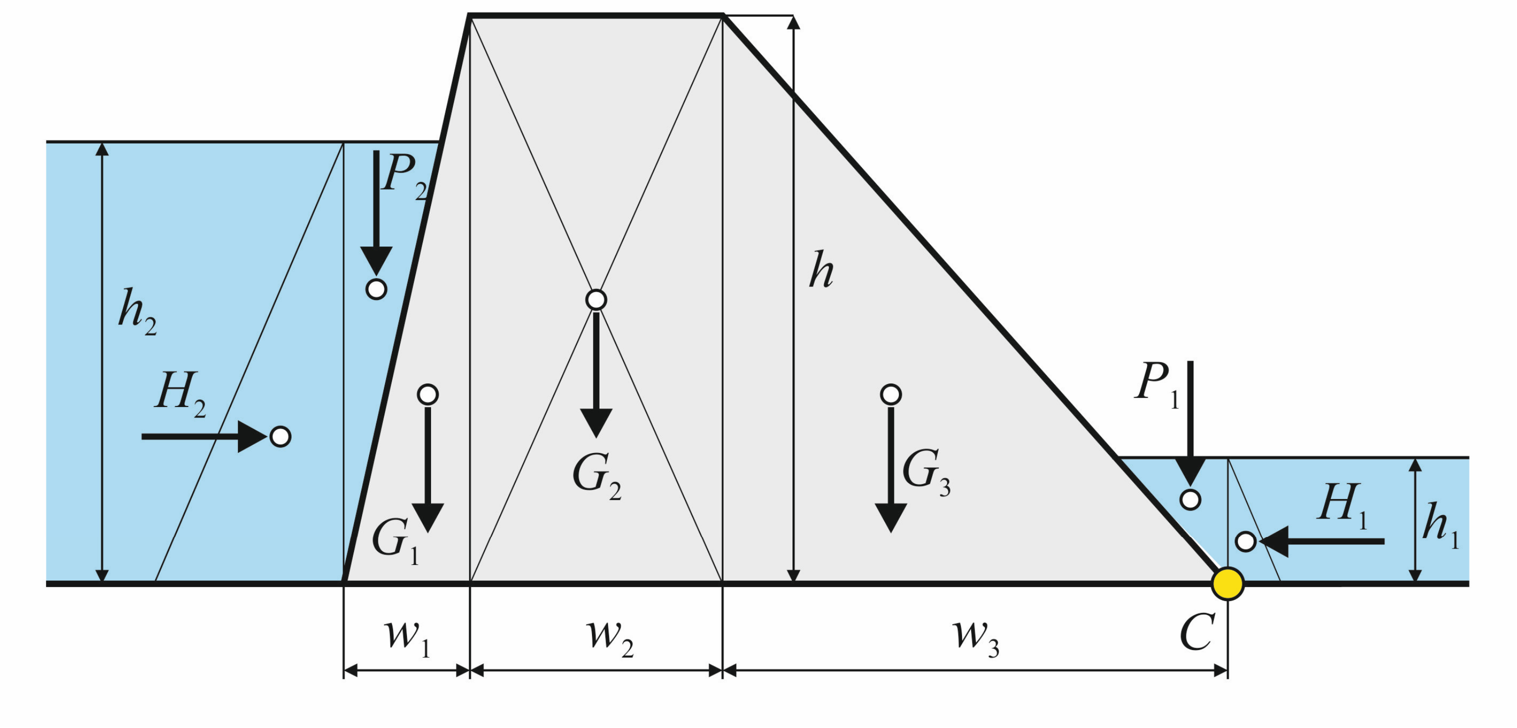
Example 10: Random input variables

Water levels h_1, h_2

Bulk density of concrete ρ_c

Coefficient of friction COF

4 random input variables



Example 10: Calculation in the AntHill program

Flipping around point C

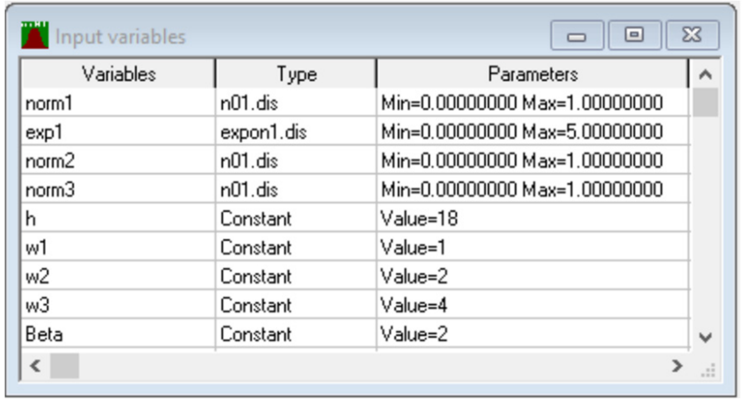
Computational model, definition

```
RF=R-E
E=H2*wl2/3
R=(P2*(w1+w2+w3-w1*wl2/h/3)+G1*(w1/3+w2+w3)+G2*(w2/2+w3)+G3*2/3*w3+P1*w3*wl1/h/3+H1*wl1/3)/Beta
; Loads
H1=wl1^2*1000*9.81/2/1000
P1=wl1*w3*wl1/h*1000*9.81/2/1000
H2=wl2^2*1000*9.81/2/1000
P2=wl2*w1*wl2/h*1000*9.81/2/1000
G1=w1*h/2*roc*9.81/1000
G2=w2*h*roc*9.81/1000
G3=w3*h/2*roc*9.81/1000
; Random input variables
wl1=3+2*norm1            ; Water level h1, normal, 3..5 m
wl2=10+(1-exp1/5)*4      ; Water level h2, exponential, 10..14 m
roc=2800+200*norm2       ; Bulk density of concrete, normal, 2800..3000 kg/m^3
```

Example 10: Calculation in the AntHill program

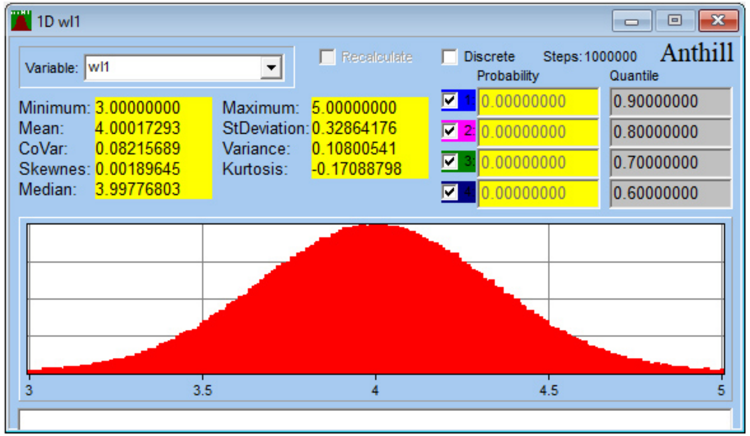
Flipping around point C

Input variables:

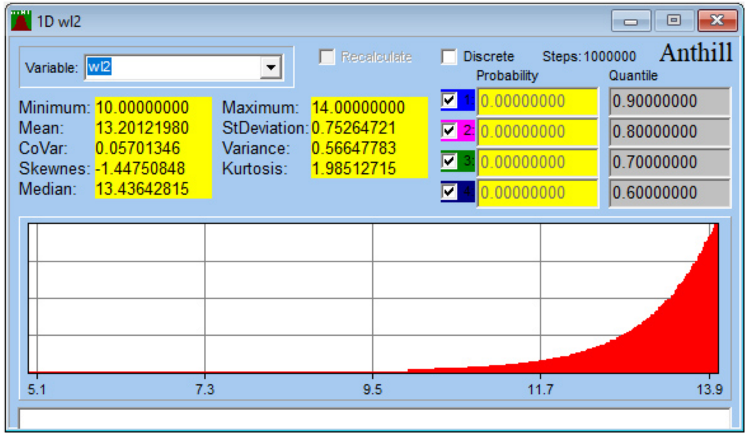


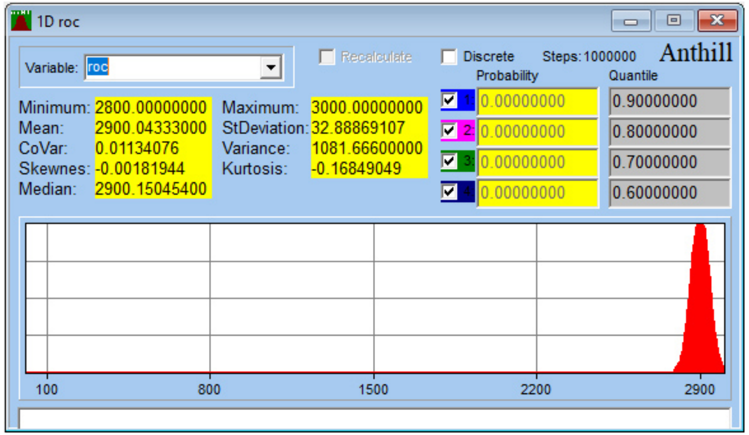
Variables	Type	Parameters
norm1	n01.dis	Min=0.00000000 Max=1.00000000
exp1	expon1.dis	Min=0.00000000 Max=5.00000000
norm2	n01.dis	Min=0.00000000 Max=1.00000000
norm3	n01.dis	Min=0.00000000 Max=1.00000000
h	Constant	Value=18
w1	Constant	Value=1
w2	Constant	Value=2
w3	Constant	Value=4
Beta	Constant	Value=2

Water level h_1, normal distribution, $3 \dots 5$ m



Water level h_2, exponential distribution, $10 \dots 14$ m



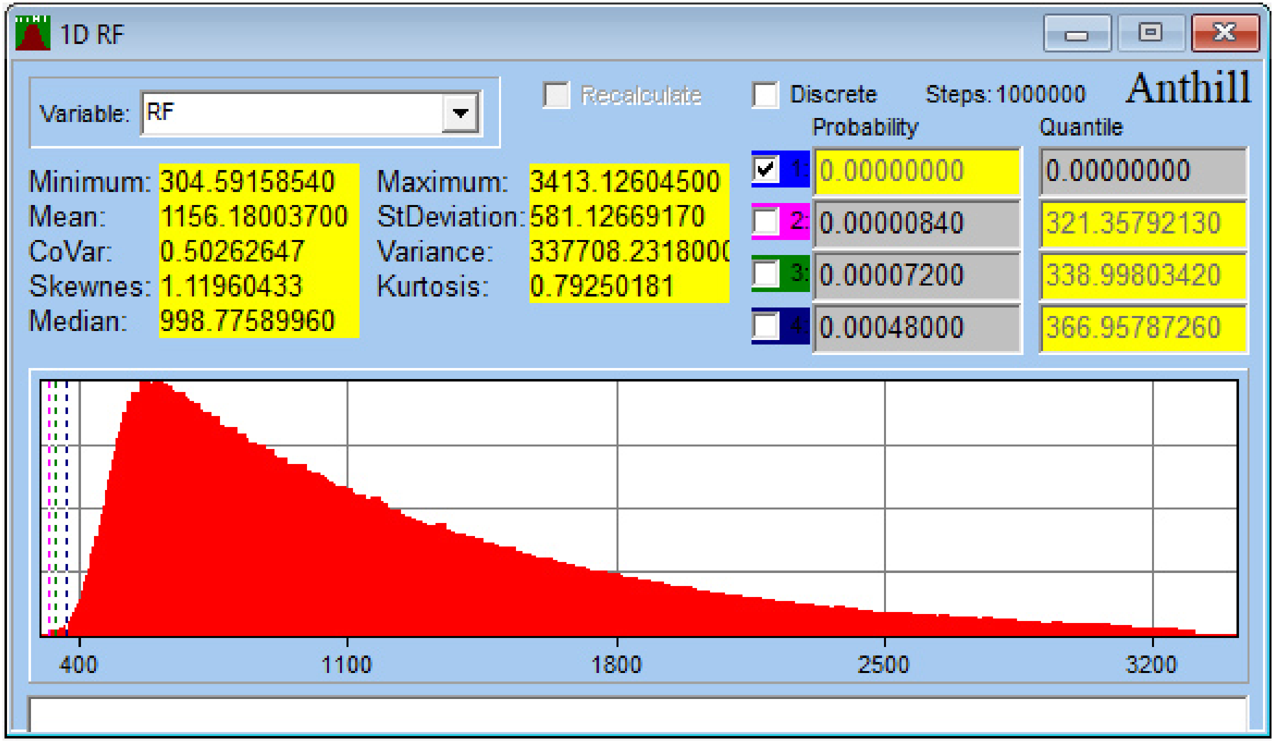


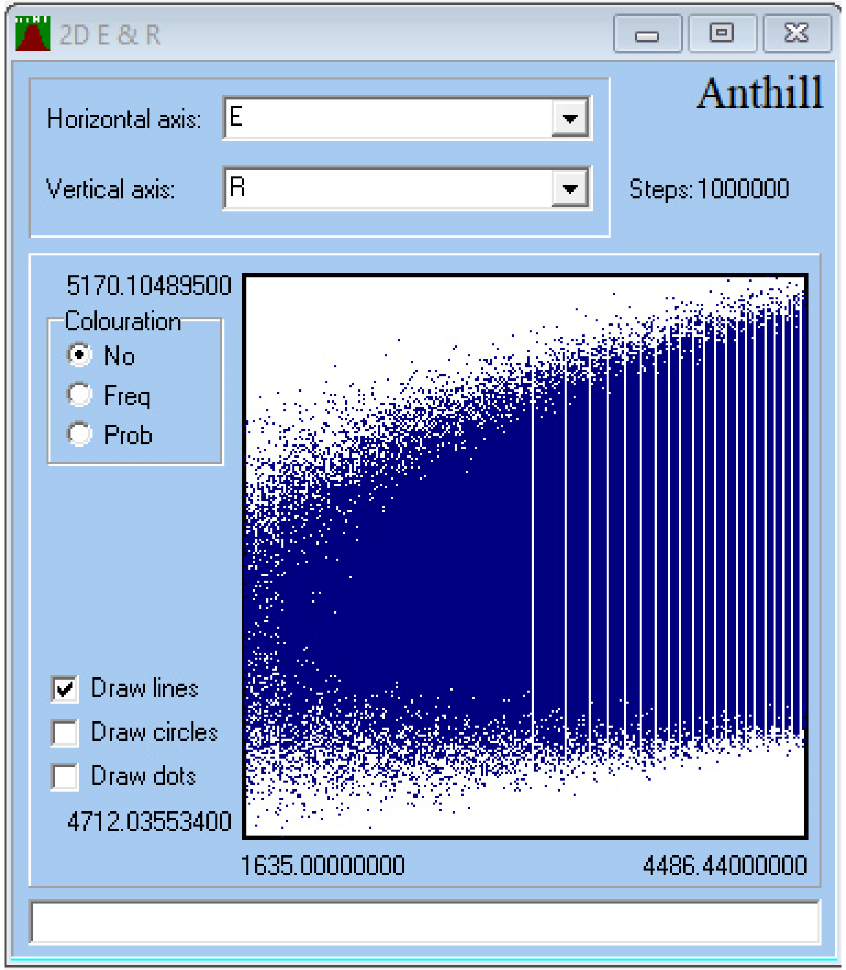
Bulk density of concrete, normal distribution, $2{,}800 \dots 3{,}000\ \mathrm{kg \cdot m^{-3}}$

Example 10: Calculation in the AntHill program

Flipping around point C

Results:





Probability of failure $P_f = 0$

Example 10: Calculation in the AntHill program

Slipping in the contact surface

Computational model, definition

```
RF=R-E
E=H2
R=(COF*(P2+G1+G2+G3+P1)+H1)/Beta
; Loads
H1=wl1^2*1000*9.81/2/1000
P1=wl1*w3*wl1/h*1000*9.81/2/1000
H2=wl2^2*1000*9.81/2/1000
P2=wl2*w1*wl2/h*1000*9.81/2/1000
G1=w1*h/2*roc*9.81/1000
G2=w2*h*roc*9.81/1000
G3=w3*h/2*roc*9.81/1000
; Random input variables
wl1=3+2*norm1                   ; Water level h1, normal, 3...5 m
wl2=10+(1-exp1/5)*4             ; Water level h2, exponential, 10...14 m
roc=2800+200*norm2              ; Bulk density of concrete, normal, 2800...3000 kg/m^3
COF=0.80+0.05*norm3             ; Coefficient of friction, normal, 0.80...0.85
```

Example 10: Calculation in the AntHill program

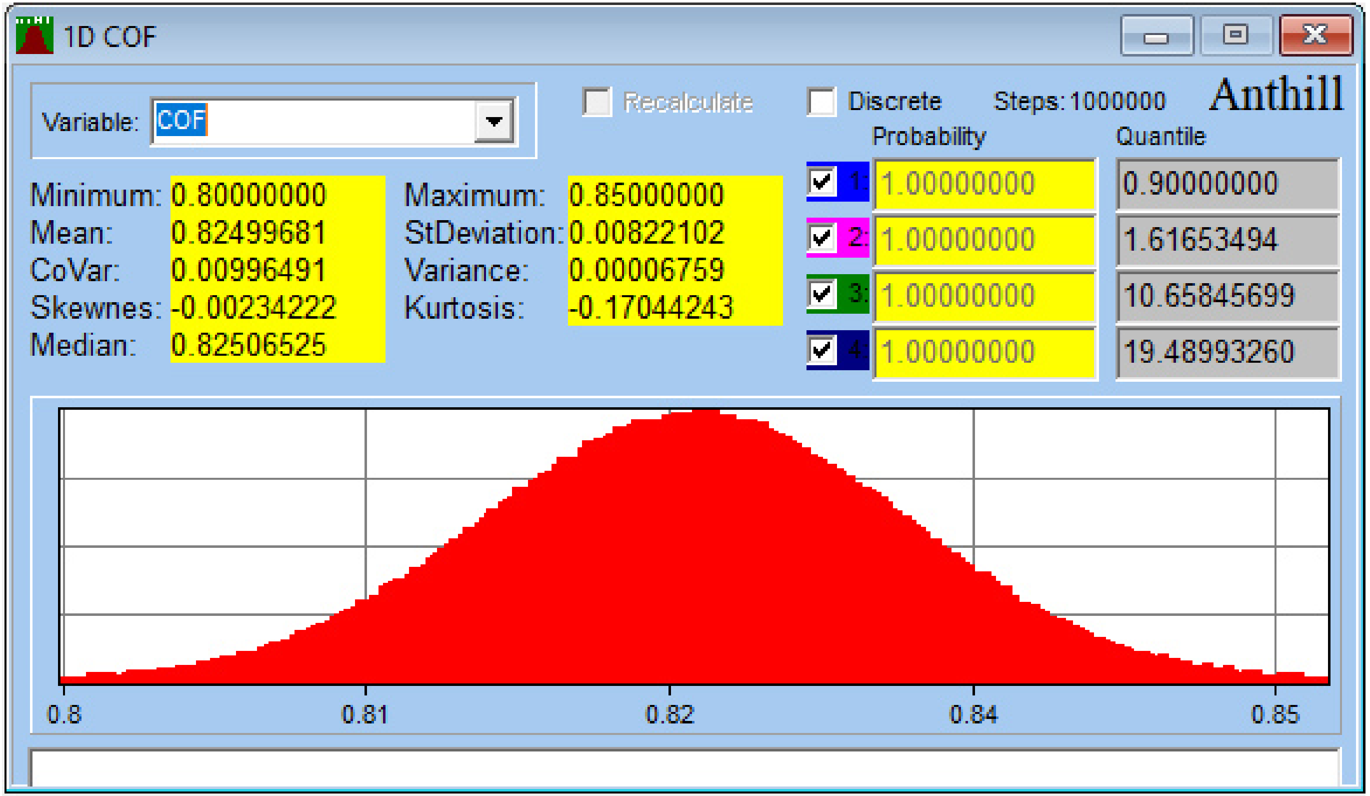
Flipping around point C

Input variables:

Input variables

Variables	Type	Parameters
norm1	n01.dis	Min=0.00000000 Max=1.00000000
exp1	expon1.dis	Min=0.00000000 Max=5.00000000
norm2	n01.dis	Min=0.00000000 Max=1.00000000
norm3	n01.dis	Min=0.00000000 Max=1.00000000
h	Constant	Value=18
w1	Constant	Value=1
w2	Constant	Value=2
w3	Constant	Value=4
Beta	Constant	Value=2

Partial output:

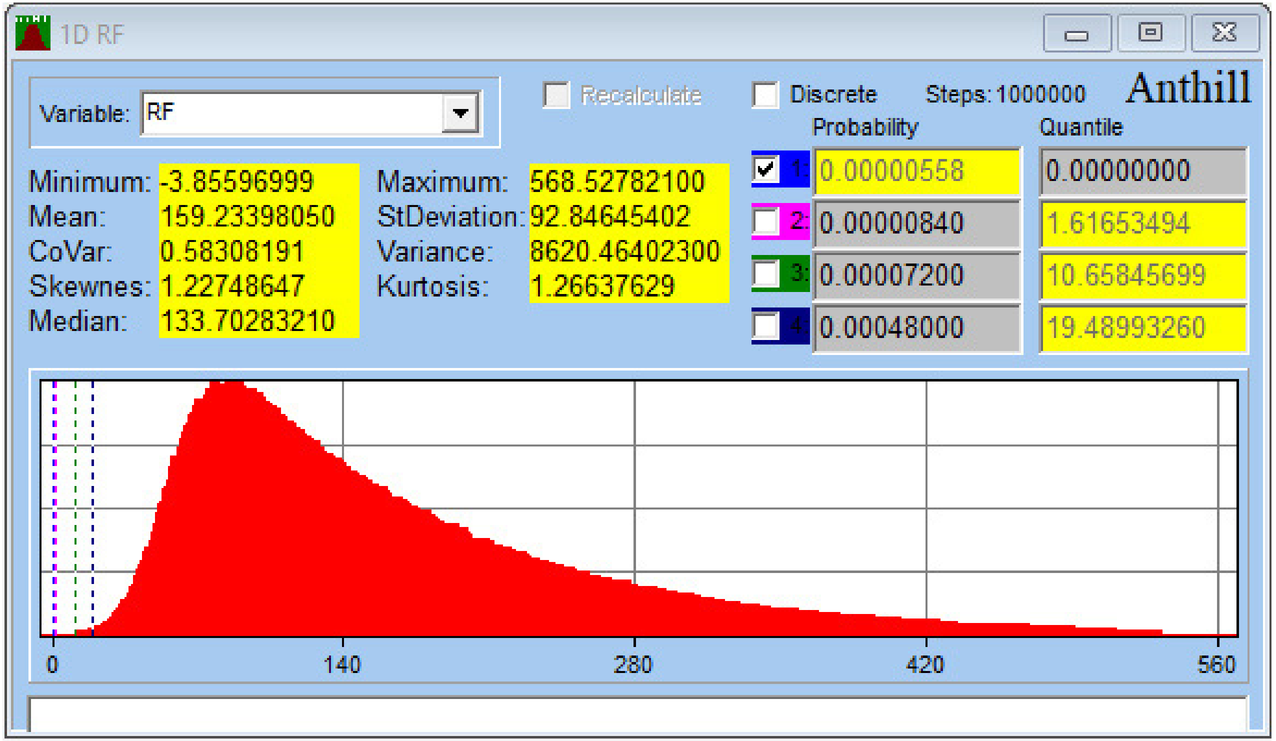


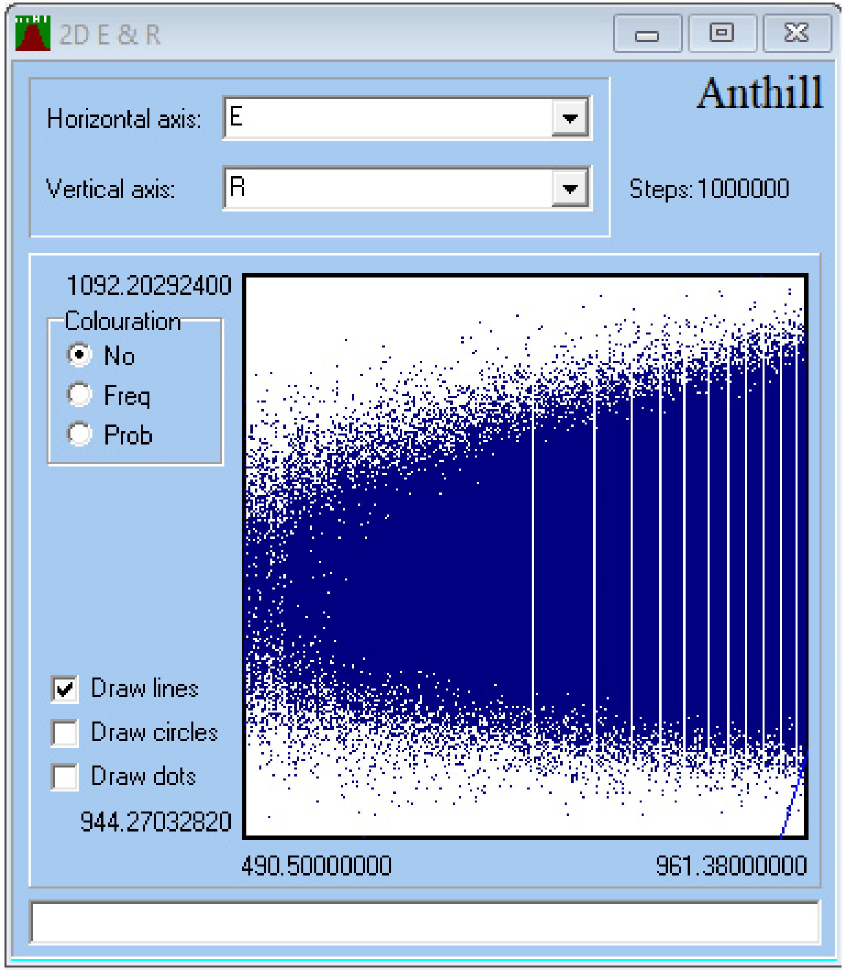
Coefficient of friction COF,
normal distribution,
0.80 ... 0.85

Example 10: Calculation in the AntHill program

Slipping in the contact surface

Results:





Probability of failure $P_f = 5.58 \cdot 10^{-6}$

Example 10: Calculation in the AntHill program

Flipping around point C and **slipping** in the contact surface, assessment of both conditions together:

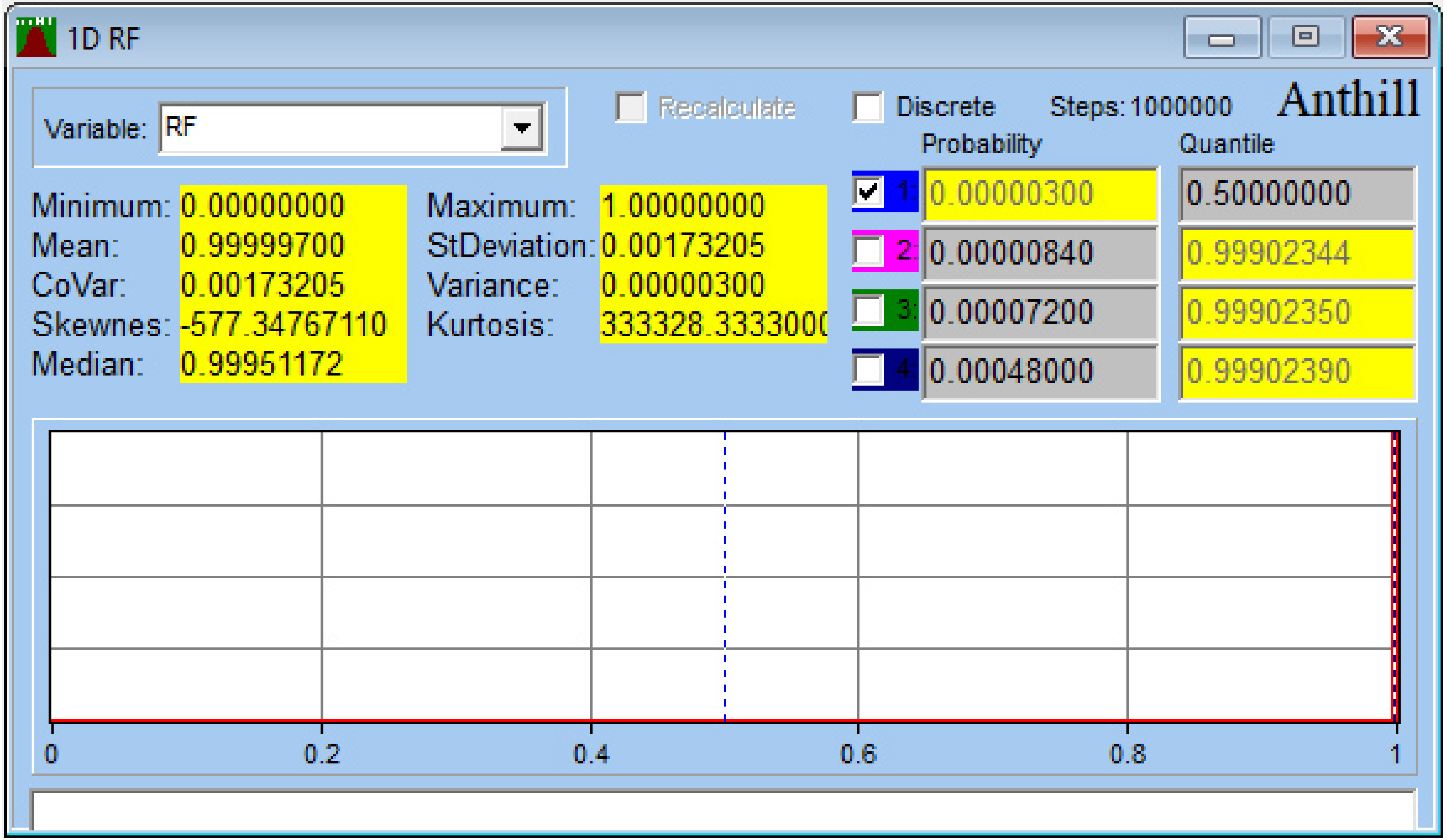
Computational model, definition

```
RF=pos(R1-E1)*pos(R2-E2)
E2=H2
R2=(COF*(P2+G1+G2+G3+P1)+H1)/Beta
E1=H2*wl2/3
R1=(P2*(w1+w2+w3-w1*wl2/h/3)+G1*(w1/3+w2+w3)+G2*(w2/2+w3)+G3*2/3*w3+P1*w3*wl1/h/3+H1*wl1/3)/Beta
; Loads
H1=wl1^2*1000*9.81/2/1000
P1=wl1*w3*wl1/h*1000*9.81/2/1000
H2=wl2^2*1000*9.81/2/1000
P2=wl2*w1*wl2/h*1000*9.81/2/1000
G1=w1*h/2*roc*9.81/1000
G2=w2*h*roc*9.81/1000
G3=w3*h/2*roc*9.81/1000
; Input random variables
wl1=3+2*norm1          ; Water level h1, normal, 3...5 m
wl2=10+(1-exp1/5)*4    ; Water level h2, exponential, 10...14 m
roc=2800+200*norm2     ; Bulk density of concrete, normal, 2800...3000 kg/m^3
COF=0.80+0.05*norm3    ; Coefficient of friction, normal, 0.80...0.85
```

Example 10: Calculation in the AntHill program

Flipping around point C and **slipping** in the contact surface, assessment of both conditions together:

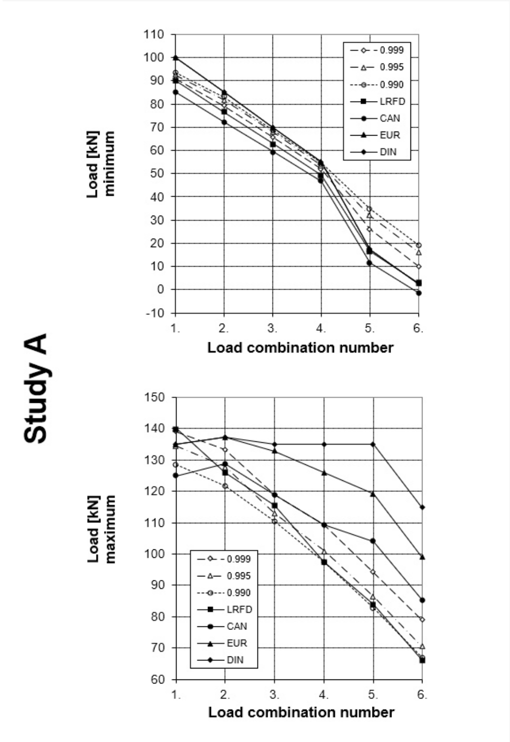
Results:

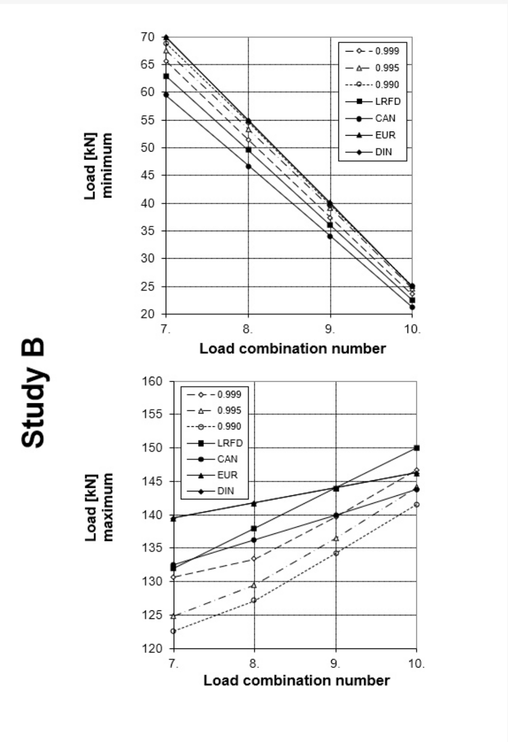


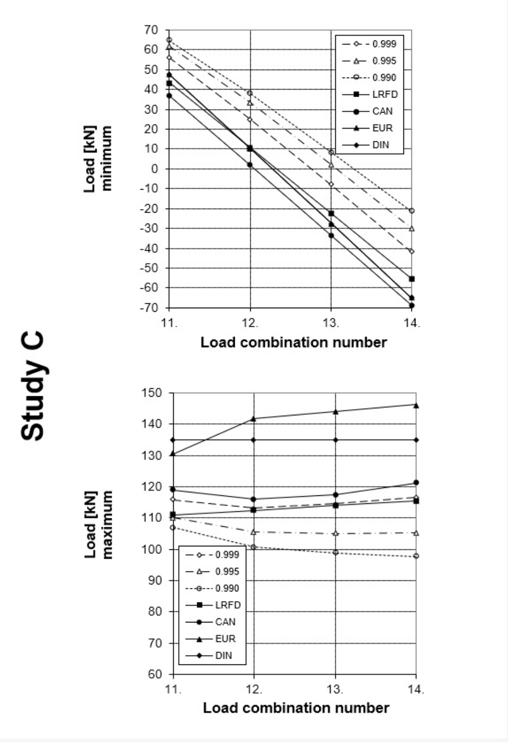
Probability of failure

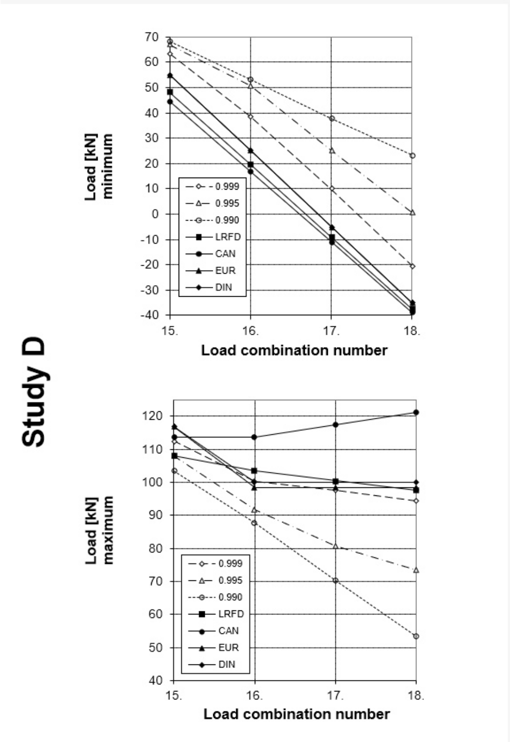
$$P_f = 3 \cdot 10^{-6}$$

Combination of Loads



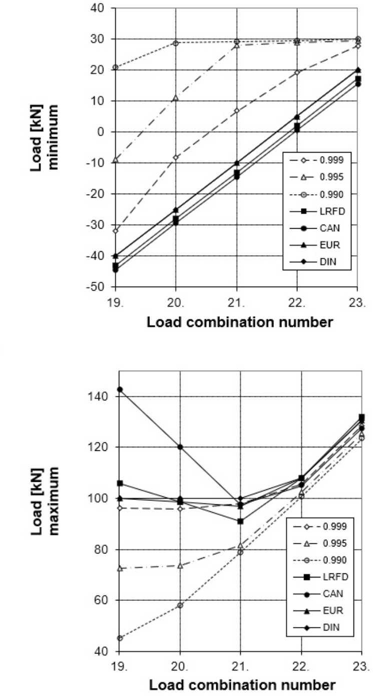




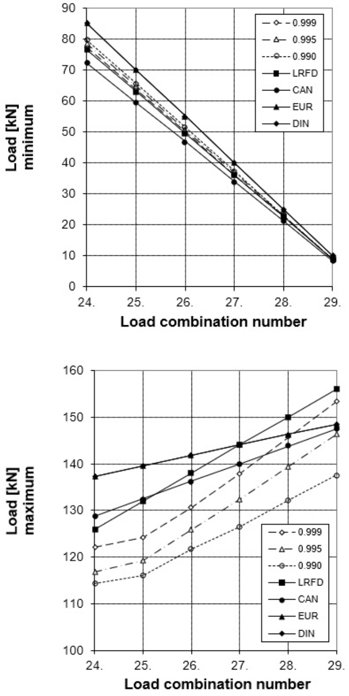


Combination of Loads

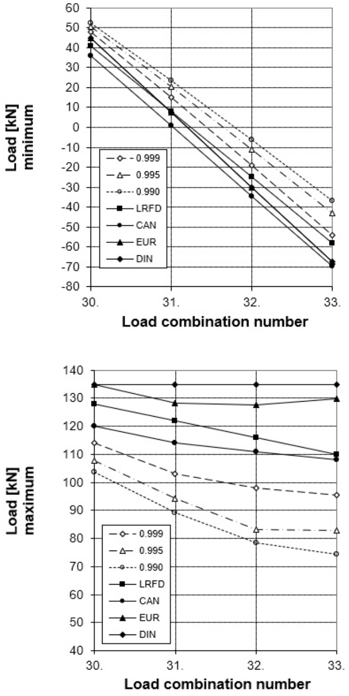
Study E



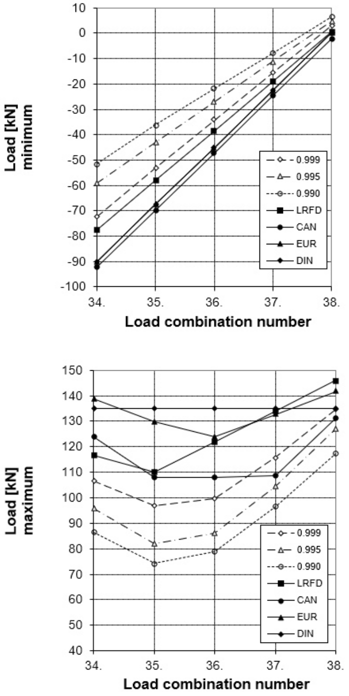
Study F



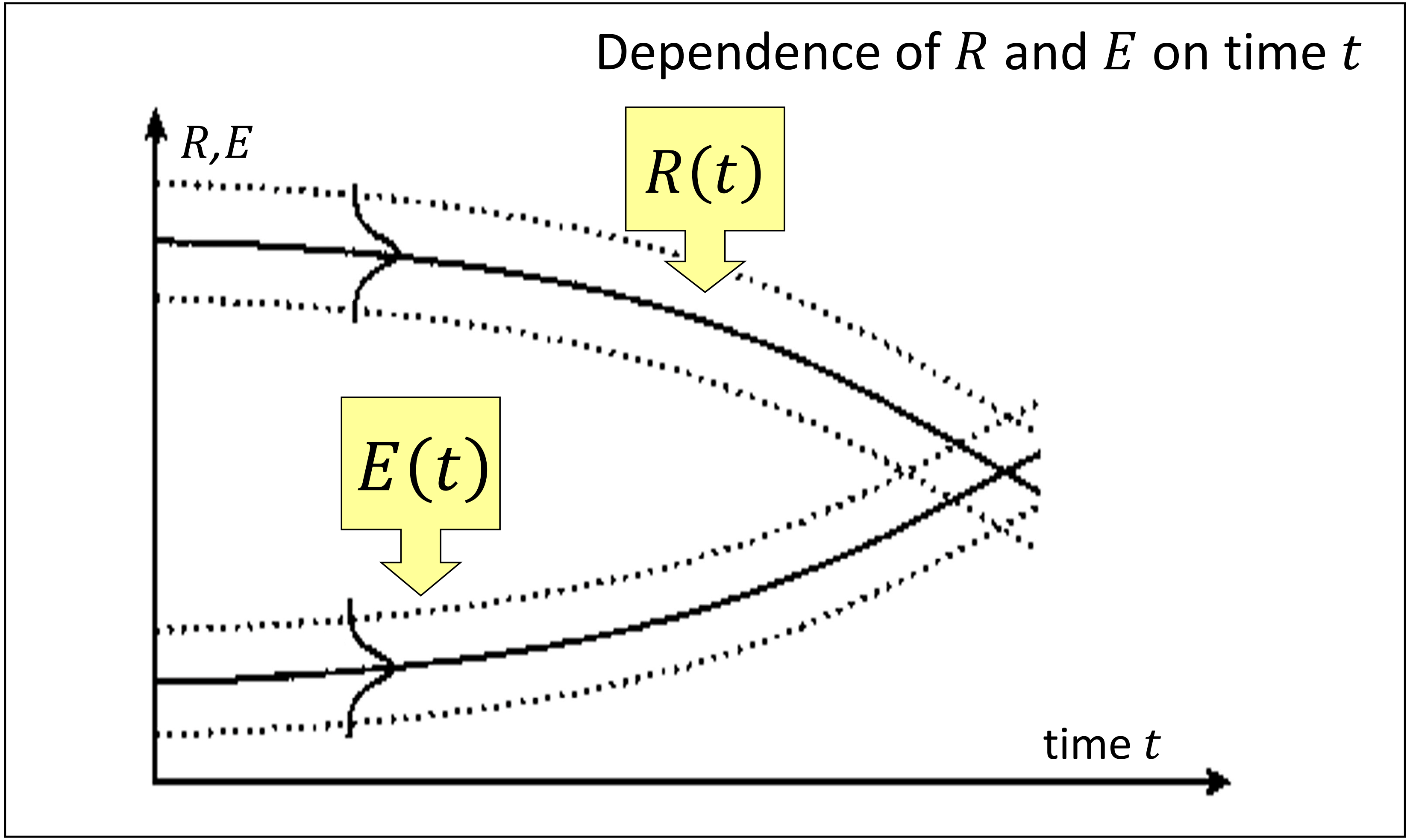
Study G



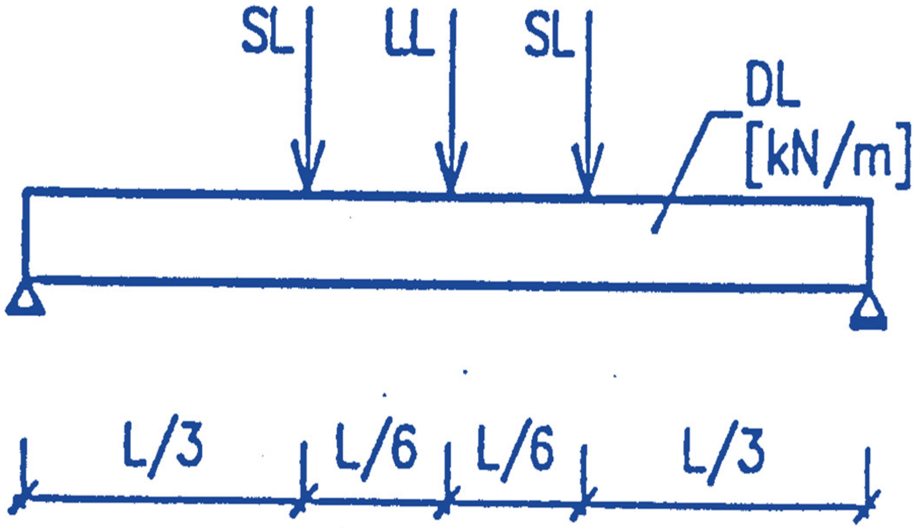
Study H

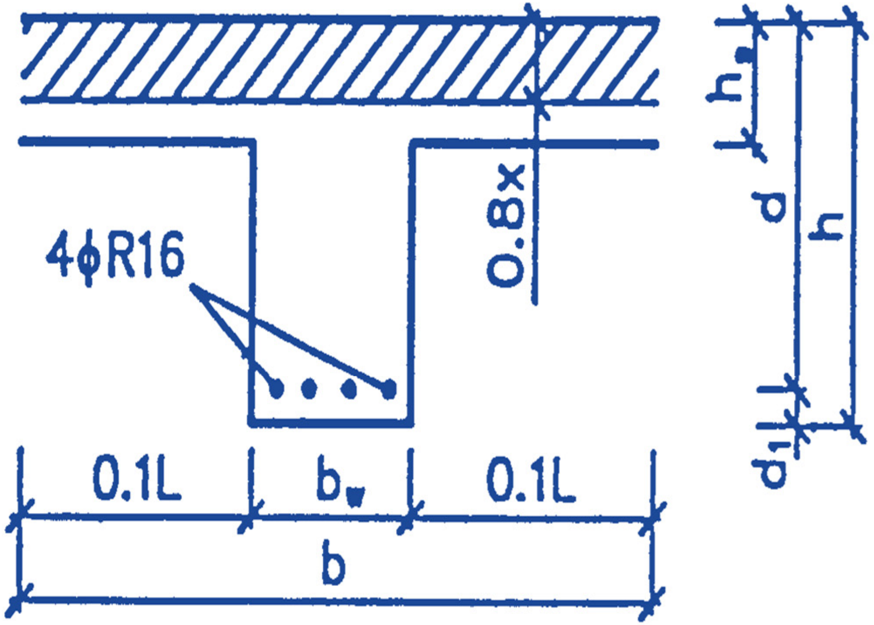


Assessment of the Structural Service Life



Calculation of Safe Beam Operation Time

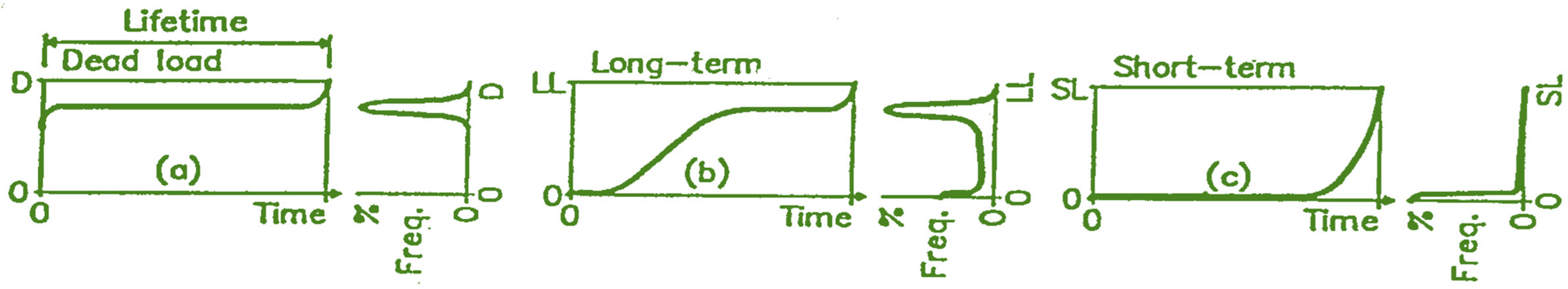




Load Effect Duration Curves

$$RF = \{M_R(t) - M_E(t)\}$$

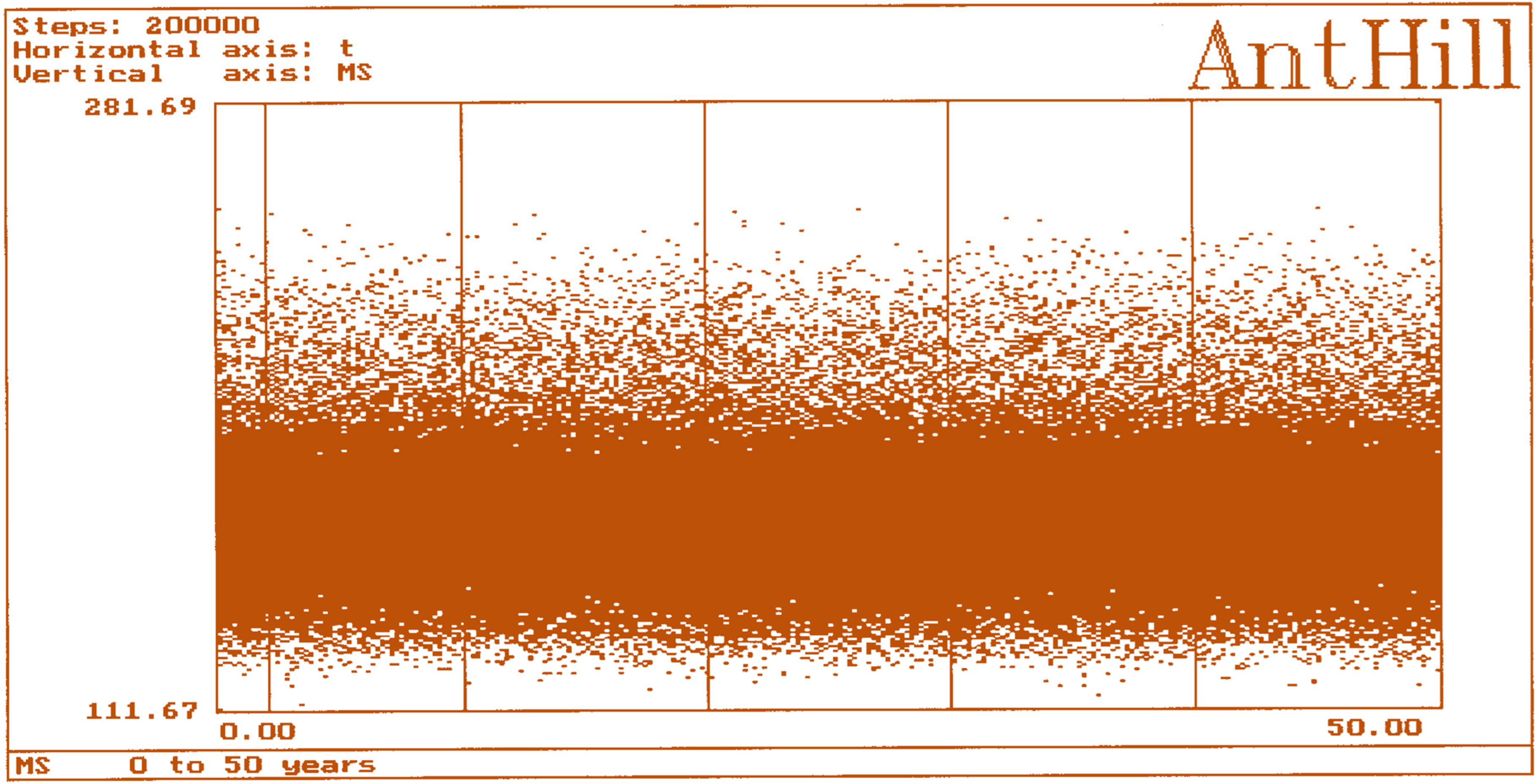
$$M_E(t) = \frac{1}{8} \cdot (DL \cdot DL_{var}) \cdot L^2 + \frac{1}{4} \cdot (LL \cdot LL_{var}) \cdot L + \frac{1}{3} \cdot (SL \cdot SL_{var}) \cdot L$$



$$M_R(t) = \left(A_s \cdot A_{s,var}\right) \cdot f_y \cdot z$$ [kN.m]

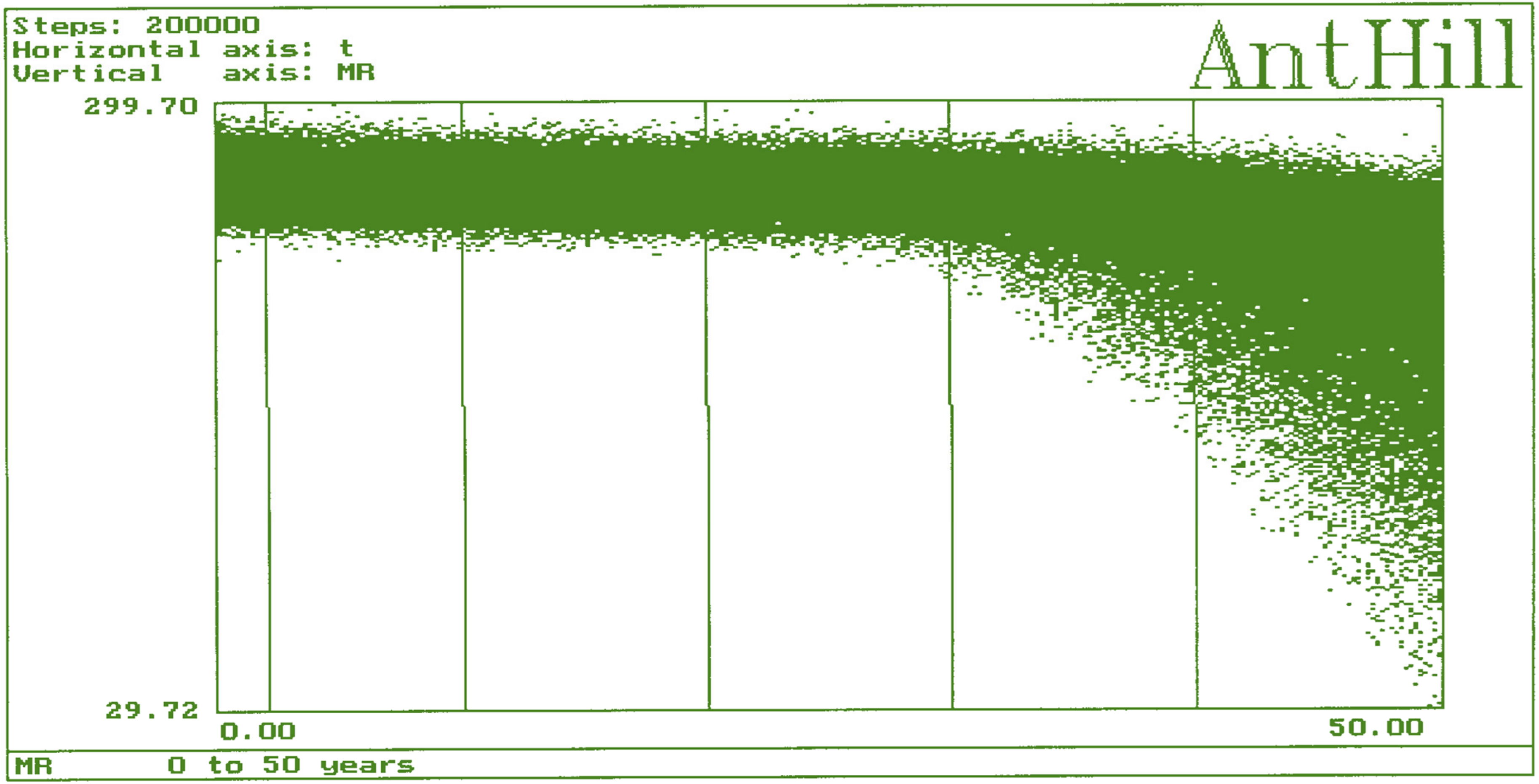
Load Effect

$M_E(t)$ [kN.m] **0 to 50 years**



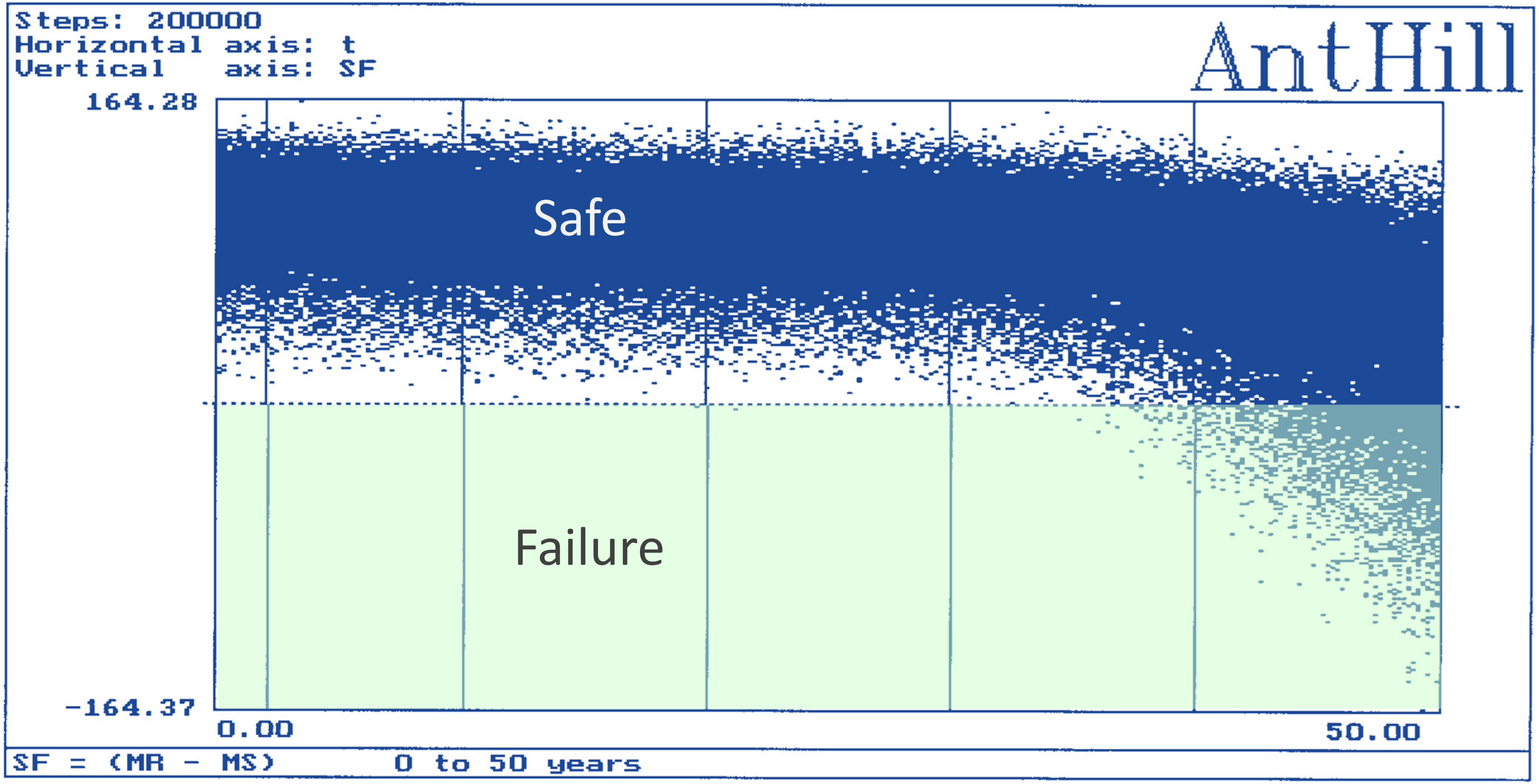
Structural Resistance

$M_R(t)$ [kN.m] **0 to 50 years**



Reliability Function

$RF(t)$ [kN.m] **0 to 50 years**



Reliability Function

$P_f = 0.00005, t = 30$ years