Introduction to the Subject

- Reliability of Building Structures
- Methods of Structural Design
- Probabilistic Approach

Reliability of Building Structures

Reliability is generally defined as a property of things serve the purpose for which it was made.

Structural reliability or reliability of supporting element - ability to perform the requirements under the specified conditions during its design life:

- **Safety**,
- **Economy**,
- **Service life** (durability) and usability of designed components and systems or under assessment.

Photo:
Assoc. Prof. Karel Kubečka

Reliability of Building Structures

The origin appearance of the Mississippi Bridge from 1967, Minneapolis

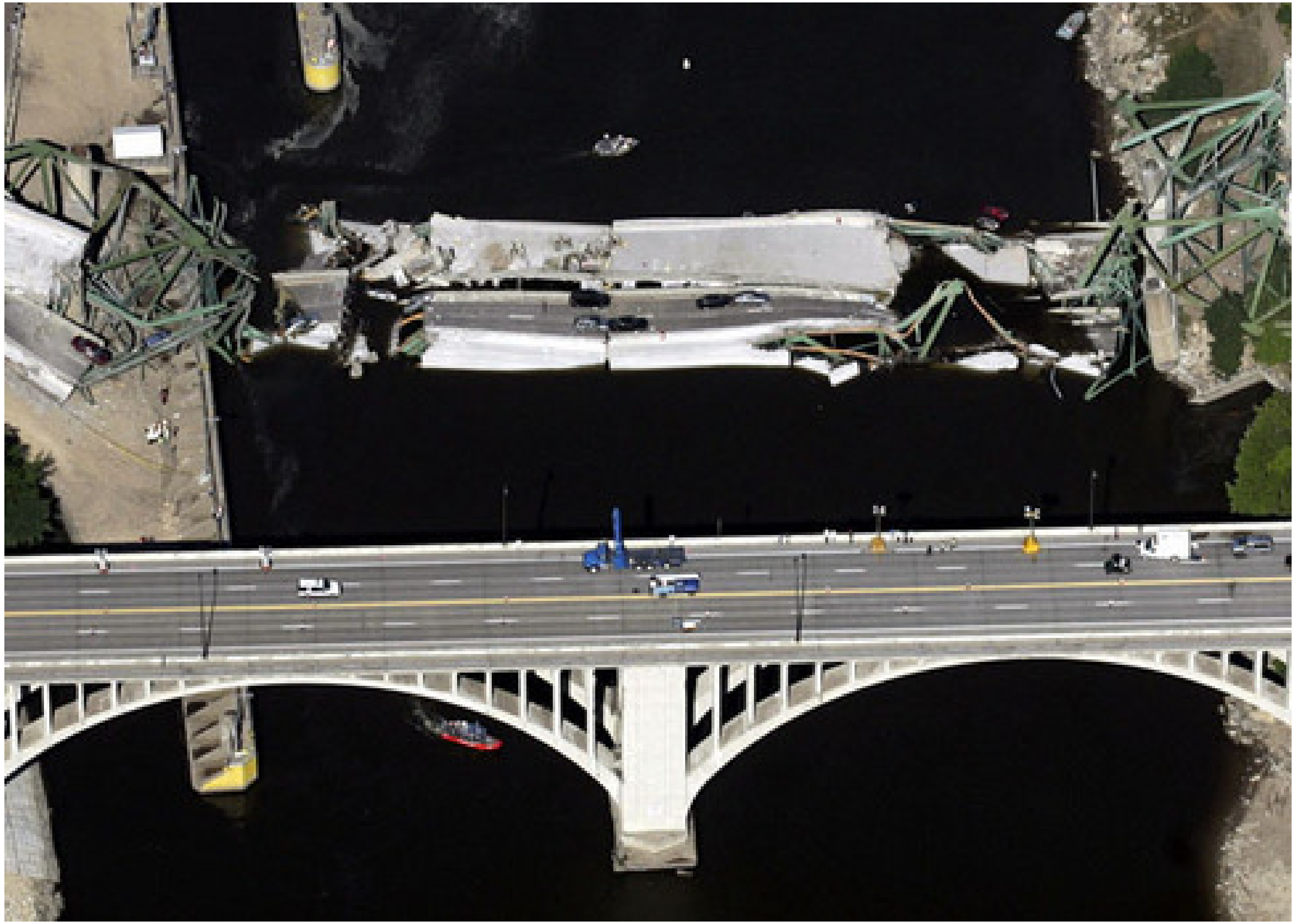
Reliability of Building Structures

Ruined bridge over the Mississippi, Minneapolis, August 2007

Reliability of Building Structures

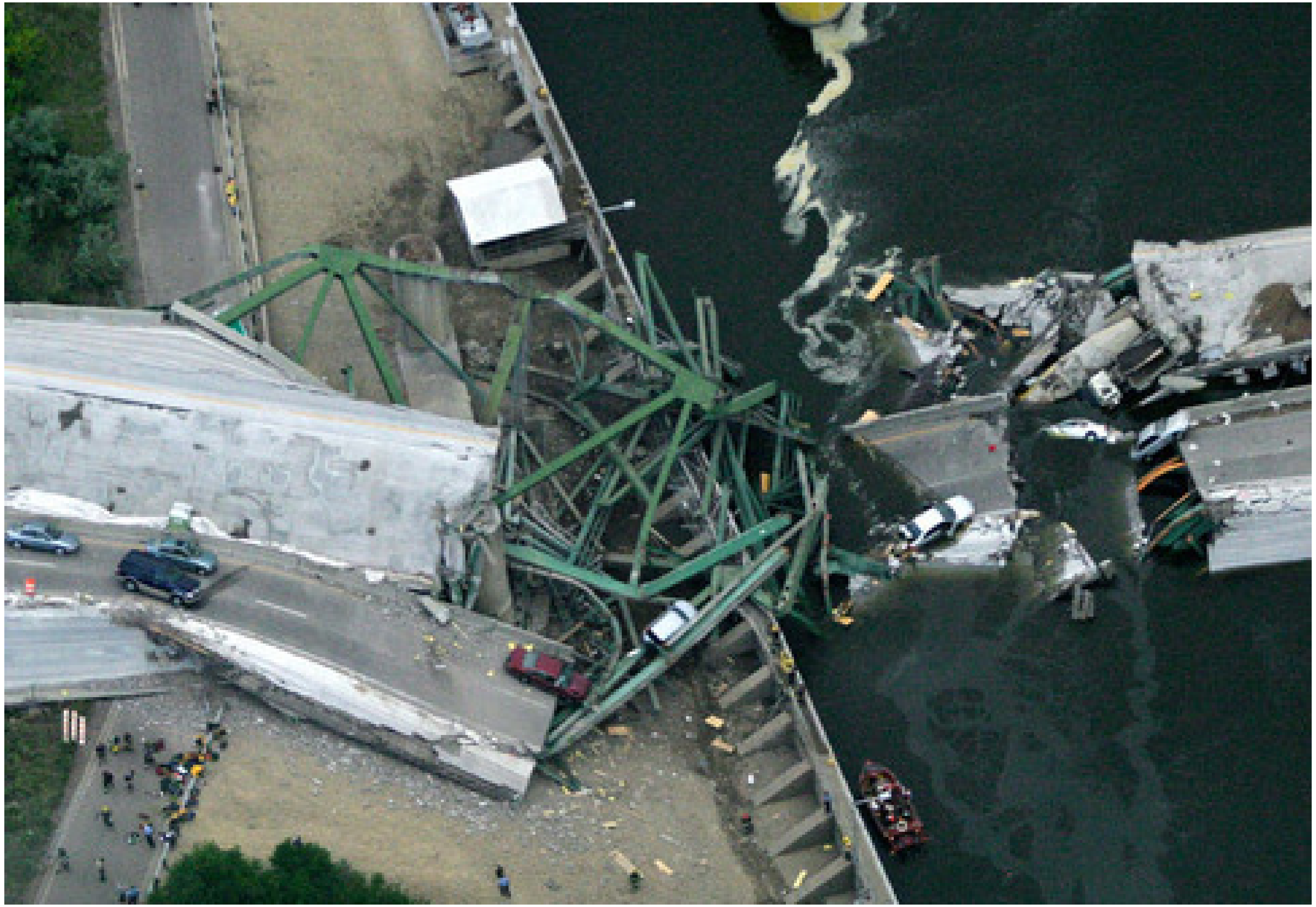
Ruined bridge over the Mississippi, Minneapolis, August 2007

Reliability of Building Structures



Ruined bridge over the Mississippi, Minneapolis, August 2007

Reliability of Building Structures



Ruined bridge over the Mississippi, Minneapolis, August 2007

Reliability of Building Structures

Ruined bridge over the Mississippi, Minneapolis, August 2007

Reliability of Building Structures

Registered office of the construction company TCHAS, Ostrava, photo:
Assoc. Prof. Karel Kubecka

Reliability of Building Structures

Registered office of the construction company TCHAS, Ostrava, photo:
Assoc. Prof. Karel Kubecka

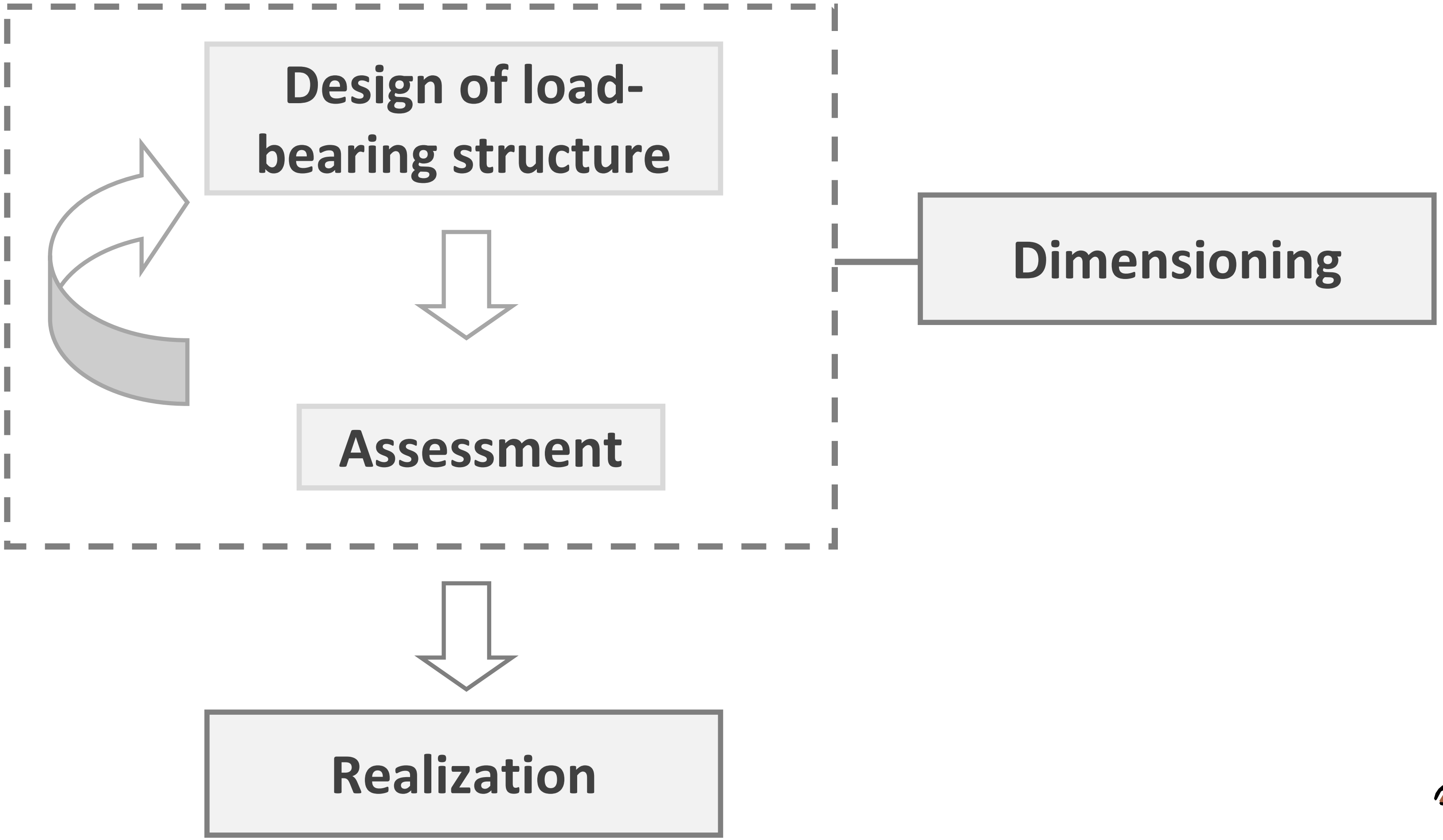
Reliability of Building Structures

Destruction of a concrete truss, photo: Prof. Radim Cajka

Reliability of Building Structures

Destruction of a wooden truss in Ostrava, photo: Assoc. Prof. Karel Kubecka

Principles of Building Structures Design



Methods of Structural Design

It must include the **uncertainties** and ensure reliability of structures under design, depending on the available **experimental and theoretical knowledge** in the field of structural mechanics, elasticity theory and mathematical statistics.

Development of various methods of designing structures gradually stabilized at three commonly used methods which are applied in various modifications in the **standards for structural design** today:

a) **Allowable Strength Design** (ASD),

b) **Load Factor** or **Ultimate Load Method**,

c) **Limit State Design** (LSD).

The acceptable limits of safety and serviceability requirements before failure occurs is called a limit state.

Allowable Strength Design (ASD)

The first worldwide method of building construction design. It is based on the condition that the stress σ in the structure - the effect of the operating load, is less than the allowable stress of the material σ_{allow} divided by the factor μ :

$$\sigma \leq \frac{\sigma_{allow}}{\mu}$$

The factor μ was determined with regard to the **uncertainties** in determining the effect of loading and resistance of the material, and should therefore ensure sufficient reliability of the entire structure.

The main shortcomings: the impossibility of individual consideration of the uncertainties of individual basic quantities and computational models for determining the effect of loading and resistance of the structure.

Load Factor, Ultimate Load Method

The second widespread method of designing building structures (e.g., concrete structures), introduced after World War II.

The method was based on the condition:

$$s = \frac{\bar{R}}{\bar{S}} > s_0$$

A method with a better description of the behavior of the element and its cross-sections, expressed by the resistance of the cross-section $\bar{R}$ and the effect of the load $\bar{S}$.

The **factor of safety** s_0 was prescribed by different values for different types of stress.

Main shortcoming: inability to take into account the uncertainties of individual basic quantities and theoretical models (as well as the method of allowable stresses).

Limit State Design

In practice implemented in half of the last century approximately (in Czech Republic since sixties, prof. Hruban).

Hangar "F" Ruzyn - the first structure in the Czech Republic, designed according the method of limit states (1966, M. Horák)

Konrád Jaroslav HRUBAN
(1893 - 1977)

Main benefits:

- Progressive approach to the **evaluation of random variables** incoming into the **conditions of of reliability**,
- A complex overview of the **load capacity** and **serviceability criteria**, which are based on **statistics** and **probability calculation**.

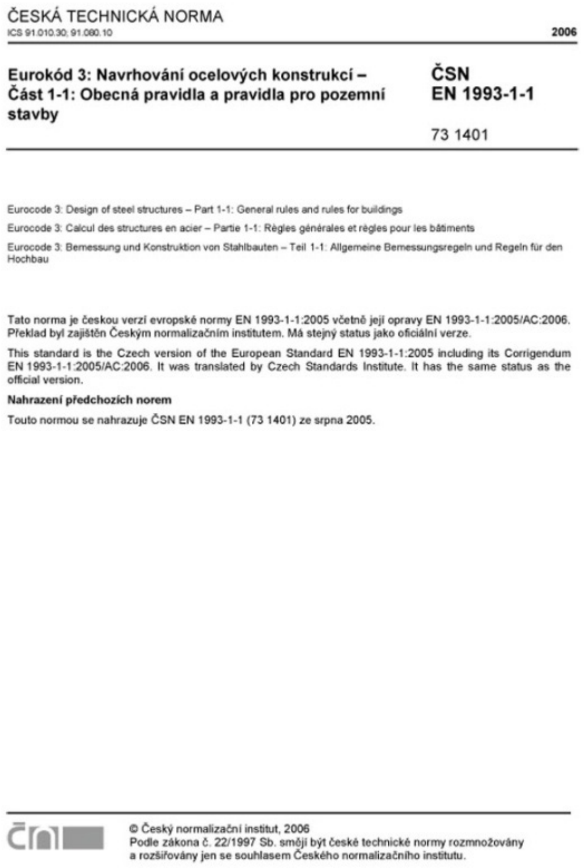
Limit State Design

Structural reliability is verified using **partial safety factors** (method of partial safety factors) - semi-probabilistic method,

Design becomes unreliable if exceeds one of limit states:

- **Ultimate Limit State**,
- **Serviceability Limit State**.

Limit State Design is used on **Eurocodes**.


ČESKÁ TECHNICKÁ NORMA

ICS 91.010.30; 91.080.10 2006

Eurokód 3: Navrhování ocelových konstrukcí – Část 1-1: Obecná pravidla a pravidla pro pozemní stavby

ČSN EN 1993-1-1

73 1401

Eurocode 3: Design of steel structures – Part 1-1: General rules and rules for buildings

Eurocode 3: Calcul des structures en acier – Partie 1-1: Règles générales et règles pour les bâtiments

Eurocode 3: Bemessung und Konstruktion von Stahlbauten – Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau

Tato norma je českou verzí evropské normy EN 1993-1-1:2005 včetně její opravy EN 1993-1-1:2005/AC:2006. Překlad byl zajištěn Českým normalizačním institutem. Má stejný status jako oficiální verze.

This standard is the Czech version of the European Standard EN 1993-1-1:2005 including its Corrigendum EN 1993-1-1:2005/AC:2006. It was translated by Czech Standards Institute. It has the same status as the official version.

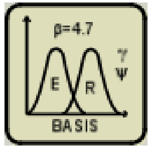
Nahrazení předchozích norem

Touto normou se nahrazuje ČSN EN 1993-1-1 (73 1401) ze srpna 2005.

© Český normalizační institut, 2006
Podle zákona č. 22/1997 Sb. smějí být české technické normy rozmnožovány a rozšiřovány jen se souhlasem Českého normalizačního institutu.

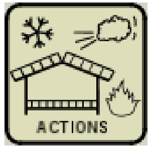
Eurocode 3: Design of steel structures - Part 1-3: General rules - Supplementary rules for cold-formed members and sheeting

Eurocodes for Structural Design



EN 1990 Eurocode: Basis of structural design

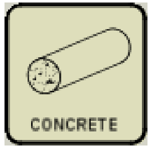
EN 1991 Eurocode 1: Actions on structures



EN 1991-1-1	General actions – Densities, self-weight, imposed loads for buildings
EN 1991-1-2	General actions – Actions on structures exposed to fire
EN 1991-1-3	General actions – Snow loads
EN 1991-1-4	General actions – Wind loads
EN 1991-1-5	General actions – Thermal Actions
EN 1991-1-6	General actions – Actions during execution
EN 1991-1-7	General actions – Accidental actions
EN 1991-2	Traffic loads on bridges
EN 1991-3	Actions induced by cranes and machinery
EN 1991-4	Silos and tanks

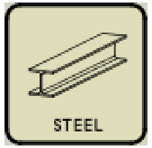
Eurocodes for Structural Design

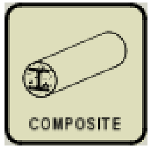
A set of **57 European standards** in total for building design



EN 1992 Eurocode 2: Design of concrete structures

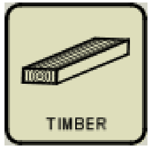
EN 1993 Eurocode 3: Design of steel structures

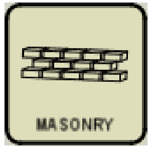




EN 1994 Eurocode 4: Design of composite steel and concrete structures

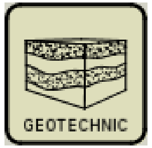
EN 1995 Eurocode 5: Design of timber structures





EN 1996 Eurocode 6: Design of masonry structures

EN 1997 Eurocode 7: Geotechnical design



EN 1998 Eurocode 8: Design of structures for earthquake resistance

EN 1999 Eurocode 9: Design of aluminium structures

Ultimate Limit State

Exceeding the limit state design has resulted in structural failure and usually trigger the need for general repairs or removal of construction:

- The **complete** or **partial collapse**,
- **Violation of the element's integrity** (breaking, fracture),
- **Loss of stability** (tilting retaining walls, landslide object).

Random Input Variables

Randomness is applied to each part of the system, including:

Structure:
- material properties
- geometric inaccuracies (imperfections, cross-section properties)

Load:
- dead
- live load
- climatic influences (wind, snow)

Environment:
- moisture (corrosion)

Partial Safety Factors

- Reduce the probability of exceeding the limit state design is done by adjusting the characteristic load values and material properties, i.e., the introduction of **design values**, **partial safety factors** γ:

$$\gamma \geq 1 \qquad E_d = E_k . \gamma \qquad R_d = \frac{R_k}{\gamma_M}$$

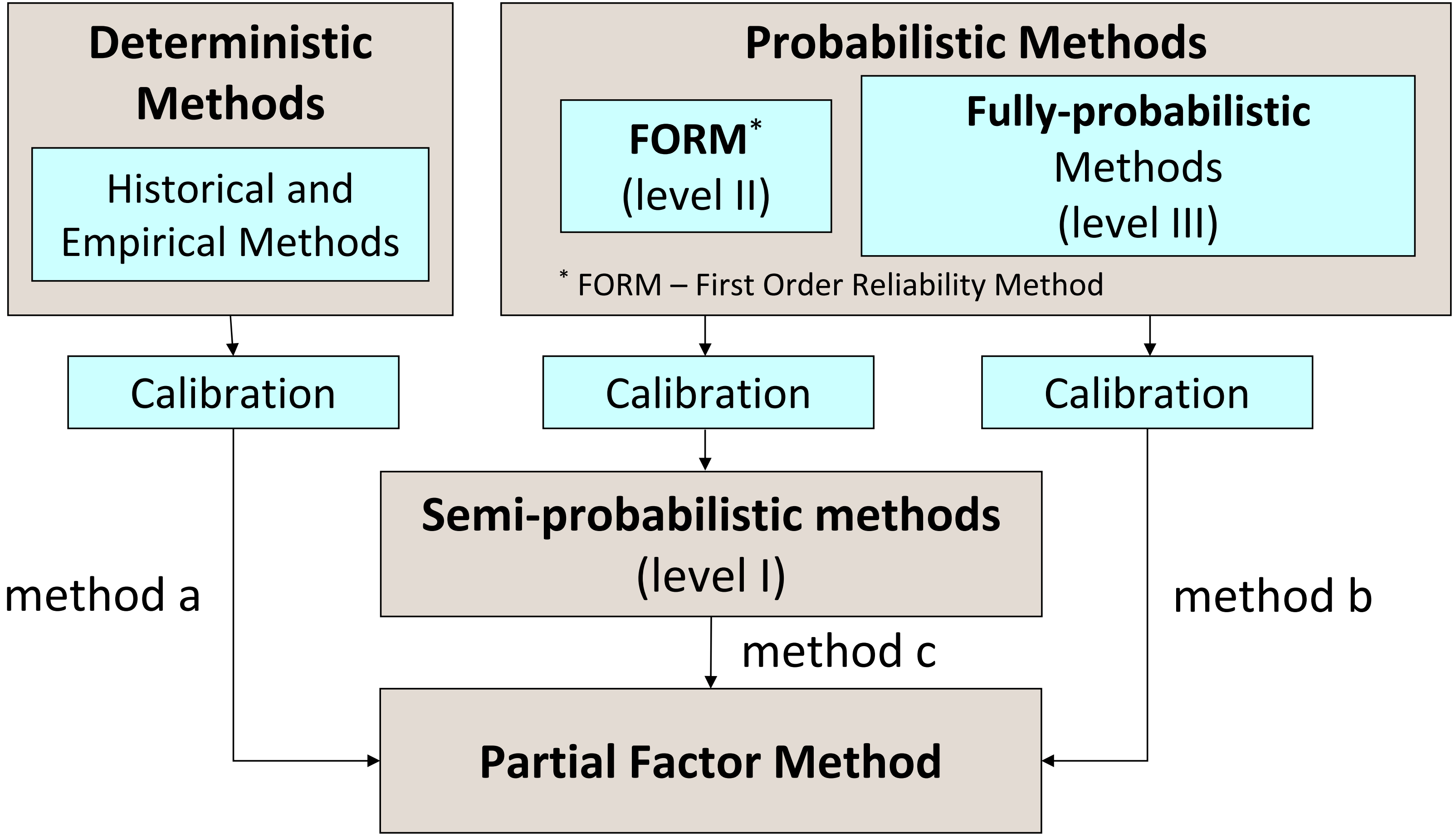
- The **characteristic values** of the basic variables (loads, geometry, material properties) are derived from statistical characteristics of these variables.
- Methodology for the calculation according to the EC introduced across EU, but some numerical values are chosen in each country individually - **National Foreword** and the **National Appendix**.

Partial Safety Factors

Partial safety factors γ are possible to obtain:

- **Calibration** of these factors based on:
    - long-time experience of building practice,
    - comparison with national standards,
    - comparative analyzes, including probabilistic approaches, which are based on probabilistic methods of reliability theory.
- **Statistical evaluation** of the experimental data and tests.

Overview of Reliability Methods



Differentiation Reliability of Structures

Based on:

- selecting values of **reliability indexes**,
- **adjustment of partial factors** for loads or properties related to resistance,
- level of **control in the design**,
- level of **control during implementation**,
- level of **inspection and compliance** with procedures referred to project documentation.

For the purposes of reliability differentiation EC recommended three classes' consequences **CC1 to CC3** (*consequences classes*).

Consequences Classes (EN 1990)

Consequences Class	*Description*	*Examples of buildings and civil engineering works*
CC3	**Serious consequences** for loss of human life, or for economic, social or environmental concerns	Grandstands, public buildings where consequences of failure are high (e.g., a concert hall)
CC2	**Moderate consequence** for loss of human life; economic, social or environmental consequences considerable	Residential and office buildings, public buildings where consequences of failure are medium (e.g., an office building)
CC1	**Low consequence** for loss of human life; economic, social or environmental consequences small or negligible	Agricultural buildings where people do not normally enter (e.g., storage buildings, greenhouses)

Probabilistic Approach

The confidence in the methods of level II and III expressed in terms of **probabilistic reliability indicators** (β reliability index, failure probability P_f).

Reliability criterion:

$$P_f \leq P_d$$

$$\beta_d < \beta$$

P_f ... probability of failure
P_d ... design value of failure probability

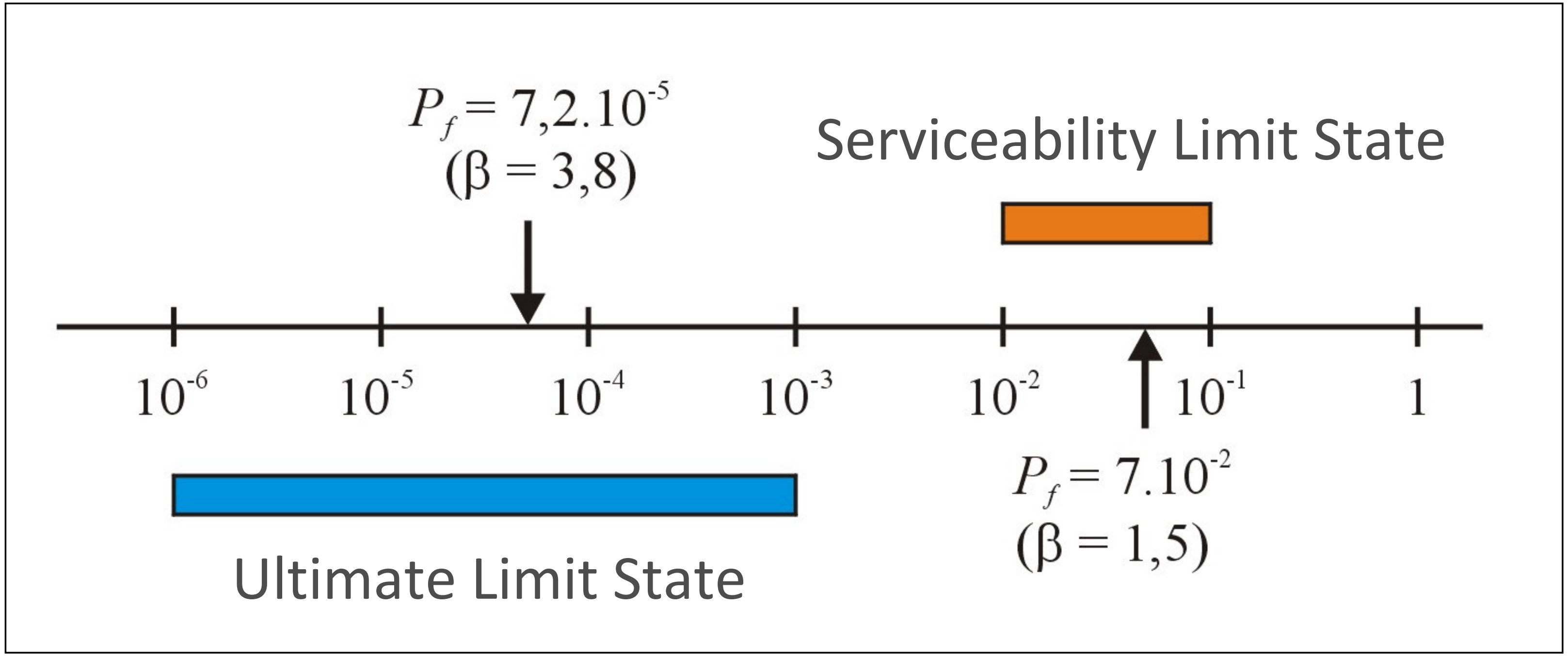
Reliability function:

$$RF = R - E$$

R ... structural resistance
E ... load effect

$$P_f = P(RF < 0) = P(R < E)$$

Indicator of Reliability



The normal range of values for the **probability of failure** P_f
for a design life of 50 years - **Ultimate Limit State** and **Serviceability Limit State**
(and recommended values of the failure probability)

Design Value of Failure Probability

Recommended minimum values of the **reliability index β** and **design value of failure probability P_d** (ultimate limit states) according to EN 1990 :

Reliability Class	Minimum Values β		P_d
	1 year reference period	50 years reference period	
RC3 (serious consequences)	5.2	4.3	$8.4 \cdot 10^{-6}$
RC2 (moderate consequence)	4.7	3.8	$7.2 \cdot 10^{-5}$
RC1 (low consequence)	4.2	3.3	$4.8 \cdot 10^{-4}$

Design Value of Failure Probability

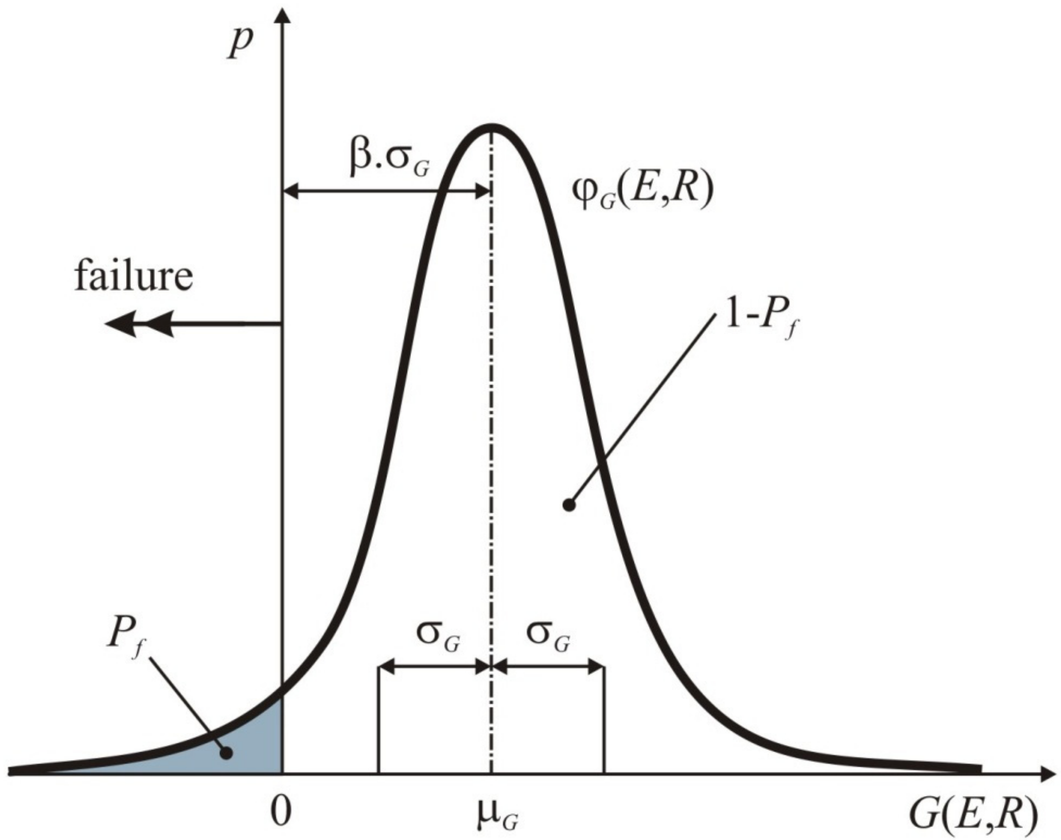
Recommended minimum values of the **reliability index β** and **design value of failure probability P_d** (serviceability limit states) according to EN 1990 :

Reliability Class	Minimum Values β		P_d
	1 year reference period	50 years reference period	
RC2 (moderate consequence)	2.9	1.5	$6.7 \cdot 10^{-2}$

Relation between β and P_f (EN 1990)

P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-7}	10^{-8}
β	1.28	2.32	3.09	3.72	4.27	4.75	5.20

Reliability Index β, Probability of Failure P_f

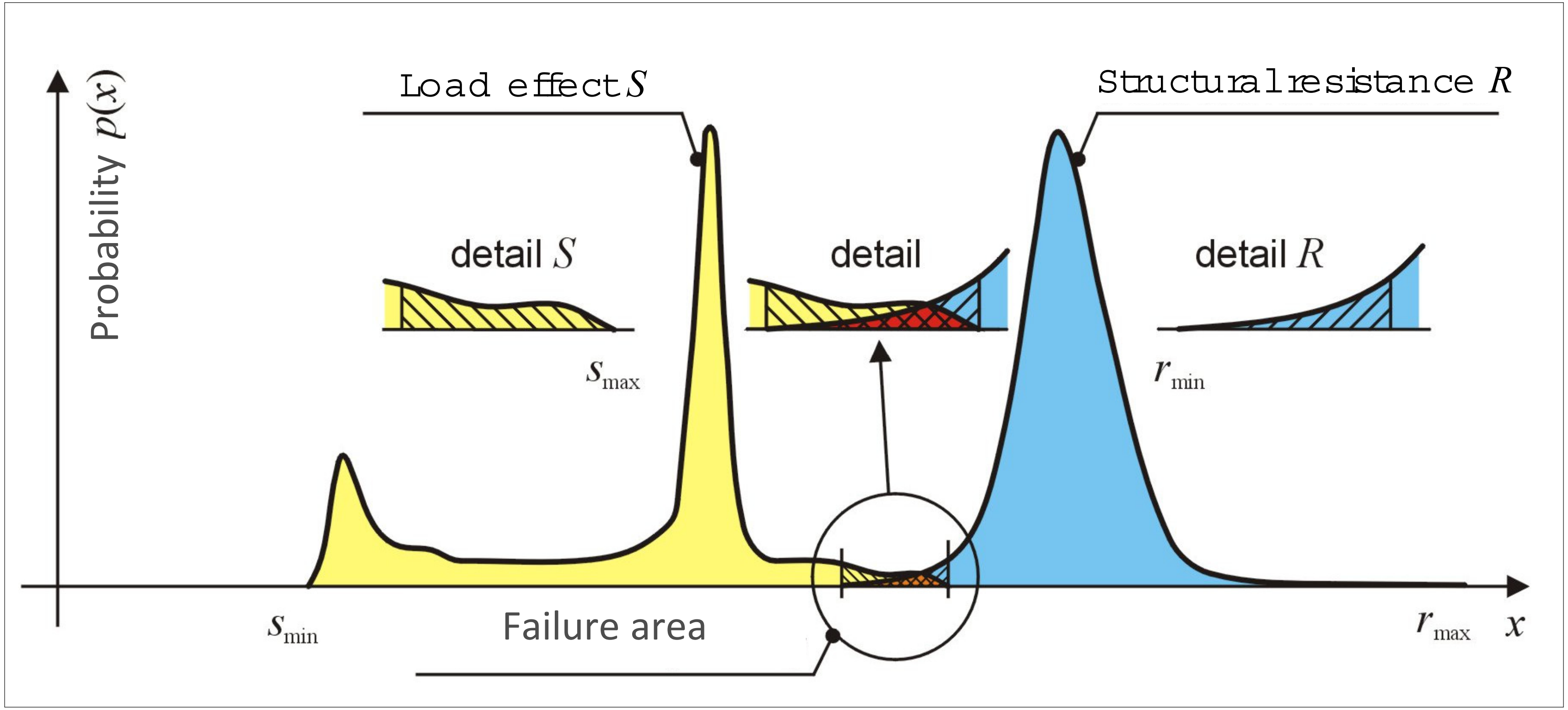


β geometrically represents the average distance μ_g of reliability function G from origin defined in standard deviation units σ_g .

$$\beta = \frac{\mu_g}{\sigma_g}$$

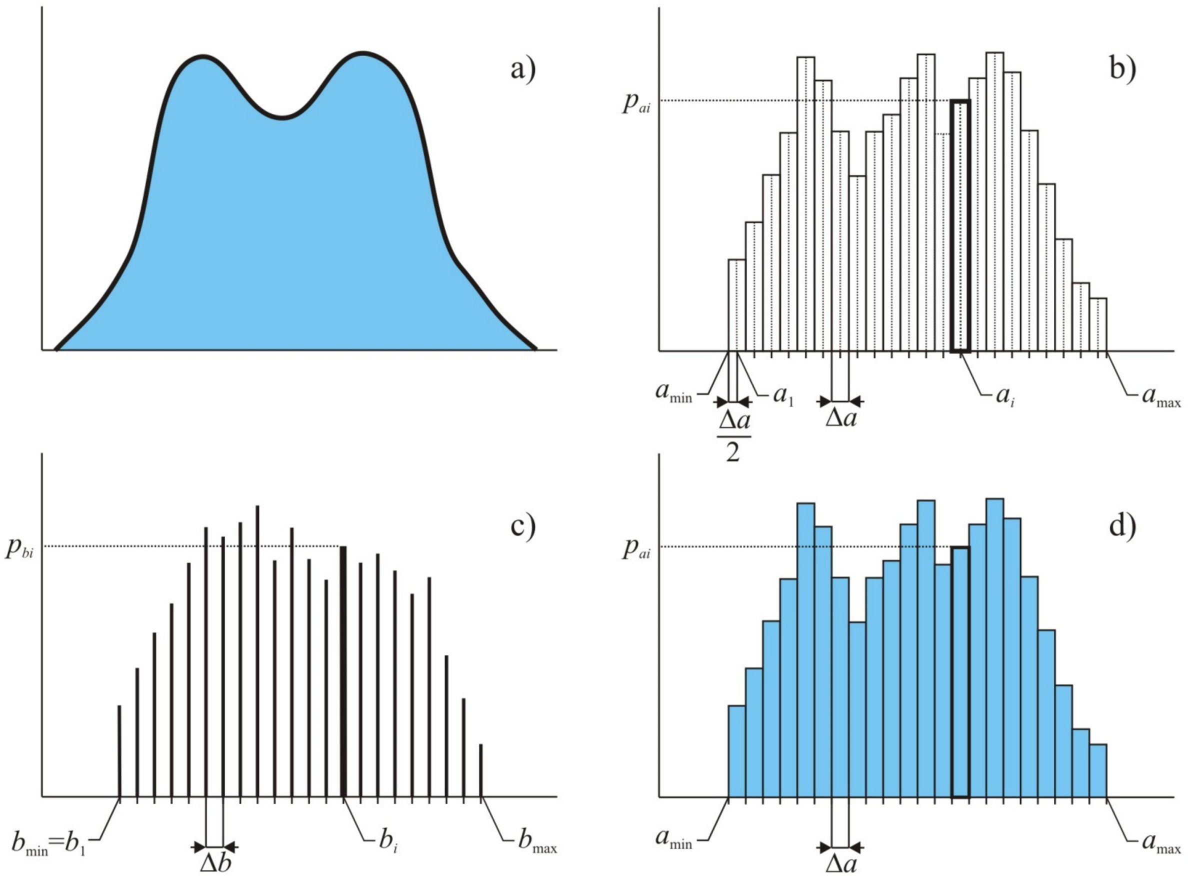
Probability of Failure - Calculation

Failure occurs when the condition is fulfilled: $RF < 0$ $RF = R - S$



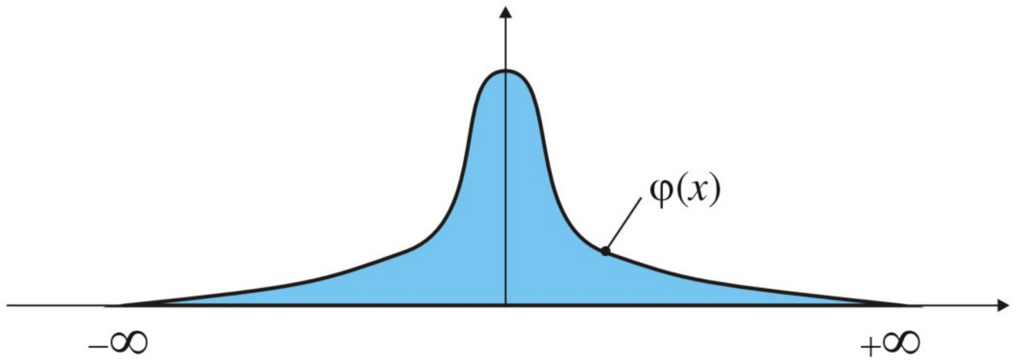
Approximation of Probability Distributions

a) **Original** approximation,
b) **Discrete** approximation,
c) **Pure discrete** approximation,
d) **Piece-wise uniform** approximation

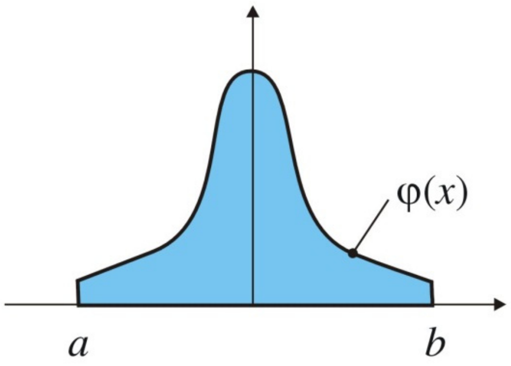


Limited Range of a Random Variable

Unlimited range of the continuous random variable



Limited range of the continuous random variable

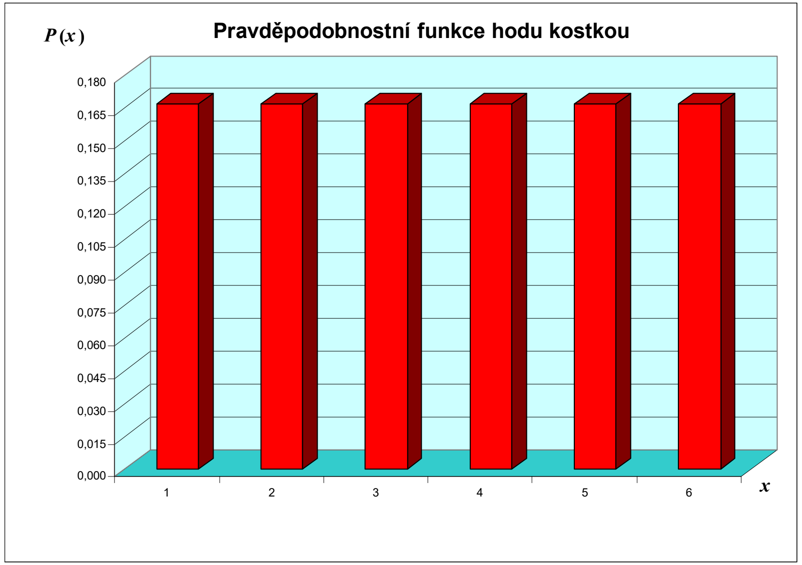


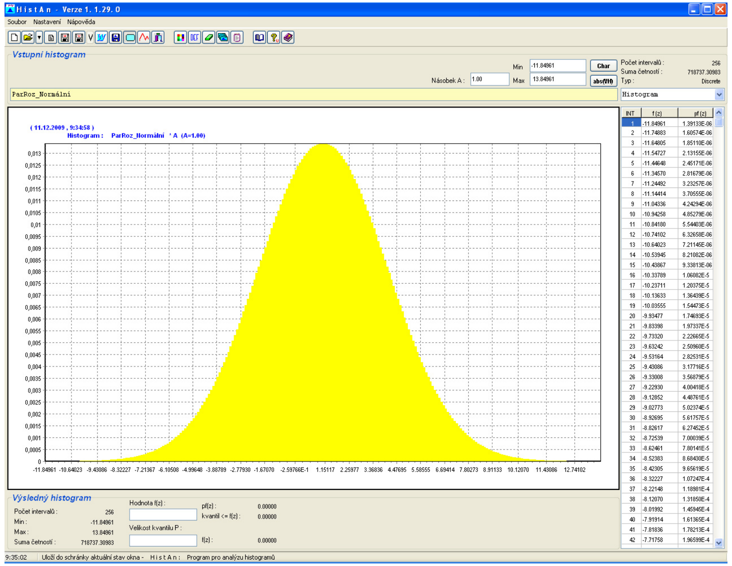
Random Variable

Random variable is defined by **probability distribution**.

Random variables:

- **discrete** - numerous number of values (finite and infinite),
- **continuous** - values from interval (finite and infinite).

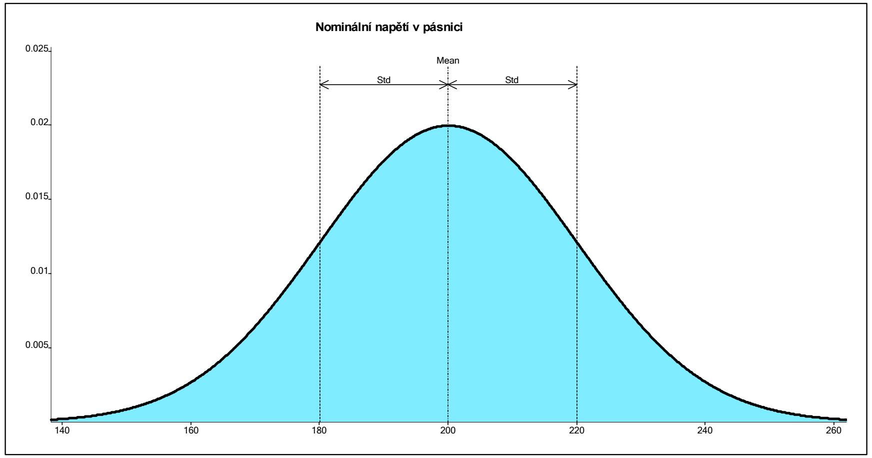




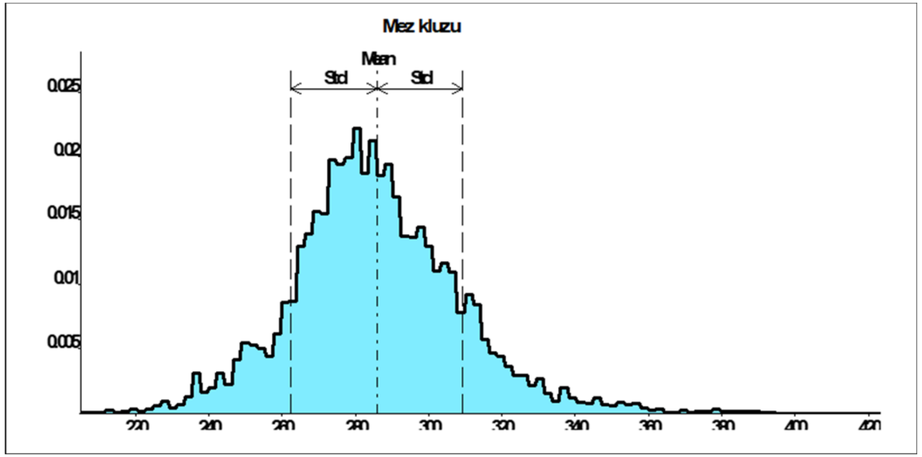
Probability Distribution

Parametric probability distribution - probabilities defined by analytical function – e.g., common expression of normal (Gaussian) probability distribution:

$$f(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



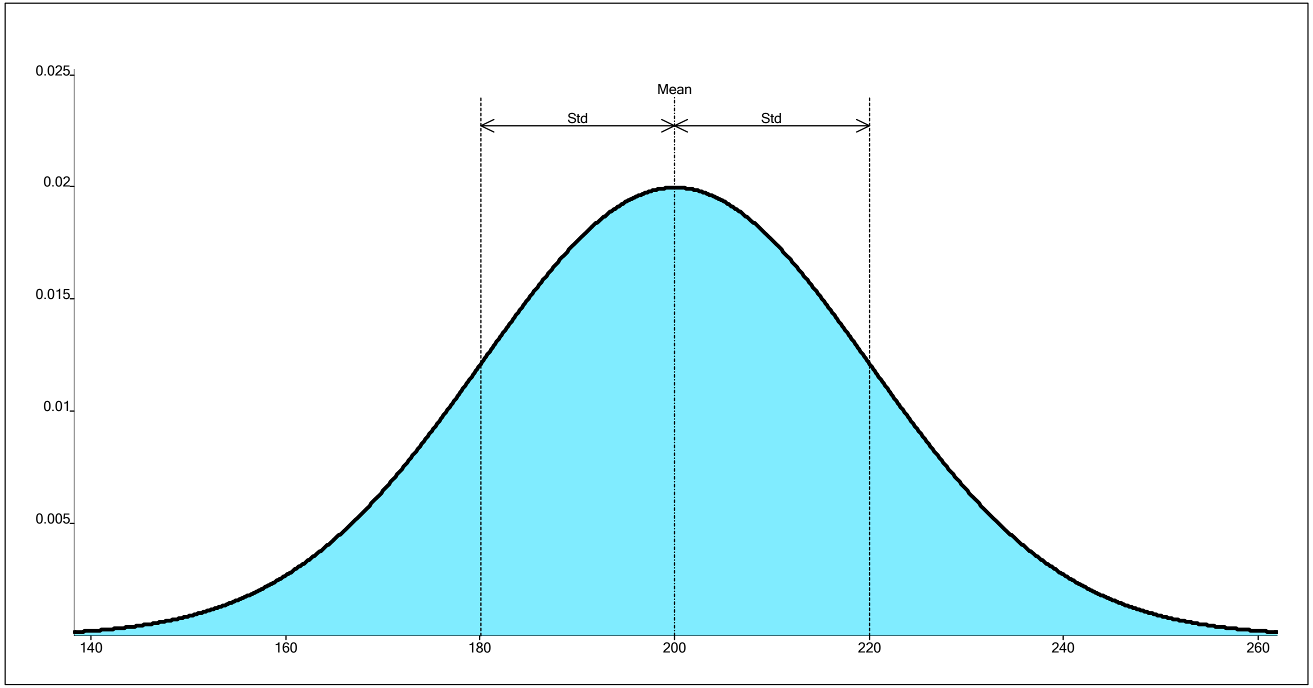
Parameters - characteristics of random variable probability distribution (e.g., μ mean value and σ standard deviation)



Non-parametric (empirical) **probability distribution** - definition based on measurements (often long-term)

Normal (Gaussian) Probability Distribution

Common expression of **normal** (**Gaussian**) **probability distribution**:



$$f(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ ... **mean value**

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

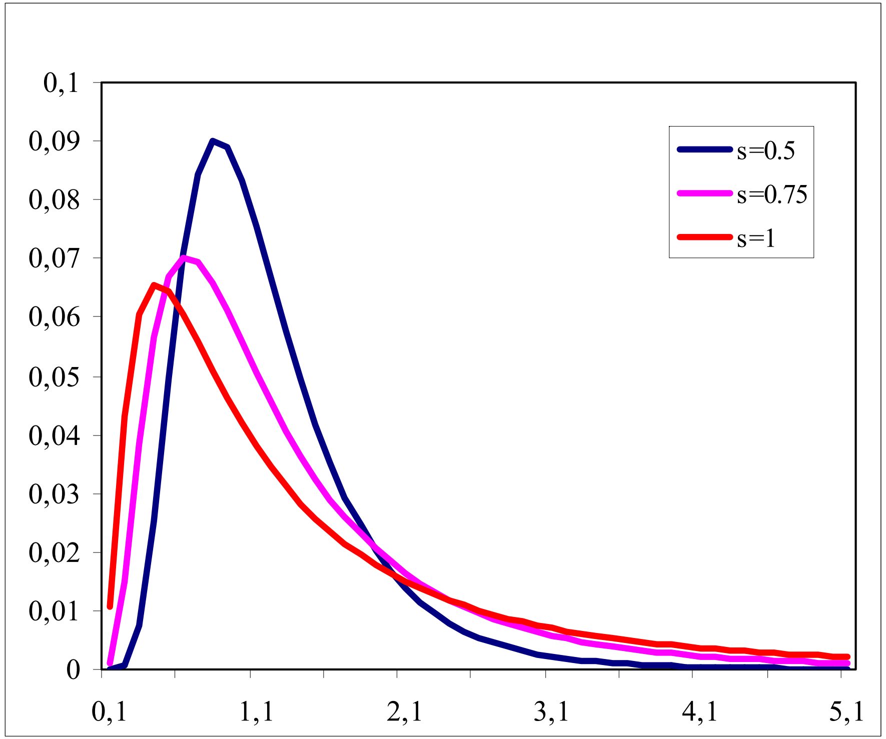
σ ... **standard deviation**

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (x_i - \mu)^2}$$

Log-Normal Probability Distribution

Common expression of **log-normal probability distribution**:

$$f(x|\mu,\sigma) = \frac{1}{x\sqrt{2\pi}\sigma} e^{\frac{-(\ln x-\mu)^2}{2\sigma^2}}$$



μ ... **mean value**

$$\mu = \frac{1}{n}\sum_{i=1}^{n} \ln(x_i)$$

σ ... **standard deviation**

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (\ln(x_i) - \mu)^2}$$

Calculation of Probability of Failure

Reliability analysis leads to estimation of the failure probability:

$$P_f = \int_{D_f} f(X_1,\, X_2, \ldots, X_n)\, \mathrm{d}X_1, \mathrm{d}X_2, \ldots,\, \mathrm{d}X_n$$

where D_f is **failure area** and $f(X_1, X_2, \ldots, X_n)$ **failure function** of n **random variables** $X_1, X_2, \ldots, X_n$ defined by their probability distributions.

General solution of failure probability P_f based on explicit integral calculation is very difficult.

Probabilistic Methods

Simulation methods

Simple simulation Monte Carlo,

Stratified simulation techniques:

Latin Hypercube Sampling – LHS,
Stratified Sampling – SC.

Advanced simulation methods:

Importance Sampling – IS,
Adaptive Sampling – AS,
Axis Orthogonal Importance Sampling,
Directional Sampling – DS,
Line Sampling – LS,
Design Point Sampling,
Subset Simulations,
Descriptive Sampling, Slice Sampling.

Approximation methods

- First (Second) Order Reliability Method - FORM (SORM),
- Response Surface Method – RSM,
- Perturbation techniques – e.g. Stochastic Finite Element Method (SFEM),
- Artificial Neural Network – ANN.

Pure numerical methods

(without simulations and approximations)

- Point Estimate Method – PEM,
- Direct Optimized Probabilistic Calculation – DOProC .

Overview e.g.:
Krejsa & Králik (2015)

Example, Computational Model

Expression and idealization of the structure under actual static or dynamic loads in space and time using mathematics-physical relationships determining the stress, strain, acceleration etc. from a time dependent load variable.

E.g.:

Reliability function RF:

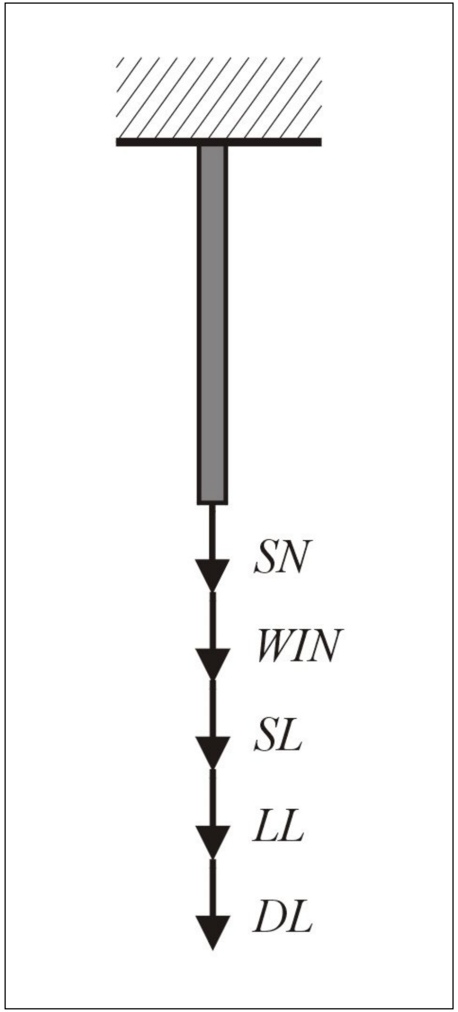
$$RF = R - \text{abs}(E)$$

Structural resistance R (axial load capacity N_{Rd}) :

$$R = N_{Rd} = A_{var} \cdot f_y$$

Load effect E (axial force N_{Ed}):

$$E = N_{Ed} = 80 \cdot DL + 293.5 \cdot LL + 80 \cdot SL + 70 \cdot WIN + 40 \cdot SN$$

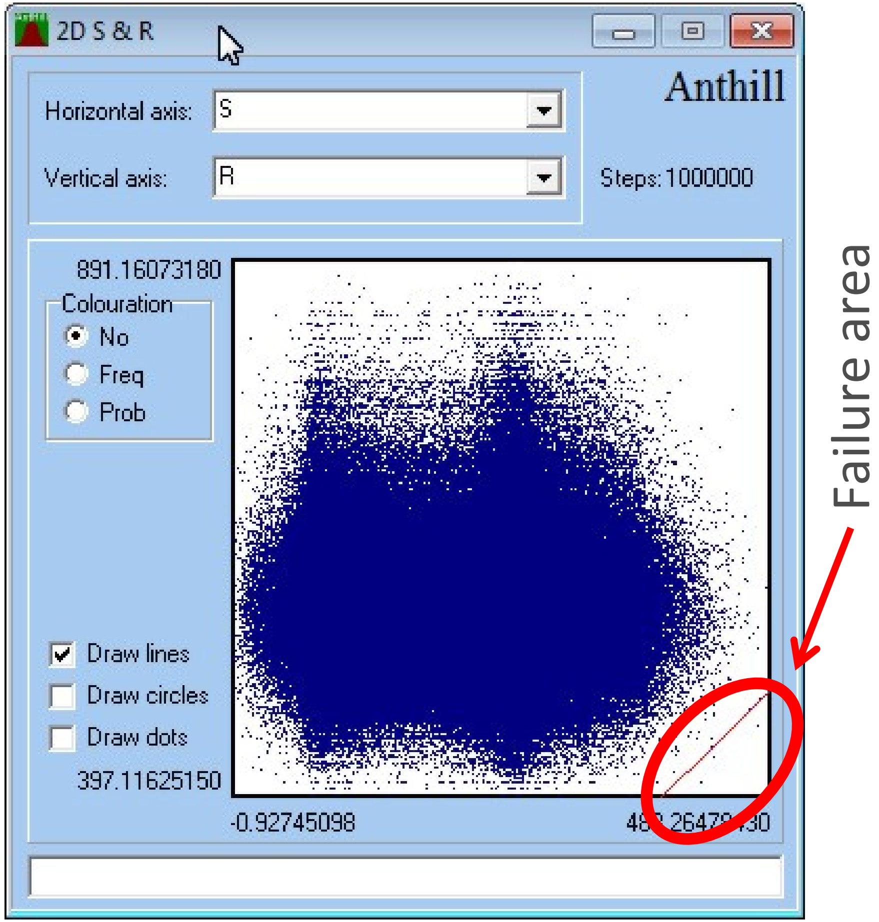


Simulation Based Reliability Assessment (SBRA)

Summary:

- Input values are defined using **non-parametric bounded histograms**,
- Analysis of reliability function using **Monte Carlo simulation**,
- Reliability is expressed:

$$p_f = \frac{N_f}{N} \leq p_d$$



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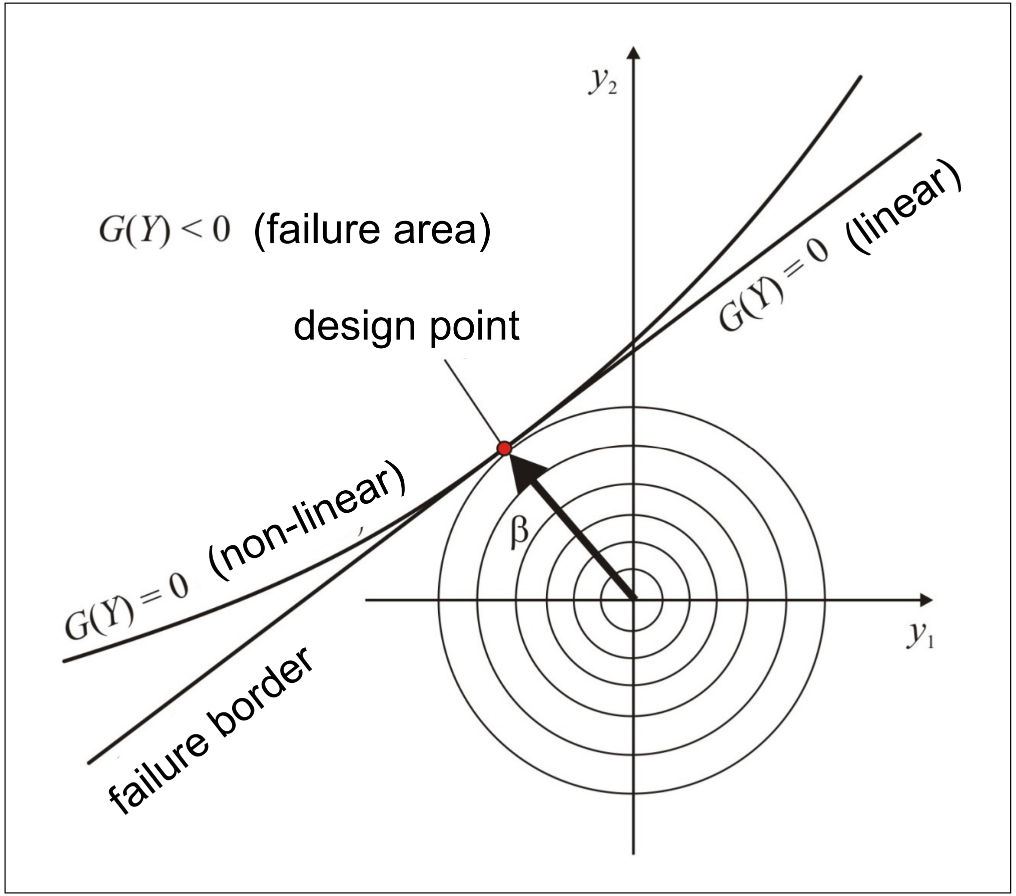
Approximation Methods FORM and SORM

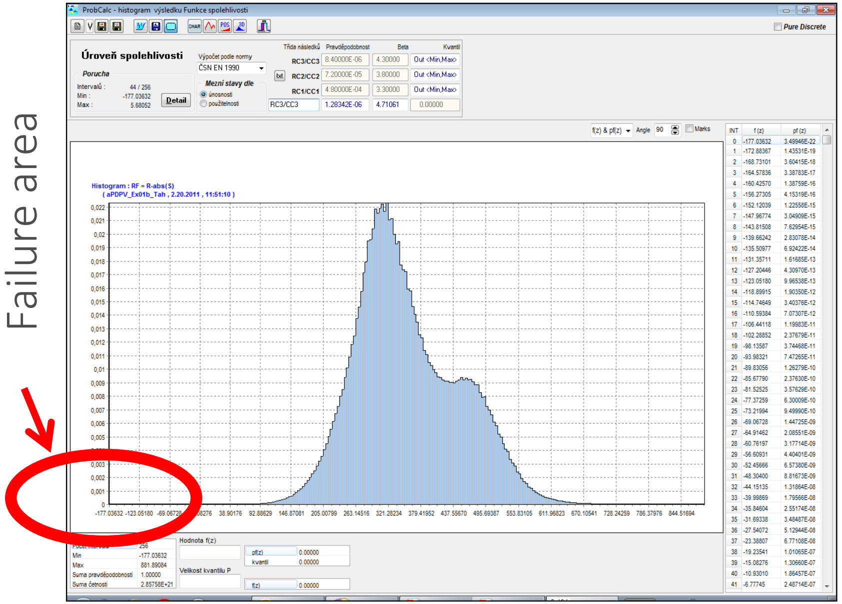


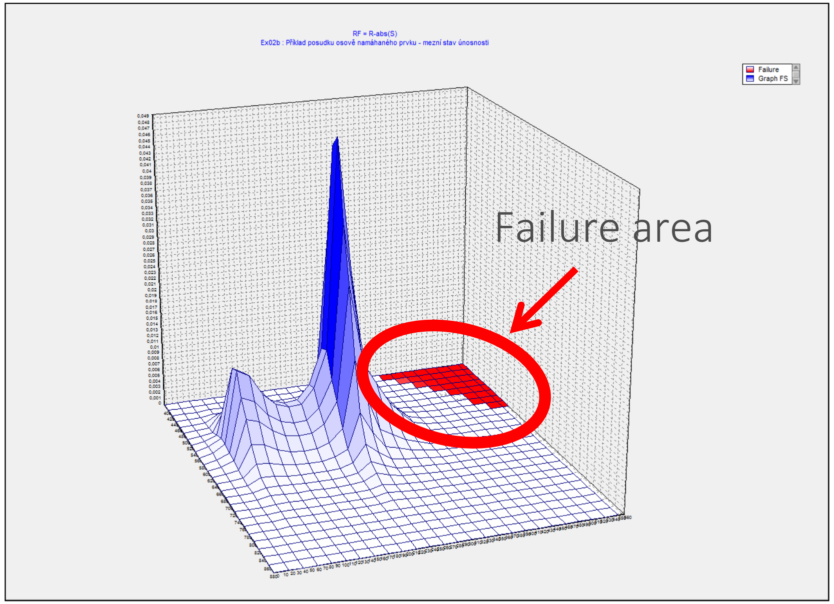
FORM (First-order Reliability Method) – linearization of failure function in design point

SORM (Second-order Reliability Method) – quadratic approximation of failure function in design point

Direct Optimized Probabilistic Calculation

The method can be used to **reliability assessments** of the structures or to the other probabilistic calculations.





The reliability function (computation model) can be expressed analytically or using dynamic libraries using numerical methods.