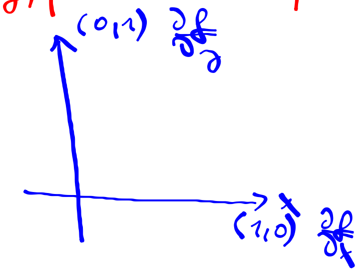
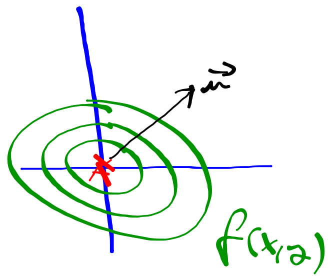
DERIVACE ve směru

$f(x,y)$, bod $A(a,b)$, $\vec{u}=(u_1,u_2)$

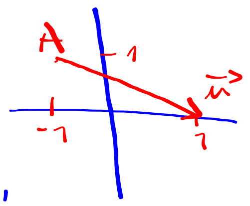




Zvolíme si pomocnou fci φ.

$$\boxed{\begin{aligned}\varphi(t) &:= f(A+t\vec{u})\\ \varphi(0) &= f(A)\end{aligned}}$$

1) $f(x,y)=x^2-3xy-2y^2$, $A=(-1,1)$, $\vec{u}=(2,-1)$



Jak vypadá $\frac{\partial f}{\partial u}(A)=?$

$f_u(A)=?$

$f(x,y)=x^2-3xy-2y^2$

$\varphi(t)=f(t\vec{u}+A)=f(2t-1,-t+1)$

$u=(2,-1)$

$A=(-1,1)$

$\Rightarrow \varphi(t)=(2t-1)^2-3(2t-1)(1-t)-2(1-t)^2=\boxed{8t^2-9t+2}$

$$\boxed{f_u(x,y)=\varphi'(t)}$$

$$\boxed{f_u(A)=\varphi'(0)}$$

$\varphi(t)=f(A+tu)$

$\varphi(0)=f(A)$

$\varphi'(t)=16t-9$

$$\boxed{f_u(A)=-9}$$

2) $f(x,y)=\sqrt{25-x^2-y^2}$, $A=(2,3)$, $\vec{u}=(2,3)$

$\boxed{\frac{\partial f}{\partial \vec{u}}(A)=\varphi'(0)}$

$\varphi(t)=f(2t+2,3t+3)=$

$=\sqrt{12-26t-13t^2}$

$\varphi'(t)=\frac{-26t-26}{2\sqrt{12-26t-13t^2}}$

$\varphi'(0)=\frac{-26}{2\sqrt{12}}$

PARCIÁLNÍ DERIVACE VYŠŠÍCH ŘÁDŮ

$$f(x,y), \quad \frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y}$$

$$\frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial x \partial y}, \quad \frac{\partial^2 f}{\partial y^2}$$

$$f(x,y) = x^3 + xy - 3xy^3$$

1. řád:

$$\frac{\partial f}{\partial x} = 3x^2 + y - 3y^3, \quad \frac{\partial f}{\partial y} = 0 + x - 9xy^2 = MK$$

VV

2. řád:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = 6x + 0 \qquad \boxed{\frac{\partial^2 f}{\partial x \partial y} = 1 - 9y^2}$$

$$\frac{\partial^2 f}{\partial y^2} = -18xy \qquad \frac{\partial MK}{\partial x} = \frac{\partial VV}{\partial y}$$

TAYLORŮV VZOREC

$f(x,y)$, střed: $A=(a,b)$, řád polynomu: m

Taylorův polynom řádu m se středem v A

$T_m(x,y,a,b)$ = Aproximace fce $f(x,y)$ na malém okolí bodu A.

$$T_m(x,y,a,b) = f(a,b) + d f_A(h,k) + \frac{1}{2!} d^2 f_A(h,k) + \dots + \underbrace{\frac{1}{m!} d^m f_A(h,k)}_{\text{diferenciál } m\text{-tého řádu}}$$

$$d f_A(h,k) = \frac{\partial f}{\partial x}(A)\cdot h + \frac{\partial f}{\partial y}(A)\cdot k$$

$$d^2 f_A(h,k) = \frac{\partial^2 f}{\partial x^2}(A)\cdot h^2 + 2\frac{\partial^2 f}{\partial x \partial y}(A)\cdot h\cdot k + \frac{\partial^2 f}{\partial y^2}(A)\cdot k^2$$

$f(x,y) = x^y$

$A=(1,2)$ $m=2$ $\boxed{h=x-1 \quad k=y-2}$

$$T_m(x,y,1,2) = f(1,2) + d f_{1,2}(h,k) + \frac{1}{2} d^2 f_{1,2}(h,k)$$

$$d f_{1,2}(x-1,y-2) = \frac{\partial f}{\partial x}(1,2)\cdot(x-1) + \frac{\partial f}{\partial y}(1,2)(y-2) =$$

$$\boxed{\frac{\partial f}{\partial x} = y\cdot x^{y-1} \qquad \frac{\partial f}{\partial y} = x^y\cdot \ln|x|}$$

$$= 2\cdot(x-1) + 0\cdot(y-2) = 2x-2$$

$$d^2 f_{(1,2)}(x-1,y-2) = \frac{\partial^2 f}{\partial x^2}(1,2)(x-1)^2 + 2\frac{\partial^2 f}{\partial x \partial y}(1,2)(x-1)(y-2) + \frac{\partial^2 f}{\partial y^2}(1,2)(y-2)^2$$

$$\frac{\partial^2 f}{\partial x^2} = y(y-1)x^{y-2}$$

$$\frac{\partial^2 f}{\partial y^2} = x^y\cdot \ln^2|x| \qquad \frac{\partial^2 f}{\partial x \partial y} = 1\cdot x^{y-1} + y\cdot x^{y-1}\cdot \ln|x|$$

$$d^2 f_{(1,2)}(x-1,y-2) = 2(x-1)^2 + 2\cdot 1\cdot(x-1)(y-2) + 0$$

$$T_2(x,y,1,2) = f(1,2) + 2x-2 + \frac{1}{2}\cdot\left(2(x-1)^2 + 2(x-1)\cdot(y-2)\right)$$

$1{,}07^{1{,}98}$ kalkulačka $\Rightarrow$

$T_2(x,y,1,2) \Rightarrow T_2(1{,}07,\ 1{,}98,\ 1,2)$