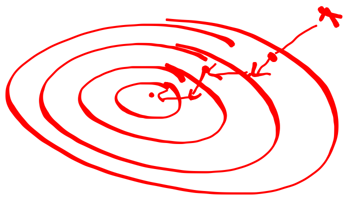
PARCIÁLNÍ DERIVACE

GRADIENT

$\frac{\partial f}{\partial x_1}$ = parciální derivace f podle x_1.

$$\operatorname{grad} f(x_1,x_2,\dots,x_k) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_k}\right)$$

$$\operatorname{grad} f(A) = \left(\frac{\partial f}{\partial x_1}(A), \frac{\partial f}{\partial x_2}(A), \dots\right)$$



grad $f(*)$

1) $f(x,y) = e^{x+1}\cdot \sin(2y)$

$\boxed{\operatorname{grad} f}$, $\operatorname{grad} f(1,1)$

i) Spočítáme parciální derivace podle všech proměnných

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(e^{x+1}\sin(2y)\right) = \underline{e^{x+1}\cdot\sin(2y)}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(e^{x+1}\sin(2y)\right) = \underline{\cos(2y)\cdot 2\cdot e^{x+1}}$$

$$\operatorname{grad} f = \left(e^{x+1}\sin(2y), 2e^{x+1}\cos(2y)\right)$$

$$\operatorname{grad} f(\underset{x}{1},\underset{y}{1}) = \left(e^2\sin(2), 2e^2\cos(2)\right)$$

2) $f(x,y) = 3x^2 y + e^{2xy}$

$A = (3,2)$

$\frac{\partial f}{\partial x} = 6x\, y + e^{2xy} \cdot 2y$

$\frac{\partial f}{\partial y} = 3x^2 + e^{2xy} \cdot 2x$

$\text{grad} f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$

$\text{grad} f(A) = \left(36 + 4e^{12}, 27 + 6e^{12} \right)$

3) $f(x) = \dfrac{x\cos(y) - y\cos x}{1 + \cos x + \sin y}$ $A = (0,0)$

$\operatorname{grad} f = \left(\dfrac{\partial f}{\partial x} , \dfrac{\partial f}{\partial y} \right)$

$f'(x) = \left(\dfrac{a(x,y)}{b(x,y)} \right)' = \dfrac{a'(x,y)\, b(x,y) - a(x,y)\, b'(x,y)}{b^2(x,y)}$

$\dfrac{\partial f}{\partial x} = \dfrac{\frac{\partial a}{\partial x} b - a \cdot \frac{\partial b}{\partial x}}{b^2} = \dfrac{\overbrace{[\cos(y) + y\sin(x)]}^{a'_x} \cdot (1 + \cos x + \sin y) - (x\cos y - y\cos x)\overbrace{(-\sin x)}^{b'_x}}{(1 + \cos x + \sin y)^2}$

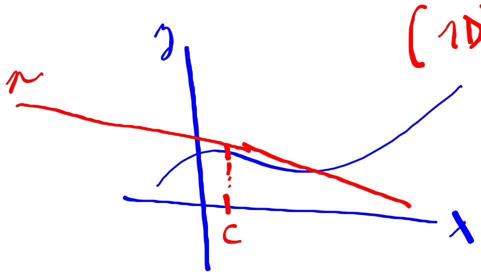
$\dfrac{\partial f}{\partial x}(0,0) = \dfrac{1}{2}$

$\dfrac{\partial f}{\partial y} = \dfrac{(-x\sin y - \cos x)(1 + \cos x + \sin y) - (x\cos y - y\cos x)(\cos y)}{(1 + \cos x + \sin y)^2}$

$\dfrac{\partial f}{\partial y}(0,0) = -\dfrac{1}{2}$ $\Rightarrow \operatorname{grad} f(0,0) = \left(\dfrac{1}{2} , -\dfrac{1}{2} \right)$

$\boxed{\text{DIFERENCIÁL (TOTÁLNÍ)}}$

(1D)



Co potřebujeme

- $f(x,y)$, která je definovaná na okolí A (spolu s derivací)
- $A \in \mathbb{R}^2$, $A = (a_1, a_2)$

$$\Rightarrow \boxed{d f_{(a_1,a_2)}(h,k) = \boxed{\alpha h + \beta k}}$$

$$\alpha = \frac{\partial f}{\partial x}(A)$$

$$\beta = \frac{\partial f}{\partial y}(A)$$

4) Spočítejte diferenciál fce $f(x,y)$: v bodě $A = (1,0)$

$$f(x,y) = \ln(x^2+y^2) \qquad d f_{(1,0)}(h,k) = \boxed{2 \cdot h + 0 \cdot k}$$

$$\frac{\partial f}{\partial x} = \frac{1}{x^2+y^2}(2x+0) \Rightarrow \alpha = \frac{\partial f}{\partial x}(1,0) = \underline{\underline{2}}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x^2+y^2}(2y+0) \Rightarrow \beta = \frac{\partial f}{\partial y}(1,0) = \underline{\underline{0}}$$

TEČNÁ ROVINA

rovnice $z = \alpha x + \beta y + \gamma$

Mějme bod $A = (a_1, a_2)$
$\qquad\qquad\qquad\quad x \quad y$

$\Rightarrow$ z bude aproximovat funkční hodnotu f na malém okolí A.

$$\boxed{z = \alpha(x - a_1) + \beta(y - a_2) + f(a_1, a_2)}$$

$\alpha = \frac{\partial f}{\partial x}(A)$

$\beta = \frac{\partial f}{\partial y}(A)$

$f(x,y) = \sqrt{x^2 + y^2}$ $\quad A = (3,4)$ (bod dotyku)

$\Rightarrow$ určete rovnici tečné roviny

$\Rightarrow$ Použijme ji k aproximaci hodnoty $\sqrt{2{,}98^2 + 4{,}05^2}$

$\frac{\partial f}{\partial x} = \frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}} \cdot 2x \Rightarrow \alpha = \frac{1}{2}(3^2 + 4^2)^{-\frac{1}{2}} \cdot 2 \cdot 3 = \boxed{\frac{3}{5}}$

$\frac{\partial f}{\partial y} = \frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}} \cdot 2y$

$\Rightarrow \beta = \frac{1}{2}(3^2 + 4^2)^{-\frac{1}{2}} \cdot 2 \cdot 4 = \boxed{\frac{4}{5}}$

$f(A) = \sqrt{9 + 16}$

Tečná rovina

$z = \frac{3}{5} \cdot (x - 3) + \frac{4}{5}(y - 4) + 5$

Lineární aproximace výrazu $\underbrace{\sqrt{2{,}98^2 + 4{,}05^2}}_{\sqrt{x^2 + y^2}} \doteq 5{,}0282..$ $\quad x = 3,\ y = 4$

$= \frac{3}{5}(2{,}98 - 3) + \frac{4}{5}(4{,}05 - 4) + 5 = 5{,}028$

$\sqrt{7{,}53^2 - 18{,}1^2}$