Když narazíte na červené rovnítko (=), popřemýšlejte o tom, jaké úpravy byly provedeny (extra důsledně)

Vypočítejte délku křivky

$$\boxed{y = \ln(\sin(x))\,,\ x \in \langle 0, \tfrac{\pi}{3} \rangle}$$

$$l = \int_0^{\frac{\pi}{3}} \sqrt{1 + \left[\left(\ln(\sin(x))\right)'\right]^2}\, dx = \int_0^{\frac{\pi}{3}} \sqrt{1 + \left(\frac{\cos x}{\sin x}\right)^2}\, dx = \int_0^{\frac{\pi}{3}} \sqrt{\frac{\sin^2 x + \cos^2 x}{\sin^2 x}}\, dx =$$

$$= \int_0^{\frac{\pi}{3}} \frac{1}{|\sin(x)|}\, dx = \int_0^{\frac{\pi}{3}} \frac{1}{\sin(x)}\, dx = \int_0^{\frac{\pi}{3}} \frac{\sin(x)}{\sin^2(x)}\, dx = \int_0^{\frac{\pi}{3}} \frac{\sin(x)}{1-\cos^2(x)}\, dx = \int_1^{\frac{1}{2}} \frac{-1}{1-t^2}\, dt =$$

$$= -\frac{1}{2} \int_1^{\frac{1}{2}} \left(\frac{1}{1-t} + \frac{1}{1+t}\right) dt = -\frac{1}{2}\Big[\ln|1+t| - \ln|1-t|\Big]_1^{\frac{1}{2}} = -\frac{1}{2}\left[\ln\left|\frac{1+t}{1-t}\right|\right]_1^{\frac{1}{2}} =$$

$$= -\frac{1}{2}\left[\ln\frac{\frac{3}{2}}{\frac{1}{2}} - \lim_{t \to 1^{-}} \ln\left|\frac{2}{1-t}\right|\right] = -\frac{1}{2}\left[\ln 3 - \infty\right] = \underline{\underline{+\infty}}$$

interval, na kterém integrujeme:

=> Blížíme se k 1 zleva