DÉLKA KŘIVKY

$$f(x)=\sqrt{a^2-x^2}\quad,\ x\in\langle 0,a\rangle\ ,\ a>0$$

$$l=\int_0^a \sqrt{1+(f'(x))^2}\,dx=\int_0^a\sqrt{1+x^2\cdot(a^2-x^2)^{-1}}\,dx$$

$$f'(x)=\left((a^2-x^2)^{\frac{1}{2}}\right)'=\left[\frac{1}{\cancel{2}}(a^2-x^2)^{-\frac{1}{2}}\cdot(-\cancel{2}x)\right]^2$$

$$(x^{\frac{1}{2}})'=\frac{1}{2}x^{-\frac{1}{2}}\qquad (a^c)^d=a^{c\cdot d}$$

$$=\int_0^a\sqrt{1+\frac{x^2}{a^2-x^2}}\,dx=\int_0^a\sqrt{\frac{a^2-x^2+x^2}{a^2-x^2}}\,dx=$$

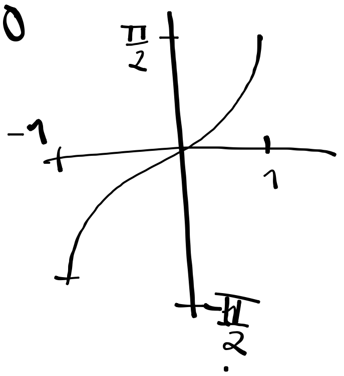
$$=\int_0^a\sqrt{\frac{a^2}{a^2-x^2}}\,dx=a\int_0^a\frac{1}{\sqrt{a^2-x^2}}\,dx=\cancel{a}\int_0^a\frac{1}{\cancel{a}\sqrt{1-\frac{x^2}{a^2}}}\,dx$$

$$a^2\left(1-\frac{x^2}{a^2}\right)$$

$$t=\frac{x}{a}\qquad dt=\frac{dx}{a}$$

$$=\int_0^1\frac{1}{\sqrt{1-t^2}}\,a\cdot dt=a\left[\arcsin(t)\right]_0^1=$$

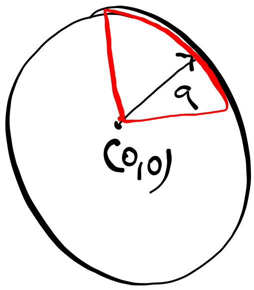
$$=a\,\frac{\pi}{2}-a\cdot 0=\boxed{a\,\frac{\pi}{2}}$$



$$x^2+y^2=a^2$$

$$y^2=a^2-x^2$$

$$y=\sqrt{a^2-x^2}$$



NEVLASTNÍ INTEGRÁLY

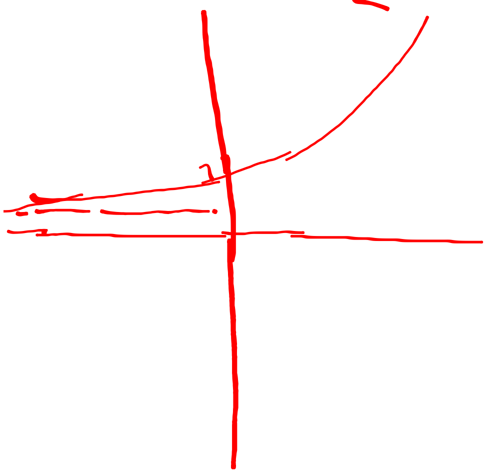
$$\int_{-\infty} \,,\quad \int^{+\infty} \,,\quad \int_{-\infty}^{+\infty} \,,\quad \int_a^b f(x)\,dx$$

$c \in (a,b),\ c \notin D_f$

$$\int_0^{+\infty} e^{-2x}\,dx = \left| \begin{matrix} -2x = t \\ -2dx = dt \end{matrix} \right| = -\frac{1}{2}\int_0^{-\infty} e^t\,dt =$$

$$= -\frac{1}{2}\left[e^t\right]_0^{-\infty} = -\frac{1}{2}\left[\lim_{t\to-\infty} e^t - e^0\right] = -\frac{1}{2}\left[0-1\right] =$$

$$= \underline{\underline{\frac{1}{2}}}$$



$$\frac{1}{e^{\infty}}$$

$$\int_{-\infty}^{+\infty} \frac{1}{x^2}\,dx = \int_{-\infty}^{-1} \frac{1}{x^2}\,dx + \boxed{\int_{-1}^{1} \frac{1}{x^2}\,dx} + \int_{1}^{+\infty} \frac{1}{x^2}\,dx =$$

$$\int \frac{1}{x^2}dx = \int x^{-2}dx = -x^{-1}$$

$$\Rightarrow \int_{-1}^{0} \frac{1}{x^2}dx + \int_{0}^{1} \frac{1}{x^2}dx$$

$$\left[-\frac{1}{x}\right]_{-\infty}^{-1} + \left[-\frac{1}{x}\right]_{0}^{1} + \left[-\frac{1}{x}\right]_{1}^{+\infty} =$$

$$+ \left[-\frac{1}{x}\right]_{-1}^{0}$$

$$\left[-\frac{1}{-1} - \lim_{x\to-\infty} \frac{1}{-x}\right] = 1 + \lim_{x\to-\infty} \frac{1}{x} = 1$$

$$\left[-\frac{1}{1} - \lim_{x\to 0_+} \frac{1}{-x}\right] = -1 + \lim_{x\to 0_+} \frac{1}{x} = -1 + \infty = +\infty$$

$$\left[\lim_{x\to\infty} -\frac{1}{x} - \frac{1}{+1}\right] = 0 - 1 = -1$$

$$+\infty + \infty - 1 + 1$$

$$\boxed{= \infty}$$

$$\left[\lim_{x\to 0_-}\left(-\frac{1}{x}\right) - \frac{1}{-1}\right] = +\infty + 1 = +\infty$$

$$-\frac{1}{x}$$

DEFINIČNÍ OBORY

$\sqrt{z} : z \geq 0$

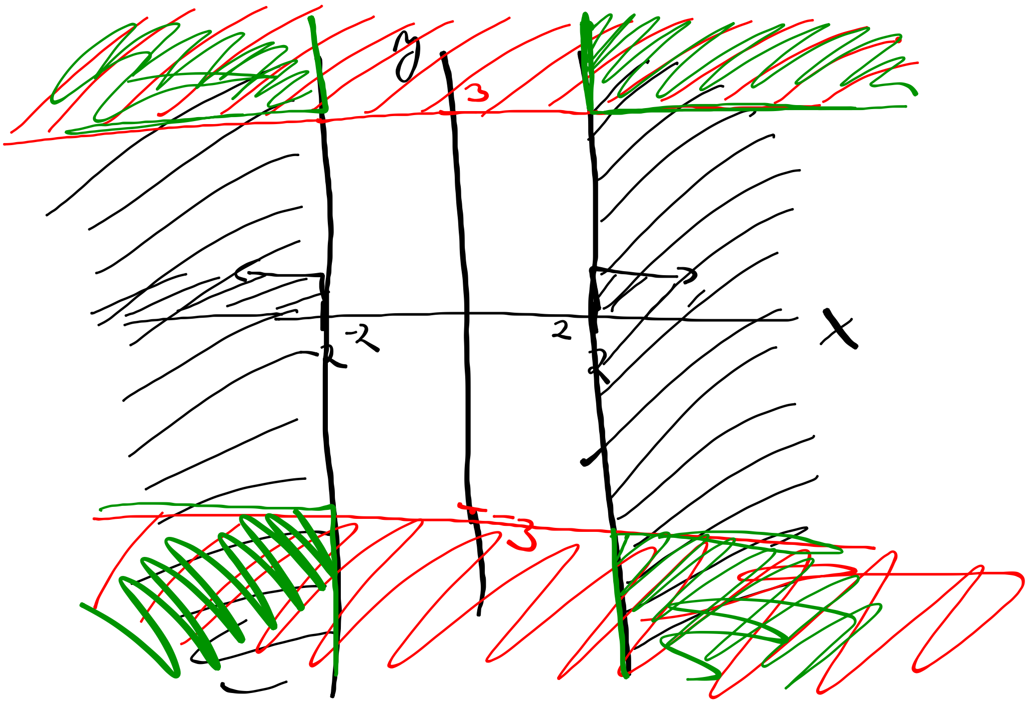
$f(x,y) = \sqrt{x^2-4} + \sqrt{y^2-9}$

$x^2 - 4 \geq 0$ $\quad$ $y^2 - 9 \geq 0$

$\sqrt{x^2} = |x|$ $\quad$ $x^2 \geq 4$ $\quad$ $y^2 \geq 9$

$|x| \geq 2$

$|y| \geq 3$



$D_f = \{ (x,y) : |x| \geq 2 \wedge |y| \geq 3 \}$
$(x,y) \in \mathbb{R}^2$

$f(x,y) = \ln(y \cdot \ln(2x-y))$, $D_f = ??$ $\qquad \ln(z) \Rightarrow z>0$

$\ln(2x-y) \Rightarrow 2x-y>0$

$y<2x$

$\ln(y\cdot\ln(2x-y)) \Rightarrow y \cdot \ln(2x-y) > 0$

a) $y>0 \wedge \ln(2x-y)>0$

b) $y<0 \wedge \ln(2x-y)<0$

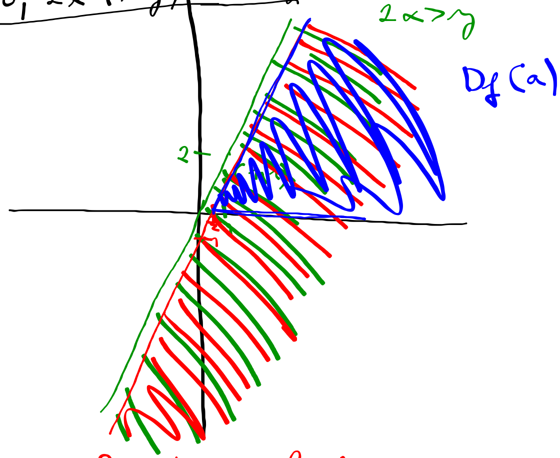
$\ln(2x-y)>0=\ln(1)$

$2x-y>1$

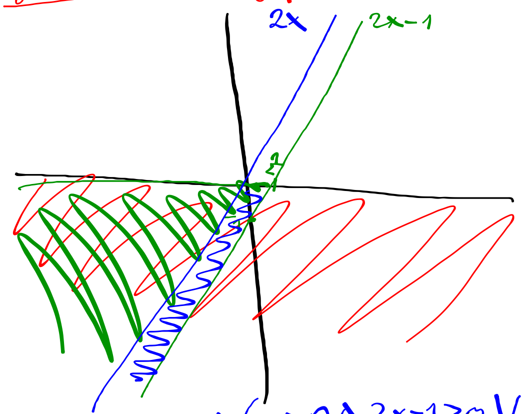
$\boxed{2x-1>y}$

$\ln(2x-y)<0 \Rightarrow \boxed{2x-1<y}$

a) $y>0,\ 2x-1>y,\ 2x>y$



b) $y<0,\ 2x-1<y,\ 2x>y$



$D_f=\{(x,y): 2x>y \wedge (y>0 \wedge 2x-1>y \vee y<0 \wedge 2x-1<y)\}$