INTEGRÁLY TYPU $\int R(e^x)\,dx$, $\int R(\ln(x))\,dx$

$$\boxed{e^x = t}$$
$$\ln(e^x) = \ln(t)$$
$$x = \ln(t)$$
$$\boxed{dx = \frac{1}{t}\,dt}$$

$$\int \frac{e^x + 1}{e^{3x} - e^{2x} + e^x - 1}\,dx = \left| \begin{array}{l} e^x = t \\ dx = \frac{1}{t}\,dt \end{array} \right|$$
$$= \int \frac{t+1}{t^3 - t^2 + t - 1} \cdot \frac{1}{t}\,dt =$$

$$\frac{t+1}{t(t^3 - t^2 + t - 1)} \Rightarrow$$ PRO JAKÁ t je nulový? $t_1 = 1$

$$\Rightarrow \boxed{\frac{t+1}{t(t-1)(t^2+1)}}$$

$$(t-1)X = t^3 - t^2 + t - 1$$
$$X = \frac{t^3 - t^2 + t - 1}{t-1}$$

$$(t^3 - t^2 + t - 1) : (t-1) = t^2 + 1$$

$$\Rightarrow \int \frac{t+1}{t(t-1)(t^2+1)}\,dt = \int \frac{A}{t} + \frac{B}{t-1} + \frac{Ct + D}{t^2+1}$$

$$t + 1 = A(t-1)(t^2+1) + B(t)(t^2+1) + (Ct+D)t(t-1)$$

$$0t^3 + 0t^2 + t + 1 = (At - A) + At^3 - At^2 + Bt + Bt^3 + Ct^3 - Ct^2 + Dt^2 - Dt$$
$$0t^3 + 0t^2 + 1t + 1 = t^3(A+B+C) + t^2(-A-C+D) + t(-D+B+A) + 1(-A)$$

$$\left.\begin{array}{l} 0 = A+B+C \\ 0 = -A-C+D \\ 1 = A+B-D \\ 1 = -A \end{array}\right\} \Rightarrow \boxed{\begin{array}{l} A = -1 \\ B = 1 \\ C = 0 \\ D = -1 \end{array}}$$

$$= \int \left(\frac{-1}{t} + \frac{1}{t-1} + \frac{-1}{t^2+1} \right) dt$$

$$-1\int \frac{1}{t}\,dt + \int \frac{1}{t-1}\,dt - \int \frac{1}{t^2+1}\,dt =$$

$$= -\ln|t| + \ln|t-1| - \operatorname{arctg}(t) =$$

$$= \left| \begin{array}{l} \text{zpětná substituce} \\ t = e^x \end{array} \right| = \boxed{-\ln|e^x| + \ln|e^x - 1| - \operatorname{arctg}(e^x)}$$

INTEGRÁLY TYPU $\int R(x, \sqrt[n]{x})\,dx$

$$\int \frac{4\cdot\sqrt{x}+1}{16x-\sqrt{x}}\,dx = \left| \begin{array}{l} x = t^2 \\ dx = 2t\,dt \end{array} \right| =$$

$$= \int \frac{4\cdot t+1}{16t^2-t}\,2t\,dt = \int \frac{8t^2+2t}{16t^2-t}\,dt$$

$$= \int \frac{\cancel{t}(8t+2)}{\cancel{t}(16t-1)}\,dt = \int \frac{1}{2}\,dt + \frac{5}{2}\int \frac{1}{16t-1}\,dt$$

$$(8t+2):(16t-1) = \frac{1}{2} + \frac{5}{2(16t-1)}$$

$$\begin{array}{l} -\frac{1}{2}(16t-1) \\ -8t+\frac{1}{2} \\ \hline (0t+\frac{5}{2}):(16t-1) \end{array}$$

$$\frac{\ln(16t-1)}{16} \quad \int \frac{1}{16t-1} \quad /:16$$

$$\boxed{\frac{1}{2}t + \frac{5}{2}\cdot\frac{1}{16}\ln(16t-1)}$$

$$\left.\begin{array}{l} t^2 = x \\ t = \sqrt{x} \end{array}\right\} \Rightarrow \boxed{\frac{1}{2}\sqrt{x} + \frac{5}{32}\ln(16\sqrt{x}-1)}$$

$$\int \frac{6}{(3+4x)^8}\,dx = \left| \begin{array}{l} t = 3+4x \\ dt = 4\,dx \\ \frac{dt}{4} = dx \end{array} \right| = \int \frac{6}{4}\,t^{-8}\,dt$$

$$\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx = \left| \begin{array}{l} -2\frac{1}{x^3}dx = dt \Rightarrow \frac{1}{x^3}dx = \frac{dt}{-2} \\ \frac{1}{x^2} = t \end{array} \right|$$

= Integrace polynomu