$$\int \frac{\sin(\ln(3x))}{x}\,dx = \int \sin(t)\,dt =$$

$g(x) = \ln(3x) = t$

$g'(x) = \frac{3}{3x}\,dx = dt$

ZPĚT. SUB.

$= -\cos(t) = -\cos(\ln(3x))$

2. SUBSTITUČNÍ METODA

$$\int f(x)\,dx = \left| \begin{array}{l} x = g(t) \\ \Downarrow \\ dx = g'(t)\,dt \end{array} \right| = \int f(g(t))\,g'(t)\,dt$$

$= G(t) = G(g^{-1}(x))$

$x = g(t) \quad / g^{-1}()$

$g^{-1}(x) = g^{-1}(g(t))$

$\boxed{g^{-1}(x) = t}$

$$\int e^{\sqrt{x}}\,dx = \int e^{t}\,\frac{2\,dt}{x^{-\frac{1}{2}}}$$

$t = \sqrt{x}$

$dt = \frac{1}{2}x^{-\frac{1}{2}}\,dx \Rightarrow$ NIKAM NEVEDE

$\frac{2\,dt}{x^{-\frac{1}{2}}} = dx$

$$\int e^{\sqrt{x}}\,dx = \left| \begin{array}{l} x = t^2 \\ dx = 2t\,dt \end{array} \right| = \int e^{t}\cdot 2t\,dt =$$

Per partes

$$= \left| \begin{array}{ll} u = 2t & v' = e^{t} \\ u' = 2 & v = e^{t} \end{array} \right| = 2te^{t} - \int 2e^{t}\,dt = 2te^{t} - 2e^{t}$$

$\int (u.v)' = \int u'v + \int v'u$

$u.v = \int u'v + \int v'u$

$\int u'v = uv - \int uv'$

$x = t^2 \Rightarrow t = \sqrt{x}$

$\Rightarrow \boxed{2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}}}$

$$\int \sqrt{1 - x^2}\,dx = \int \sqrt{1 - \sin^2(t)}\cdot\cos(t)\,dt =$$

$x = \sin(t)$

$dx = \cos(t)\,dt$

$\sin^2(x) + \cos^2(x) = 1$

$\Rightarrow 1 - \sin^2(x) = \cos^2(x)$

$$= \int |\cos(t)|\cos(t)\,dt = \int \cos^2(t)\,dt =$$

$$= \left| \begin{array}{ll} u = \cos(t) & v' = \cos(t) \\ u' = -\sin(t) & v = \sin(t) \end{array} \right| = \cos(t)\sin(t) + \int \sin^2(t)\,dt$$

$$\int \sin^2(t)\,dt = \left| \begin{array}{ll} u = \sin(t) & v' = \sin(t) \\ u' = \cos(t) & v = -\cos(t) \end{array} \right| =$$

$= -\sin(t)\cos(t) + \int \cos^2(t)\,dt$

$$\int \frac{x^2+1}{x^3-x^2+x-1}\,dx = \int \frac{1}{x-1}\,dx = \ln|x-1|$$

$$\frac{x^2+1}{x^3-x^2+x-1} = \frac{\cancel{x^2+1}}{(x-1)\cancel{(x^2+1)}} = \frac{1}{x-1}$$

$$(x^3-x^2+x-1):(x-1)=x^2$$
$$-x^2(x-1)$$
$$(x-1):(x-1)=1$$
$$(x^3-x^2+x-1):(x-1)=x^2+1$$

$$\int \frac{x^2+2}{x^3+2x^2-5x-6}\,dx = \boxed{\int \frac{x^2+2}{(x-2)(x+3)(x+1)}}$$

$$(x^3+2x^2-5x-6):(x-2)=x^2+4x+3$$
$$-x^2(x-2)$$
$$-x^3+2x^2$$
$$0x^3+(4x^2-5x-6):(x-2)=+4x\ +3$$
$$-(+4x(x-2))$$
$$-4x^2+8x$$
$$0x^2+(3x-6):(x-2)=3$$

$$x^2+4x+3=(x+3)(x+1)$$

$$\frac{x^2+2}{(x+3)(x+1)(x-2)} = \frac{A}{x+3}+\frac{B}{x+1}+\frac{C}{x-2} \quad / \text{jm.}$$

$$x^2+2 = A(x+1)(x-2)+B(x+3)(x-2)+C(x+3)\cdot(x+1)$$

DOSADÍME KOŘENY

$x=2 \Rightarrow 2^2+2 = A\cdot 0+B\cdot 0+C\cdot 15$

$C=\frac{6}{15}\ \frac{2}{5}$

$x=-3$ $\quad A=\frac{11}{10} \Rightarrow 11 = A\,10$

$x=-1$ $\quad B=-\frac{1}{2} \quad 3=B(2\cdot -3)$

$$\int \frac{x^2+2}{x^3+2x^2-5x-6}\,dx = \int \frac{\frac{11}{10}}{x+3}\,dx + \int \frac{-\frac{1}{2}}{x+2}\,dx + \int \frac{\frac{6}{15}}{x-1}\,dx =$$

$$\frac{11}{10}\int \frac{1}{x+3}\,dx - \frac{1}{2}\int \frac{1}{x+2}\,dx + \frac{2}{5}\int \frac{1}{x-1}\,dx$$

$=$