

1. Substitucijski Metoda

$$\int f(g(x)) g'(x) dx = \int f(u) du = F(u) = F(g(x))$$

$g(x) = u$   
 $g'(x) dx = du$

$$\int \frac{18x^2 \cdot \sqrt[4]{6x^3+5}}{6x^3+5} dx = \int u^{\frac{1}{4}} du =$$

$g(x) = 6x^3+5$   
 $g'(x) = 18x^2 dx \Rightarrow 18x^2 dx = du$   
 $6x^3+5 = u$

$$= \frac{4}{5} u^{\frac{5}{4}} = \frac{4}{5} (6x^3+5)^{\frac{5}{4}}$$

$$\int \left(1 - \frac{1}{x}\right) \cdot \cos(x - \ln x) dx =$$

$\int f(g(x)) \cdot g'(x) dx$   
 $g(x) = x - \ln(x) = u$   
 $g'(x) = \left(1 - \frac{1}{x}\right) dx = du$

$$= \int \cos(u) du = \sin(u) = \sin(x - \ln(x))$$

$$\int 3(8x-1) \cdot e^{4x^2-x} dx = \int 3e^u du =$$

$g(x) = 4x^2 - x = u$   
 $(8x-1) dx = du$

$$= 3 \cdot e^u = 3e^{4x^2-x}$$

$$\int x^2 (3-10x^3)^4 dx = \int \frac{1}{30} u^4 du =$$

$g(x) = 3-10x^3 = u$   
 $g'(x) = (-30x^2) dx = du$   
 $x^2 dx = \frac{du}{-30}$

$$= -\frac{1}{30} \frac{u^5}{5} = -\frac{1}{30} \frac{(3-10x^3)^5}{5}$$

$$\int \sin(1-x) (2-\cos(1-x))^4 dx =$$

$g(x) = 2 - \cos(1-x) = u$   
 $g'(x) = -\sin(1-x) dx = du$

$$= -\int u^4 du = -\frac{u^5}{5} = -\frac{(2-\cos(1-x))^5}{5}$$

$$\int \frac{3}{5x^2+4} dx = \int \frac{3}{4} \cdot \frac{1}{\frac{5}{4}x^2+1} \cdot \frac{2}{\sqrt{5}} dx =$$

$$\int \frac{1}{x^2+1} dx = \arctan(x) \quad \begin{matrix} g(x) = \frac{\sqrt{5}}{2}x = 4 \\ g(x) = \frac{\sqrt{5}}{2}x = u \\ dx = \frac{2}{\sqrt{5}} du \end{matrix}$$

$$\frac{3}{5x^2+4} = \frac{3}{4} \cdot \frac{1}{\frac{5}{4}x^2+1} = \frac{3}{4} \cdot \frac{1}{\left(\frac{\sqrt{5}}{2}x\right)^2+1}$$

$$= \frac{3}{4} \cdot \frac{2}{\sqrt{5}} \cdot \arctan(u) =$$

$$= \frac{6}{4\sqrt{5}} \arctan\left(\frac{\sqrt{5}}{2}x\right)$$

ROZKLAD NA PARCIAŁNI ZŁOMKY

$$\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} \quad | \cdot x(x-1)$$

$$\frac{1}{x(x-1)} = -\frac{1}{x} + \frac{1}{x-1}$$

$$1 = A(x-1) + Bx \quad \begin{matrix} x=1 \Rightarrow 1 = A \cdot 0 + B \\ B=1 \\ x=0 \Rightarrow 1 = A(-1) \\ A=-1 \end{matrix}$$

$$\frac{4}{(x+2)(2x+1)} = \frac{A}{x+2} + \frac{B}{2x+1} \quad | \cdot (x+2)(2x+1)$$

$$4 = A(2x+1) + B(x+2)$$

$$x=-2 \Rightarrow 4 = A(-3) \Rightarrow A = -\frac{4}{3}$$

$$x=-\frac{1}{2} \Rightarrow 4 = B \cdot \frac{3}{2} \Rightarrow B = \frac{8}{3}$$

$$\Rightarrow -\frac{4}{3} \cdot \frac{1}{x+2} + \frac{8}{3} \cdot \frac{1}{2x+1}$$

$$\frac{2x+5}{(x-7)(x^2+x+1)} = \frac{A}{x-7} + \frac{Bx+C}{x^2+x+1} \quad | \cdot (x-7)$$

$$2x+5 = A(x^2+x+1) + (Bx+C)(x-7)$$

$$x=7 \Rightarrow 2 \cdot 7 + 5 = A \cdot (49+7+1) + 0$$

$$19 = A \cdot 57$$

$$\frac{1}{3} = A$$

$$2x+5 = \frac{1}{3}(x^2+x+1) + (Bx+C)(x-7) \quad | - \frac{1}{3}(x^2+x+1)$$

$$\left(-\frac{1}{3}\right)x^2 + x\left(2-\frac{1}{3}\right) + 5-\frac{1}{3} = Bx^2 - 7Bx + Cx - 7C$$

$$-\frac{1}{3} = B \quad x \cdot B + x(C-7B) + 1 \cdot (-7C)$$

$$2-\frac{1}{3} = C-7B \quad \frac{5}{3} = -\frac{2}{3} + \frac{7}{3} = \frac{5}{3} \checkmark$$

$$5-\frac{1}{3} = -7C \Rightarrow \frac{-2}{3} = C \quad \text{ZŁOŹSKA OK}$$