INTEGRÁLY ELEM. FCÍ

$\int e^x\,dx = e^x$ $\int x^n\,dx = \frac{x^{n+1}}{n+1}$

$\int \frac{1}{x}\,dx = \ln(x)$ $\int \cos(x)\,dx = \sin(x)$

$\int \sin(x)\,dx = -\cos(x)$ $\int a^x\,dx = \frac{a^x}{\ln(a)}$

$$\int (9x^6 + 6x^3 - 14\sin x)\,dx = \frac{9x^7}{7} + 6\frac{x^4}{4} + 14\cos(x)$$

$$\int \left(\frac{1}{x+2} + \frac{1}{x^3}\right)dx = \ln(x+2) + \frac{x^{-2}}{-2}$$

$\frac{1}{x^3} = x^{-3}$

$$\int \left(\sin(x+80) - 2\cdot 5^x + 1\right)dx = -\cos(x+80) - \frac{2\cdot 5^x}{\ln(5)} + x$$

$$\int \left(4\sqrt[3]{x} - \frac{30}{x+3} + \frac{5}{\cos^2(x)}\right)dx =$$

$\sqrt[3]{x} = x^{\frac{1}{3}}$

$$= 3x^{\frac{4}{3}} - 30\cdot\ln(x+3) + 5\cdot \mathrm{tg}(x)$$

$$\int \left(\frac{9^{x-2} + 12^{x+3}}{3^{x+1}}\right)dx = \int \frac{3^{2x-4} + 3^{x+3}\cdot 4^{x+3}}{3^{x+1}}$$

$(a\cdot b)^x = a^x \cdot b^x$

$(a\cdot a)^x = a^x \cdot a^x = a^{2x}$

$$= \int \frac{3^{2x-4}}{3^{x+1}} + 4^{x+3}\cdot\frac{3^{x+3}}{3^{x+1}} =$$

$$\frac{3^{2x-4}}{3^{x+1}} = 3^{x-5} \qquad \frac{3^{x+3}}{3^{x+1}} = 9$$

$$= \int 3^{x-5} + 4^{x+3}\cdot 9 =$$

$$= \frac{3^{x-5}}{\ln(3)} + \frac{4^{x+3}}{\ln(4)}\cdot 9$$

$$\int 3\cos(3x+5)\,dx = \underline{\underline{\sin(3x+5)}}$$

$$\int\left(4\cos(5x+20) - 3\sin(8x-2) + \frac{1}{2x+80}\right)dx$$

$$\int 4\cos(5x+20)\,dx = \frac{4}{5}\sin(5x+20)$$

$$\frac{4}{5}\left(\sin(5x+20)\right)' = 4\cos(5x+20) \quad / :5$$

$$-3\int \sin(8x-2)\,dx = \frac{3}{8}\cos(8x-2)$$

$$3\left(\cos(8x-2)\right)' = -24\sin(8x-2) \quad / :8$$

$$\frac{3}{8}\left(\cos(8x-2)\right)' = -3\sin(8x-2)$$

INTEGRACE PER PARTES

$$\int f(x)\cdot g(x)\,dx = ??$$

$$(f\cdot g)' = f'g + f\cdot g' \quad / \cdot\int$$

$$\int (f\cdot g)' = \int f'g + \int f\cdot g'$$

$$f\cdot g = \int f'g + \underline{\underline{\int f g'}}$$

$$\boxed{\int f g' = f\cdot g - \int f' g}$$

$$\int (x\cdot e^x)\,dx = \left| \begin{matrix} f = x & g' = e^x \\ f' = 1 & g = e^x \end{matrix} \right| =$$

$$= f\cdot g - \int f'g = x\cdot e^x - \int 1\cdot e^x\,dx = \underline{\underline{x\cdot e^x - e^x}}$$

$$\int (x\cdot \sin x)\,dx = \left| \begin{matrix} f = x & g' = \sin x \\ f' = 1 & g = -\cos x \end{matrix} \right| =$$

$$= f\cdot g - \int f'g = -x\cdot\cos x + \int \cos x\,dx =$$

$$= \underline{\underline{-x\cdot\cos(x) + \sin(x)}}$$

$$\int \ln(x)\,dx = \int 1\cdot\ln(x)\,dx = \left| \begin{matrix} f = \ln(x) & g' = 1 \\ f' = \frac{1}{x} & g = x \end{matrix} \right|$$

$$= x\cdot\ln(x) - \int \underbrace{\frac{1}{x}\cdot x}_{1}\,dx = \underline{\underline{x\cdot\ln x - x}}$$

$$\underbrace{\int \left(e^x \cdot \sin(x)\right) dx}_{Y} = \left| \begin{array}{ll} f = \sin(x) & g' = e^x \\ f' = \cos(x) & g = e^x \end{array} \right| =$$

$$= \sin(x)\cdot e^x - \int \left(\cos(x)\cdot e^x\right) dx = \left| \begin{array}{ll} f = \cos x & g' = e^x \\ f' = -\sin(x) & g = e^x \end{array} \right| =$$

$$= \sin(x)\cdot e^x - \left(\cos(x)\cdot e^x + \int \sin(x)\cdot e^x\right)$$

$$Y = \sin x\, e^x - \cos x\, e^x - Y \quad /+Y$$

$$2\int \sin(x)\cdot e^x dx = e^x\left(\sin(x) - \cos(x)\right) \quad /:2$$

$$\int \sin(x) e^x dx = \frac{1}{2} e^x \left(\sin(x) - \cos(x)\right)$$