$[\sin(3x^2+5)]^{\cos(x\cdot\sin x)} = f(x)$, $f'(x) = ?$

Součet fcí: $(f(x)+g(x))' = f'(x)+g'(x)$

1). $f(x) = x^5 + \cos(x)$

$f'(x) = 5x^4 + (-\sin(x))$

2) $f(x) = 3\ln(x) + 3e^x$

$f'(x) = \frac{3}{x} + 3e^x$

Součin a podíl fcí: $(f(x)\cdot g(x))' = f'g + fg'$

$\left(\frac{f(x)}{g(x)}\right)' = (f(x)\cdot g^{-1}(x))' =$

1) $f(x) = \frac{x+1}{x-1}$, $f'(x) = \frac{1\cdot(x-1)-(x+1)\cdot 1}{(x-1)^2}$

2) $(\sin(x)\cos(x))' = \sin'(x)\cos(x) + \sin(x)\cos'(x)$
$= \cos^2(x) - \sin^2(x)$

3) $(x^5\cdot 3e^x)' = 5x^4\, 3e^x + x^5\cdot 3e^x$

4) $(\underbrace{3^x}_{f(x)}\cdot\underbrace{\sin(x)\cdot x^2}_{g(x)})' =$

$= 3^x\cdot\ln(3)\cdot\sin(x)\cdot x^2 +$
$+ 3^x\cdot(\sin(x)\cdot x^2)' =$

$(e^x)' = e^x$

$(a^x)' = a^x\cdot\ln(a)$

$(3^x)' = 3^x\cdot\ln(3)$

$(\sin(x)\cdot x^2)' = \cos(x)x^2 + 2\sin(x)x$

$\Rightarrow 3^x\ln(3)\cdot\sin(x)\cdot x^2 + 3^x(\cos(x)\,x^2 + 2x\sin(x))$

Složené fce: $(f(g(x)))' = f'(g(x))\cdot g'(x)$

$f(x) = 2x$

$f'(x) = 2x^0\cdot 1$

1) $f(x) = \sin(2x^2+1)$

$f'(x) = \cos(2x^2+1)\cdot(2x^2+1)' =$

$= \cos(2x^2+1)\cdot 4x$

2) $f(x) = (3x^2+8)^2$

$f'(x) = 2(3x^2+8)\cdot(3x^2+8)' = 2(3x^2+8)\cdot 6x$

3) $f(x) = \sqrt[3]{\sin(x)}$

$f'(x) = \frac{\cos(x)}{\sqrt[3]{\sin^2(x)}\cdot 3}$

$\left(x^{\frac{1}{3}}\right)' = \frac{1}{3}\cdot x^{-\frac{2}{3}}$

$f'(x) = \frac{1}{3}\sin^{-\frac{2}{3}}(x)\cdot\cos(x)$

4) $(\ln^3(\ln^2(x)))' =$

4) $f(x)=\ln^3(\ln^2(x))$

$f'(x)=3\ln^2(\ln^2(x))\cdot(\ln(\ln^2(x)))'$

$(x^3)'=3x^2\cdot x'$

$(\ln(\ln^2(x)))'=\frac{1}{\ln^2(x)}\cdot(\ln^2(x))'$

$(\ln(x))'=\frac{1}{x}$

$(\ln^2(x))'=2\ln(x)\cdot(\ln(x))'=$

$=2\frac{\ln(x)}{x}$

$f'(x)=\frac{3\ln^2(\ln^2(x))\cdot\ln(x)\cdot 2}{\ln^2(x)\cdot x}$

$f(x)=\ln(\cos(x^2+3x))$

$f'(x)=\frac{1}{\cos(x^2+3x)}\cdot(-\sin(x^2+3x)\cdot(2x+3))$

Mocnění fcí: $\left(f(x)^{g(x)}\right)'=\left(e^{\ln(f(x)^{g(x)})}\right)'$

$h(x),\ h^{-1}(x)$ $\qquad e^{\ln(x)}=x$

$h(h^{-1}(x))=x$ $\qquad \ln(x^a)=a\ln(x)$

$\boxed{\left(e^{g(x)\cdot\ln(f(x))}\right)'}$

1) $f(x)=x^{3x^2+1}$

$f'(x)=\left(e^{(3x^2+1)\ln(x)}\right)'=e^{(3x^2+1)\ln(x)}\cdot[(3x^2+1)\cdot\ln(x)]'$

$((3x^2+1)\ln(x))'=6x\ln(x)+\frac{3x^2+1}{x}$

$f'(x)=x^{3x^2+1}\cdot\left(6x\ln(x)+\frac{3x^2+1}{x}\right)$

2) $f(x)=x^{\ln x}$

$f'(x)=x^{\ln x}\cdot(\ln^2(x))'=x^{\ln x}\,2\frac{\ln(x)}{x}$

3) $f(x)=\sin(x)^{\cos(x)}$

$f'(x)=\sin(x)^{\cos(x)}[\cos(x)\ln(\sin(x))]'=$

$=\sin(x)^{\cos(x)}\cdot\left[-\sin(x)\ln(\sin(x))+\frac{\cos^2(x)}{\sin(x)}\right]$

4) $\left(x^{\frac{1}{x}}\right)'=x^{\frac{1}{x}}\cdot\left(\frac{\ln(x)}{x}\right)'=x^{\frac{1}{x}}\cdot\left(\frac{1-\ln(x)}{x^2}\right)$

$\left(\frac{\ln(x)}{x}\right)'=\frac{\frac{1}{x}\cdot x-\ln(x)\cdot 1}{x^2}$