Definiční obory funkcí více proměnných

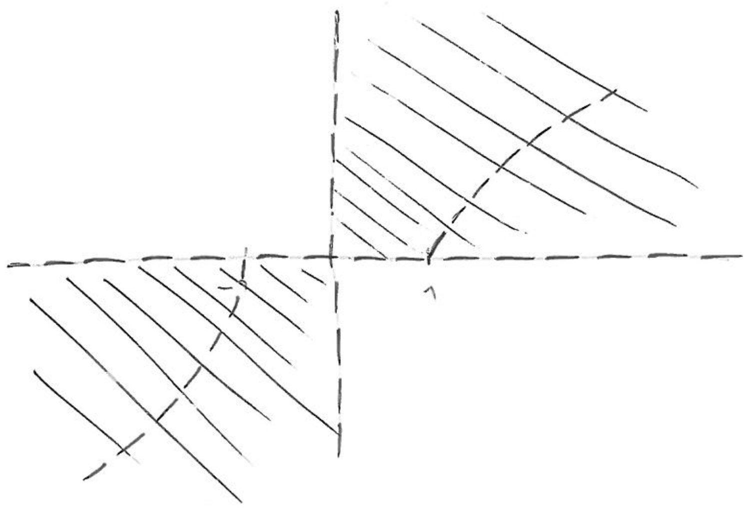
① $f(x,y) = \dfrac{\ln(xy)}{x^2-y^2-1}$

$$xy > 0 \quad \wedge \quad x^2-y^2-1 \neq 0$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$(x>0 \wedge y>0) \vee (x<0 \wedge y<0) \qquad x^2-y^2 \neq 1$$

$$D f = \{(x,y) \in \mathbb{R}^2 : \; xy > 0 \;\wedge\; x^2-y^2 \neq 1\}$$



POZNÁMKA:
grafem rovnice
$x^2-y^2=1$
je dvojice
hyperbol

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② $f(x,y) = \arcsin(x+y)$

$$-1 \leq x+y \leq 1$$

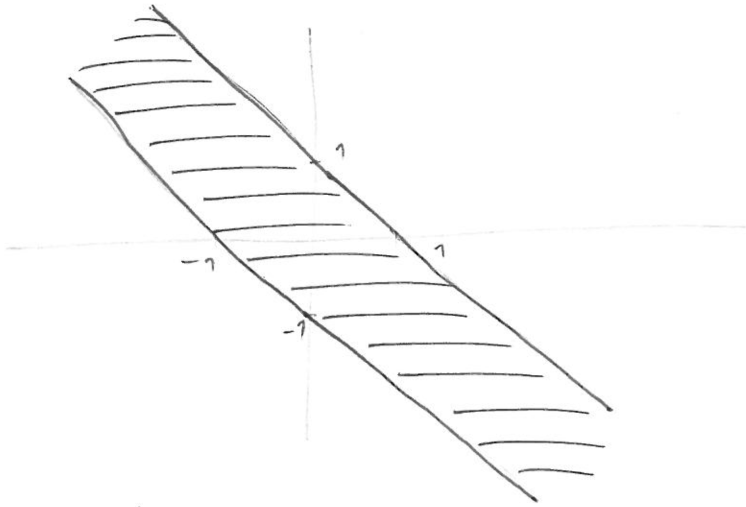
$$\Updownarrow$$

$$-1 \leq x+y \quad \wedge \quad x+y \leq 1$$

$$\Updownarrow$$

$$y \geq -x-1 \quad \wedge \quad y \leq -x+1$$

$Df = \{(x,y) \in \mathbb{R}^2 : -1 \leq x+y \leq 1\}$



③ $f(x,y) = \dfrac{1}{\sqrt{y-\sqrt{x}}}$

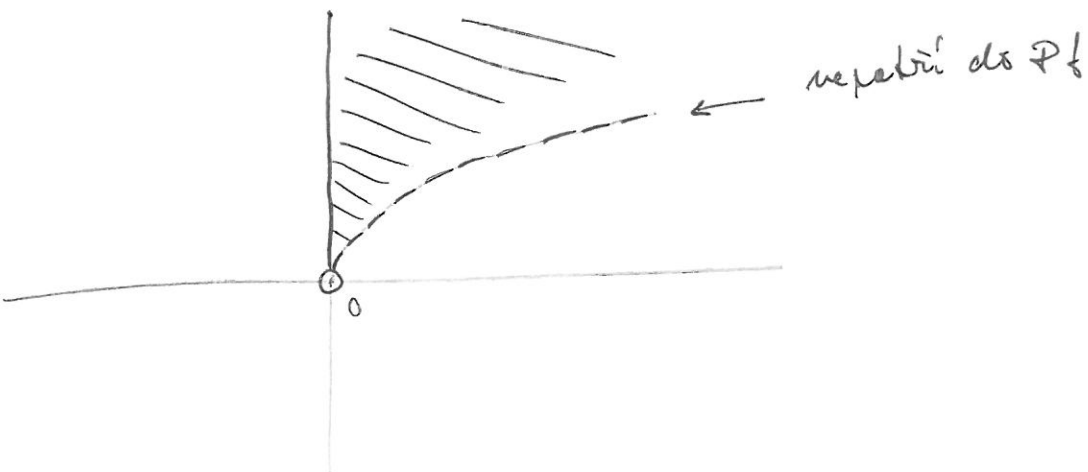
$\sqrt{y-\sqrt{x}} \neq 0$

$y - \sqrt{x} > 0$

$y > \sqrt{x}$

zároveň musí platit $x \geq 0$, aby byla definována $\sqrt{x}$

$Df = \{(x,y) \in \mathbb{R}^2 : x \geq 0 \wedge y > \sqrt{x}\}$



④ $f(x,y) = \sqrt{\frac{1+x}{1-y}} \cdot \ln\left(x^2 + 4y^2 - 1\right)$

$$\underbrace{\frac{1+x}{1-y} \geq 0}_{(1)} \quad \wedge \quad \underbrace{x^2 + 4y^2 - 1 > 0}_{(2)}$$

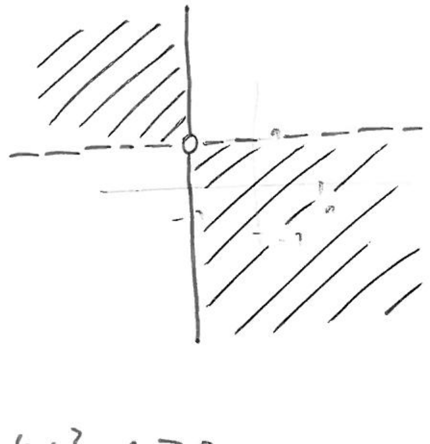
(1): $\frac{1+x}{1-y} \geq 0$

$$\Updownarrow$$

$$(1+x \geq 0 \ \wedge\ 1-y > 0) \ \vee\ (1+x \leq 0 \ \wedge\ 1-y < 0)$$

$$\Updownarrow$$

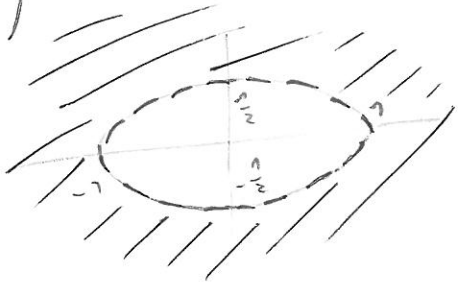
$$(x \geq -1 \ \wedge\ y < 1) \ \vee\ (x \leq -1 \ \wedge\ y > 1)$$



(2): $x^2 + 4y^2 - 1 > 0$

$x^2 + 4y^2 > 1$

$\left(\frac{x}{1}\right)^2 + \left(\frac{y}{\frac{1}{2}}\right)^2 > 1$

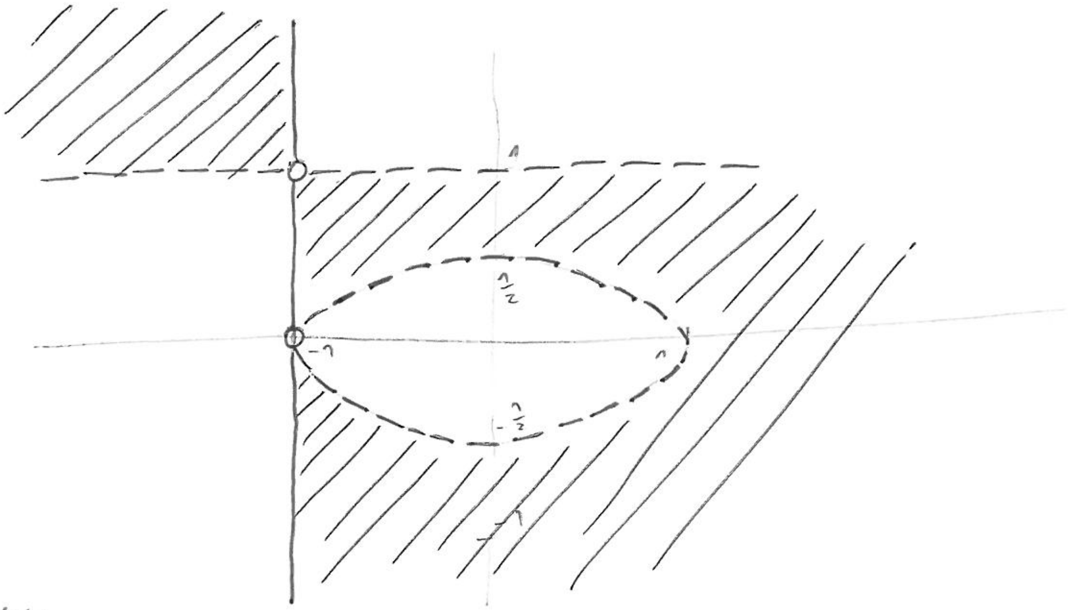


POZNÁMKA:

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ je

rovnice elipsy s hlavní poloosou délky a a vedlejší poloosou délky b (pro $a, b > 0$)

$$D_f = \{(x,y) \in \mathbb{R}^2 : ((x \geq -1 \wedge y < 1) \vee (x \leq -1 \wedge y > 1)) \wedge x^2 + 4y^2 > 1\}$$



POZNÁMKA:

definiční obor je **průnikem** řešení nerovnic (1) a (2)