

## Types of Exam Problems Mathematical Analysis (470-2110/02,04,06) - Instr. Tereza Kovářová

Each Example – 10pts, together – 70 pts at maximum, time limit – 100 min

### Exam Topics

1. Limits of sequences, Limits of functions, Domain of a function
2. Function behavior- intervals of strict monotonicity, local extremes, global extremes
3. Function behavior - intervals of strict convexity/concavity, asymptotes
4. Approximation of a function by Taylor polynomial or Maclaurin polynomial or differential
5. Solving indefinite integral
6. Evaluation of definite integral and its applications (area of a plane region, length of a plane curve, volume of a solid of revolution)
7. Theoretical true-false questions aimed to test understanding of basic notions and their main properties.

### Sample Exam - Assignment

1. Determine the limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x}.$$

2. On the set  $M = [-\pi; 2\pi]$ , determine the global extremes of the function

$$f(x) = 4 \sin \frac{x}{2} + (2\pi - 2x) \cos \frac{x}{2}.$$

3. Find all asymptotes of the function

$$f(x) = \frac{(x+2)^2}{x+1}.$$

4. Write the Taylor polynomial of the second degree of the function

$$f(x) = \tan x$$

at the point  $x_0 = \frac{\pi}{4}$ .

5. Solve the indefinite integral

$$\int \frac{\ln^2 x}{2x} dx.$$

6. What is the area of the plane region bounded by the curves

$$y = \cos x, x = 0, x = \pi, y = \frac{2}{\pi}x - 1.$$

7. Determine which of the following statements are True or False.

- (a) The function  $\cos x$  is the inverse function to the trigonometric function  $\sin x$ .
- (b) If  $f$  is strictly monotonic function, then  $f$  is injective.
- (c) If  $\lim_{x \rightarrow -1^+} f(x) = -\infty$ , then  $f$  has asymptote  $x = -1$ .
- (d) If  $f''(x) = x - 1$  for any  $x \in (2, 5)$ , then the function  $f$  is strictly convex in the interval  $(2, 5)$ .
- (e) Each increasing sequence is not bounded.

Instructions how to work out this task. You will get 2 pts for the correct answer and you will lose 2 pts for the incorrect answer. For unanswered question you get 0 pts. The maximum pts is 10 and the minimum 0. You will not get negative points on this problem, but choose your answers wisely. (It does not pay off to bet.)

For instance, if you are not sure about questions (a) and (c), write:

- (a)
- (b) True
- (c)
- (d) False
- (e) True