

## Exercises – Part 1

The problems given below are similar to those, that will appear on your semester tests.

### 1. Supremum, Infimum, Maximum, Minimum, Bounded Set

If exist, determine  $\min M$ ,  $\max M$ ,  $\inf M$  and  $\sup M$ , where:

1.  $M = \{x \in \mathbb{R} : \frac{x+5}{5x-1} \geq 1\}$ .
2.  $M = \{0.4, 0.44, 0.444, 0.4444, \dots\}$ .
3.  $M = \{x \in \mathbb{R} : x \geq 0 \wedge \log_2 x^2 + \log_2 x < 3\}$ .
4.  $M = \{x \in \mathbb{R} : 0 \leq x \leq \frac{\pi}{2} \wedge \sin x < \frac{1}{2}\}$ .
5.  $M = \left\{x \in \mathbb{R} : \frac{(x-1)^2(x-3)^3}{(x+4)^2(x+2)} \leq 0\right\}$ .

### 2. Properties of Functions

1. Determine the domains of the following functions:

- (a)  $f(x) = \frac{1}{\log_5(x^2 - 1)}$
  - (b)  $f(x) = \sqrt{|x - 2| - |x + 2|}$
  - (c)  $f(x) = 2 \arcsin \frac{1}{x + 2}$
  - (d)  $f(x) = \frac{1}{\sqrt[3]{x^2 - 4}} + e^{3x+1} + \frac{\sqrt{x+3}}{x^2 + x - 6}$
  - (e)  $f(x) = \sqrt[5]{\ln(\tan x)}$
  - (f)  $f(x) = 2^{\frac{1}{\sqrt{3x}}}$
  - (g)  $f(x) = \log_2(1 - |3 - x|)$
  - (h)  $f(x) = \ln(1 - \ln x)$
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2. Sketch a graph of a function  $f$ , if you know that:

- $D(f) = \mathbb{R}$ ,  $f$  is continuous, even and periodic with the period  $T = 2$ ,
- $f(0) = 1$ ,  $f(1) = 0$ ,
- $f$  is a linear function on the interval  $[0, 1]$

Calculate  $f(1000)$ ,  $f(\frac{3}{2})$  and  $f(-5)$ .

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3. Sketch a graph of a function  $f$  given by:

$$f(x) = \begin{cases} \sin(x - 1) + 1 & \text{if } x \in (-\infty, -1) \\ x & \text{if } x \in [-1, 1] \\ \sin(-x + 1) - 1 & \text{if } x \in (1, \infty) \end{cases}$$

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4. Sketch a graph of a function  $f$  given by:

$$f(x) = |x| - 3|x + 2| + x$$

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5. Find the equation of a quadratic function  $g : \mathbb{R} \mapsto \mathbb{R}$  so that:

$$g(0) = 3, \quad g(1) = 0, \quad g(2) = -1.$$