

## 2 Project – submit your solution to tereza.kovarova@vsb.cz

### Combinatorics

- 2.1. A restaurant employee is creating a drink menu for which he chooses from the selection of drinks offered by drink supplying company. He will choose 5 brands of wine, so that two are white wines, two are red wines and one is a sparkling wine. Next, he will choose 1 brand of draft beer and 3 brands of bottled beer, so that bottled beers are of different brand than the chosen draft beer. And last he will choose 5 kinds of fruit juice.

The drink supplying company offers 15 brands of red wine, 16 brands of white wine, and 10 brands of sparkling wine. Next, it offers 12 kinds of fruit juice. Among draft beers offered are brands: Radegast, Bernard, and Pilsner. Among bottled beers offered are brands: Radegast, Pilsner, and 12 other brands, but not Bernard.

In how many ways can the employee set up the drink menu? Also, find out, in how many ways can the employee arrange bottles of all kinds of drinks on the menu with price tickets in a row on the shelf above the bar. Assume all wine bottles are supposed to be next to each other as well as beer bottles are grouped together and so are juice bottles? (3 b)

- 2.2. Suppose you decide to play in a lottery with the following rules: From numbers ranging from 1 up to 30 are 4 chosen arbitrarily, no number can repeat and the order is not important. Two additional numbers are chosen from numbers between 1 and 20, while the number can repeat and the order does matter.

The first price is paid to a player if all 6 numbers are guessed correctly including the proper order of complementing numbers.

The second price is paid to a player if all first 4 numbers are guessed correctly and one of the complementing numbers is guessed in correct order.

The third price is paid to a player if the first 4 numbers are guessed correctly (in any order) and none of the complementing numbers is guessed.

Calculate the probability, that you win one of the first three prices. (2 b)

### Graph Theory

- 2.3. Find the metric (the distance matrix) of Hershel graph  $H$ . Is it possible to add only one edge  $e$  to Hershel graph  $H$  so that in the new graph  $G$  the maximum distance between vertices is 2? If it is possible, construct  $G$  and write the metric of  $G$ ,  $G = H \cup e$ . Otherwise prove that by adding one edge  $e$  into  $H$  the maximum distance can not decrease below 3.

(What is Herschel graph find by yourselves on the Internet. It's picture is to be found also in the textbook of graph theory by P.Kovar: [http://homel.vsb.cz/~kov16/files/skriptum\\_teorie\\_grafu.pdf](http://homel.vsb.cz/~kov16/files/skriptum_teorie_grafu.pdf)) (2 b)

- 2.4. Find the minimum spanning tree of a weighted graph  $G$ , which is obtained by omitting one edge from  $K_8$ . Vertices of a graph  $K_8$  are assigned integers from 1 to 8 and to obtain a graph  $G$  the edge between vertices numbered with 1 and 2 is deleted. The weight of each edge in  $G$  is given by the product of numbers assigned to its end vertices. To find the minimum spanning tree, use Jarnik's or greedy algorithm. Next, determine if the spanning tree is isomorphic to a tree with binary code (0010101010100111). The given code is not necessarily minimal code. Use correctly the algorithm for trees isomorphism verification. (3 b)

## Guidelines

Write the project using a computer, include the title with your name, student ID, number of the project, year and a grading table (see the sample project). The project will contain a detailed description of your solution for each problem. Show your work by explaining the steps carefully. If you skip a problem, mark it clearly in the text by saying „*I did not solve the problem number X*“.

Submit your project to [tereza.kovarova@vsb.cz](mailto:tereza.kovarova@vsb.cz) as an uncompressed PDF file, use your student ID in the name of your submitted file.

You will be awarded 0 upto 2 or 0 upto 3 points for each of the problems.

Submit your project no later than on **Monday December 10th 2018 at 23:59**.