Quantum Chemistry Seminar 5

Approximate methods I Variational methods

Exercise 1 (Shamal)

Prove the equalities given on page 3 of lesson 5, $\langle \psi | \psi \rangle = \sum_{k=0}^{+\infty} |c_k|^2$ and $\langle \psi | \hat{H} | \psi \rangle = \sum_{k=0}^{+\infty} E_k |c_k|^2$, provided that the Hamiltonian eigenfunctions, $|\varphi_k\rangle$, are orthonormal, $\langle \varphi_j | \varphi_k \rangle = \delta_{jk}$.

Exercise 2 (Anila)

Show that $|\psi(\tau)\rangle = \sum_{k=0}^{+\infty} c_{k0} e^{-\frac{E_k}{\hbar}\tau} |\varphi_k\rangle$, where $\hat{H} |\varphi_k\rangle = E_k |\varphi_k\rangle$, represents a solution of the imaginary-time Schrödinger equation, $\hat{H} |\psi(\tau)\rangle = -\hbar \frac{\partial |\psi(\tau)\rangle}{\partial \tau}$.