Simulation of the parametric excitation of the cantilever beam vibrations

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Abstract — The effect of parametric vibration damping is theoretically well described and the first experiments are published. The paper focuses on the design and verification of the system functionality which is based on the use of piezoactuators as an element of the controlled stiffness which varies periodically in time according to a sinusoidal function. The principle of governing the piezoactuator stiffness is based on proportional control of the actuator displacement. The transfer function of a disturbance force to the actuator displacement is affected by the controller gain. Periodic changes of the controller gain should have a damping effect on vibration of the cantilever beam according to the simulation in Matlab-Simulink environment. The paper presents a simulation study of this method of damping vibrations.

Keywords — Parametric resonances; vibration damping; piezoactuators; cantilever beam; variable controller gain; simulation

I. INTRODUCTION

A cantilever beam is a simple example of a mechanical structure that can serve as an object for testing the active vibration control. The promising way to reduce vibration of mechanical structures is parametric excitation, which is fundamentally different from the active vibration control systems developed for example by Krenk [1]. Unlike active vibration damping, which is based on the use of linear methods for control of the linear time invariant systems, the new way of damping uses a periodic change of a parameter, usually spring stiffness. Such a system becomes non-linear and potentially unstable for an interval of the frequency of the mentioned parameter variation. A fundamental research in the field of instability of the non-linear mechanical systems was done by Tondl [2], [3]. Tondl's main conclusions were published many times [4]. They state formulas for calculating the frequency of parametric resonances and describe the instability intervals for the frequency of parametric excitation if the mechanical system contains only one element with the periodic change of the parameter according to the sinusoidal function of time.

Now we will use the summary of a paper written by Petermeier and Ecker [5] who distinguish between a Principle Parametric Resonances at frequencies $\omega_{j/n}^{\text{Pr}}$ and Combination

Parametric Resonances at frequencies $\omega_{j\pm k/n}^{\text{Cr}}$. These frequencies are defined as follows

$$\omega_{j/n}^{\text{Pr}} = \frac{2\Omega_j}{n}, \quad \omega_{j\pm k/n}^{\text{Cr}} = \frac{|\Omega_j - \Omega_k|}{n}, \quad j, k = 1, 2, \dots, n = 1, 2, \dots \quad (1)$$

where Ω_j and Ω_k are the j -th and k -th natural frequency of the linear system. The denominator n represents the order of the parametric resonances. Horst Ecker from the Vienna University of Technology is developing methods for parametric damping of mechanical systems and carries out experiments in this field damping.

Unlike the paper [5], which uses the beam model of the Timoshenko type, this paper examines the effect of parametric control based on the lumped parameter model of the Euler-Bernoulli type. The simulation of the beam vibration decay and the random excitation is a tool for analyzing the mentioned effect. The cantilever beam dimensions correspond to a laboratory test bench which is prepared to experiments. The main objective of the article is to demonstrate a positive effect of parametric excitation on vibration damping.

II. ELECTRICALLY CONTROLLED STIFFNESS

High-frequency periodic changes in stiffness can be made only with piezoactuators or electrodynamic exciters. The use of the piezoactuators in the position control will be considered in this chapter. Parametric damping requires a periodic change in stiffness. The way to achieve this is shown in Fig. 1.

Fig. 1. Variable stiffness based on piezoactuators

We assume that the difference of the piezoactuator travel is proportional to the difference of the input voltage

$\Delta y_1 = k_1 \Delta V$, where k_1 is a coefficient of proportionality. An effect of the difference of the external force F is a difference of the displacement $\Delta y_2 = k_2 \Delta F$, where k_2 is a compliance of the piezoactuator. The linear model of the piezoactuator is as follows

$$\Delta y = \Delta y_1 + \Delta y_2 = k_1 \Delta V + k_2 \Delta F. \quad (2)$$

The difference of the voltage follows the difference of the total displacement of the piezoactuator

$$\Delta V = K_c(0 - \Delta y) = -K_c \Delta y, \quad (3)$$

where K_c is a gain of the proportional controller. The stiffness of the piezoactuator which is a part of the closed loop is given by the following formula

$$\frac{\Delta F}{\Delta y} = \frac{1}{k_2} + \frac{k_1}{k_2} K_C = [\text{initial stiffness}](1 + k_1 K_C). \quad (4)$$

A piezoactuator such as P-845.60 made by the PI Company has stiffness 38 N/ μm , and because it is the low voltage type, then the parameter k_1 is equal to about 90 μm / 100 V.

Periodic variation in stiffness is achieved by periodically changing the gain of the proportional controller. Connection of the piezoactuator into the position control loop with a time variable gain is just a proposal to prove the feasibility of this type of excitation.

III. MATHEMATICAL MODEL OF CANTILEVER BEAMS

The mathematical model should be simple enough that it could be created in the Matlab-Simulink environment as a lumped parameter model. The resonant frequency and deflection of a beam of the discrete elements should match the beam as a continuum. Because it does not concern a beam, which was thick, the Euler-Bernoulli theory is best to use for design a model. It is assumed that the beam is combined from rigid elements which are connected by flexible links formed by torsion springs as it is shown in Fig. 2. It is also assumed that the bending stiffness $K_\delta = \tau/\Delta\delta$ of the flexible links of the adjacent elementary beams relates the applied bending moment τ to the resulting relative rotation $\Delta\delta$ of the elementary beams and the corresponding potential energy. The bending stiffness of the flexible links will be identical except for the one connection whose stiffness would be periodically increased and decreased. The periodic changes in the bending stiffness can be achieved by using the patch piezoactuator which is glued to the surface of the beam and is not connected with the rigid frame. The effect of this phenomenon will be discussed at the end of this chapter. This assumption will change the system on nonlinear and non-stationary.

The multibody system in Fig. 1 is associated with the Cartesian coordinates x, y, z . The cantilever beam is clamped at the xy -plane and its centerline is parallel to the z -axis. It is assumed only a planar motion of the cantilever beam in the yz -

plane. The link of a pair of the adjacent beam elements is considered in the mentioned plane as free with the mentioned torsion spring. To avoid the additional set of constraints for the link of the individual beam elements in one point the coordinate system is chosen in such a way that describes motion of the meeting points of two adjacent elementary beams called nodes [6]. The vertical coordinates of these nodes are designated by $y_1, y_2, \dots, y_N$. The angle of rotation of the elementary beams with respect to the horizontal axis can be designated by $\delta_1, \delta_2, \dots, \delta_N$ and their measure in radians can be expressed by $\delta_n = (y_n - y_{n-1})/\Delta L$ if all angles are small enough. For $n=1$ it is valid $\delta_1 = y_1/\Delta L$ and therefore it is assumed that $\delta_0 = 0$.

The coordinates of the beam equidistant points in the Cartesian coordinates and the independent generalized coordinates for Lagrange's equations of motion are identical. For further derivation it makes sense only the motion in the direction of the y -axis. Because they are assumed small deformations, the shifts of the nodes in the direction of the z -axis are neglected.

Fig. 2 Coordinates of elements of a cantilever beam

The derivation of the equations of motion has been published previously, so it will include only the most important formulas [7, 8] with the use of Lagrange's equation. After introduction symbols $\mathbf{M}$ for a mass square matrix, $\mathbf{K}$ for a stiffness square matrix and $\mathbf{y} = [y_1, y_2, \dots, y_N]^T$ for a coordinate column vector into the matrix equation of motion we obtain the equation for free vibration

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{0}. \quad (5)$$

It is supposed that the cross section of the beam is a rectangular. The moment of inertia of the beam element about the horizontal x -axis and perpendicular to the centerline of the beam is calculated according to the formula $J_x = \Delta m (\Delta L^2 + h^2) / 12$ where h is height and Δm is mass of the element. The Lagrange's equation gives the mass matrix of the following form

$$\mathbf{M} = \begin{bmatrix} B & A & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ & A & B & A & \\ & \vdots & \vdots & \vdots & \vdots \\ & & & A & B/2 \end{bmatrix}, \quad (6)$$

where

$$A = \frac{\Delta m}{4} \left(1 - \frac{1}{3} \left(1 + \left(\frac{h}{\Delta L} \right)^2 \right) \right), B = \frac{\Delta m}{2} \left(1 + \frac{1}{3} \left(1 + \left(\frac{h}{\Delta L} \right)^2 \right) \right). \quad (7)$$

Similarly as for the mass matrix $\mathbf{M}$ the stiffness matrix $\mathbf{K}$ is created with the use of the Lagrange's equations. Assuming that the bending stiffness of all the element joint of is the same, then we can write

$$\mathbf{K} = \frac{K_\delta}{\Delta L^2} \begin{bmatrix} 6 & \dots & & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & 1 & -4 & 6 & -4 & 1 \\ & \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & \dots & 1 & \end{bmatrix}. \quad (8)$$

Handbooks of mechanics [9] state formulas for the deflection of the cantilever beam of the length L at the free end. The beam is considered as a continuum. This deflection due to the force F_N acting at the free end in the direction of the z -axis and corresponds to the coordinate y_N in Fig. 2 therefore we may state

$$y_N = \frac{F_N L^3}{3EI_x}, \quad (9)$$

where $E = 2.14 \times 10^{11} \text{ N/m}^2$ is Young's modulus of the beam material, $I_x = bh^3/12$ is the area moment of inertia of the beam cross-section about the horizontal x -axis.

The stationary bending stiffness of each flexible link in Fig. 2 is designated as K_n , $n=1, 2, \dots, N$. The stationary values of all the bending stiffness is the same and equal to

$$K_\delta = \frac{\tau}{\Delta \delta} = \eta \frac{EI_x}{\Delta L}, \quad (10)$$

where η is a multiplication factor which will be discussed in the next paragraph section.

TABLE I. MULTIPLICATION FACTOR FOR CALCULATION OF BENDING STIFFNESS

Number N	1	2	5	10
Factor η	3	1.8750	1.3200	1.1550
Number N	20	50	100	200
Factor η	1.0763	1.0302	1.0150	1.0075

We have prepared a specimen of the cantilever beam of the following parameters: $L = 0.5 \text{ m}$, $b = 0.04 \text{ m}$, $h = 0.005 \text{ m}$ to be tested. The value of the multiplicative factor in equation (10) was designed so that the beam deflection at the free end of the lumped-parameter model fits the deflection, which was calculated using the formula (9). The deflection of the beam along its length can be calculated with the use of the formula

$$\mathbf{y}_0 = \mathbf{K}^{-1} \mathbf{F}_0. \quad (11)$$

where $\mathbf{F}_0 = [0, 0, \dots, F_N]^T$ is a force acting at the center of gravity. It was found out that the value of this factor depends on the number N of elementary beams. The results are shown in Table 1. Increasing the number of elements means that the factor η tends to one which means that the bending stiffness tends to $K_\delta = EI_x / \Delta L$ and the discrete cantilever beam tends to a continuous beam. Selection of $N = 5$ will be used in the simulations and therefore the correction factor η plays an important role. The positive consequence of the introduction of the correction factor is also that the resonant frequencies of a beam calculated for the beam as a continuum coincide with frequencies which are calculated for the beam which is divided in the elements with the use of the eigenvalues $\lambda = \Omega^2$ of the matrix $\mathbf{K}^{-1} \mathbf{M}$.

IV. EFFECT OF THE STIFFNESS PERIODIC CHANGE

As mentioned earlier, the part of the total stiffness of the $(n+1)$ -th spring in Fig. 2 is a sinusoidal function of time

$$K_{n+1} = K_\delta (1 + 2\mu \cos(\omega_0 t)), \quad (12)$$

where ω_0 is an angular frequency of the parametric excitation and μ is a parameter which represents the amplitude of changes in stiffness. In this paper, this amplitude was searched experimentally.

The potential energy V of the deflected beam is as follows

$$V = \sum_{n=0}^{N-1} \frac{1}{2} K_{n+1} (\Delta \delta_n)^2 = \frac{1}{2\Delta L^2} \sum_{n=0}^{N-1} K_{n+1} (y_{n+1} - 2y_n + y_{n-1})^2. \quad (13)$$

The first derivatives of the potential energy V in the Lagrange's equations with respect to the variables y_k , $k=1, 2, \dots, N$ and $\dot{y}_k$, $k=1, 2, \dots, N$ were used to create the stiffness matrix. The non-stationary stiffness K_{n+1} depends neither on y_k nor on $\dot{y}_k$ which is missing in the formula (13). The only problem that rigidity stiffness in the three equations of motion as shown by the following formulas [5]

$$\begin{aligned} n-1: & \dots + K_{n+1} \Delta L^2 (y_{n+1} - 2y_n + y_{n-1}) + \dots = 0 \\ n: & \dots - 2K_{n+1} \Delta L^2 (y_{n+1} - 2y_n + y_{n-1}) + \dots = 0 \\ n+1: & \dots + K_{n+1} \Delta L^2 (y_{n+1} - 2y_n + y_{n-1}) + \dots = 0. \end{aligned} \quad (14)$$

After the substitution from (10) and (12) it can be obtained

$$\begin{aligned} n-1: & \dots + 6K_\delta \Delta L^{-2} y_{n-1} + \dots = -K_\delta \Delta L^{-2} \delta(t) \\ n: & \dots + 6K_\delta \Delta L^{-2} y_n + \dots = +2K_\delta \Delta L^{-2} \delta(t) \\ n+1: & \dots + 6K_\delta \Delta L^{-2} y_{n+1} + \dots = -K_\delta \Delta L^{-2} \delta(t) \end{aligned} \quad (15)$$

where

$$\begin{aligned} \delta(t) &= 2\mu(y_{n+1} - 2y_n + y_{n-1})\cos(\omega_0 t) = \\ &= 2\mu[(y_{n+1} - y_n) - (y_n - y_{n-1})]\cos(\omega_0 t). \end{aligned} \quad (16)$$

Parametric excitation may be replaced by three forces that introduce a periodic bending moment whose amplitude is proportional to the angle between the adjacent beam elements.

The presence of viscous damping, such as a dissipative force, extends the left side of the equation of motion by an additional term which is proportional to velocity

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{F}(t), \quad \mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}, \quad (17)$$

where the matrix of proportionality $\mathbf{C}$ for Rayleigh damping is a linear combination of the mass and stiffness matrices. The relationship to the damping ratio ξ can be seen using the formula $\xi = \pi(\alpha/f + \beta f)$, where f is the frequency in hertz [10], where the constant of proportionality are as follows $\alpha = 0.159$ [Hz] and $\beta = 0.0000411$ [Hz⁻¹].

The vector of forces in the right side of the equations of motion (15) has periodic components of the same frequency and phase. The static component of this vector is missing. The amplitude of the sinusoidal functions is proportional to the coordinate of the appropriate link point.

$$\mathbf{F}_e = K_\delta \Delta L^{-2} \delta(t) [\dots, 0, -1, +2, -1, 0, \dots]^T. \quad (18)$$

Fig. 3 An equivalent arrangement of the force excitation to the parametric excitation

The parametric excitation with the use of the periodic change in the bending stiffness can replace three periodic forces with an amplitude proportional to the instantaneous angle between the pair of the n -th and $(n+1)$ -th beam

elements. These forces act on the mentioned elements in the direction of the y -axis. The model of the beam is supplemented by three feedbacks with a gain which varies periodically over time as is shown in Fig. 3.

V. MATLAB-SIMULINK MODEL

The equation of motion (16) is the second order ordinary differential equation. After the introduction of this substitution $\mathbf{x}_1 = \mathbf{y}$, and $\mathbf{x}_2 = \dot{\mathbf{y}}$, then the second order equation of motion is divided into two ordinary differential equations of the first order

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{x}_2 \\ \dot{\mathbf{x}}_2 &= \mathbf{M}^{-1}\mathbf{F} - \mathbf{M}^{-1}\mathbf{C}\mathbf{x}_2 - \mathbf{M}^{-1}\mathbf{K}\mathbf{x}_1 \end{aligned} \quad (19)$$

where $\mathbf{M}^{-1}\mathbf{K}$, $\mathbf{M}^{-1}\mathbf{C}$ or $\mathbf{M}^{-1}$ are parameters in the form of matrices. An arrangement of the subsystem which models the cantilever beam for an arbitrary number of elements in the Matlab-Simulink environment is shown in Fig. 4. Entering the simulation is complete to the initial conditions $\mathbf{x}_1(0) = \mathbf{y}(0)$ and $\mathbf{x}_2(0) = \dot{\mathbf{y}}(0)$. The blocks of the Gain type contain matrix and their input is a vector, and therefore the output is a vector as well. Simulation of the parametric resonance of the beam which is divided into 5 elements and the first joint of variable bending stiffness K_1 is in the clamping point. It is expected that this method of parametric excitation will have the greatest effect. The Matlab-Simulink model is shown in Fig. 5. Under this assumption, the system has only one feedback. The model of the beam in Fig. 5 is marked as a Subsystem in Fig. 4. The first derivative of the beam element displacements with respect to time is not needed for parametric excitation.

The effect of active vibration control is often demonstrates on the vibration decay of the beam which is bent into a stationary deflected position by acting the force of 10 N and then is suddenly released. The initial conditions are as follows

$$\begin{aligned} \dot{\mathbf{y}}(0) &= \mathbf{0} \\ \mathbf{y}(0) &= \mathbf{y}_0 = -\mathbf{K}^{-1}[0, 0, \dots, 10]^T. \end{aligned} \quad (20)$$

The beam is divided into 5 elements, and therefore has 5 resonant frequencies. The value of these frequencies in Hz and in radians per second are listed in Table 2.

Fig. 4 Matlab-Simulink lumped parameter model of the cantilever beam for arbitrary number of elements

TABLE II. RESONANT FREQUENCIES

Index k	1	2	3	4	5
f_k [Hz]	16.14	103.04	291.9	563.9	839.4
Ω_k [rad/s]	101.4	647.4	1834.1	3543.1	5274.4

Fig. 5 Matlab-Simulink model of the beam for five elements and feedbacks

VI. SIMULATION OF VIBRATION DECAY

The effect of parametric excitation will be evaluated on the base of the decay rate from the initial beam deformation. The decay of vibration at the free end of the cantilever beam with excitation switch-off is shown in Fig. 6. The decay of vibration with excitation switch-off is shown on the upper panel A) in Fig. 6. The lower panel B) shows the decay in dB

$$dB = 20 \times \log \left[\frac{y(5)}{\log y(5)} \right]_{t=0} \quad (20)$$

The time interval of the decrease in the amplitude of instantaneous vibration from the level of 0 dB to -40 dB may be a measure of damping efficiency and can be called as the decay time.

Fig. 6 Free vibration decay of the free end

Firstly, the effect of the Combination Parametric Resonances is analyzed. The effect of the parametric excitation on the decay rate for the frequency of $\omega_{2-1/1}^{Cr} = \Omega_2 - \Omega_1$, i.e. 86.9 Hz, is shown in Fig. 8. The panels

A) and B) have the same meaning as in Fig. 6. Dependence of the decay time for a decrease of -40 dB on the parameter μ is shown in Fig. 7. The most efficient damping is achieved for $\mu \geq 0.08$. The decay time for the excitation frequency $\omega_{3-2/1}^{Cr}$ is almost constant for the interval of $0.08 \leq \mu < 0.2$ as is shown in Fig. 9. The effect of the individual value of the parameter on the frequency spectrum composition is not the same as is shown in the next chapter.

Fig. 7 Decay time vs. parameter μ for $\omega_{2-1/1}^{Cr}$

Fig. 8 Vibration decay of the free end for $\omega_{2-1/1}^{Cr}$

The effect of the parametric excitation on the decay rate for the frequencies $\omega_{3-1/1}^{Cr} = \Omega_3 - \Omega_1$ and $\omega_{3-2/1}^{Cr} = \Omega_3 - \Omega_2$ of the first order of the Combination parametric resonances is shown in Fig. 9. The decay time for the excitation frequency $\omega_{3-2/1}^{Cr}$ decreases up to the value of $\mu = 0.22$ while this decay time increases for $\mu > 0.22$.

Fig. 9 Decay time vs. parameter μ for $\omega_{3-1/1}^{Cr}$ and $\omega_{3-2/2}^{Cr}$.

The effect of the parametric excitation on the decay rate for the Principle Parametric Resonance Frequencies is almost without a positive effect.

VII. SIMULATION OF RESPONSE TO RANDOM FORCE

The vibration decay from the deflected shape of the cantilever beam assesses the efficiency of damping in the time domain. The damping effect can be observed in the frequency domain. It is assumed that a random force with a frequency spectrum that is close to white noise acts at the free end of the beam. The frequency spectrum of the deflected endpoint of the beam is calculated for the identical configuration of the feedback. All the above mentioned spectra are shown in the panels of Fig. 10.

As is evident from frequency response spectra in Fig. 10 the dominant peak of the spectrum is split in two sub-peaks while this change of the frequency spectrum is accompanied by decreasing the magnitudes of the sub-peaks. The parametric excitation frequency is set at the antiresonant frequency $\omega_{2-1/1}^{Cr}$. The gain of the feedback was the largest possible as found experimentally.

Fig. 10 Frequency spectrum of the excitation and response at the beam free end for $\omega_{2-1/1}^{Cr}$

VIII. CONCLUSIONS

The parametric excitation is one of the tools to increase the efficiency of vibration damping. The paper examined the Principle and Combination Parametric Resonance frequencies and their effect on the decay rate for the cantilever beam. It

was found that the greatest effect on vibration damping has the difference frequency between the first and second resonance frequency of the cantilever beam. This frequency difference is a Combination Parametric Resonance frequency of the first order. The optimum size of half the amplitude of excitation for this frequency was also determined. Other parametric resonance frequencies are without effect on the vibration damping. The frequency spectra clearly explain why increases damping of the parametrically damped systems. The dominating peak in the spectrum splits into two adjacent peaks and their magnitude is reduced.

The objective of this paper was to demonstrate that parametric damping reduces vibration. Amplitude changes in stiffness were chosen by the simulation approach.

ACKNOWLEDGMENT

This research has been supported by the Czech Grant Agency project No. P101/12/2520 “Active vibration damping of rotor with the use of parametric excitation of journal bearings” and elaborated in the framework of the project Opportunity for young researchers, reg. no. CZ.1.07/2.3.00/30.0016, supported by Operational Programme Education for Competitiveness and co-financed by the European Social Fund and the state budget of the Czech Republic.

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